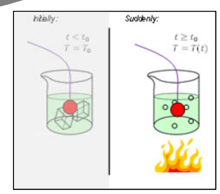
Michigan Tech

CM3215
Fundamentals of Chemical Engineering Laboratory

Unsteady Heat Transfer to a Sphere—
Measuring the heat transfer coefficient (fitting PDE soln)

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University



1

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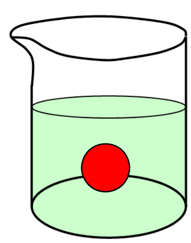
Objective:	Measure the heat-transfer coefficient to a $1.0\ inch$ brass sphere dropped into in a well-stirred beaker of hot water.

Strategy: ?



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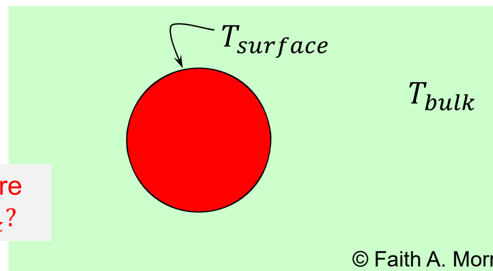
Objective: Measure the heat-transfer coefficient to a 1.0 *inch* brass sphere dropped into in a well-stirred beaker of water.

Strategy: **?**

To determine our experimental strategy, we begin with: ***What is heat transfer coefficient?***

Newton's Law of Cooling:
(a boundary condition)

$$\left|\frac{q_r}{A}\right| = h\left|T_{bulk} - T_{surface}\right|$$



Perhaps: measure $T_{surface}$ and T_{bulk}?

3

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How to experimentally measure heat-transfer coefficient?

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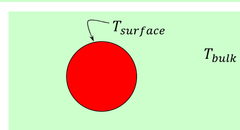

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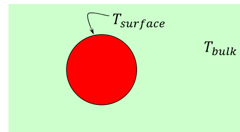

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But they will be equal!

Need *heat transfer* to be taking place.

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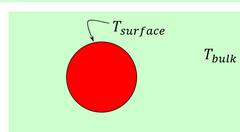

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Unsteady state experiment, then.

It would be awkward to try to measure the surface temperature.

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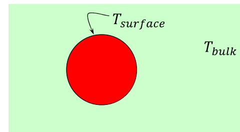

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We can embed a thermocouple in the center perhaps?

How is the temperature at the center related to what we seek to measure, h?

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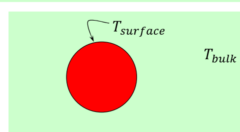

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How is the temperature at the center related to what we seek to measure, h?

We can model the situation and see what the connection is.

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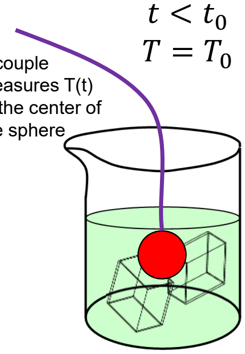
<u>Experiment</u>: Measure T(t) at the center of a sphere:

Initially:

$t < t_0$

$T = T_0$



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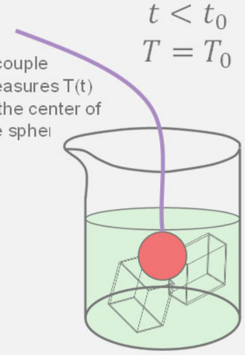
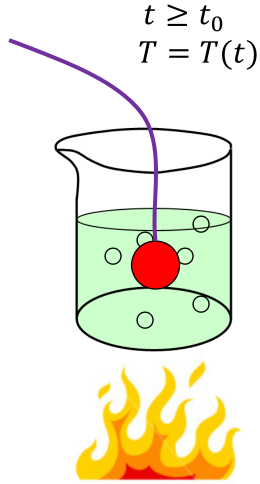


Suddenly:

$t \geq t_0$

$T = T(t)$



Heat transfer takes place.

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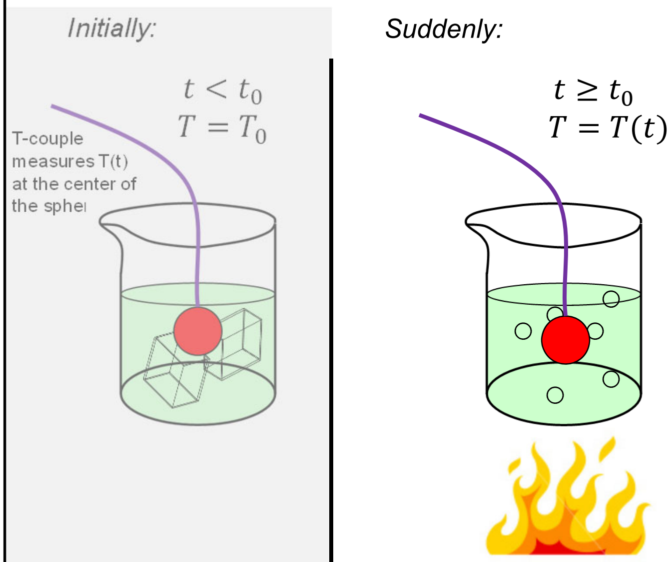
<u>Experiment</u>: Measure T(t) at the center of a sphere:



(already done for you)

<u>Excel:</u>

t(s)	T(°C)
9.50E-02	7.46E+00
2.11E-01	7.44E+00
3.09E-01	7.44E+00
4.09E-01	7.57E+00
5.24E-01	7.46E+00
6.23E-01	7.49E+00
7.39E-01	7.53E+00
8.37E-01	7.46E+00
9.54E-01	7.59E+00
1.05E+00	7.53E+00
1.15E+00	7.58E+00
1.27E+00	7.48E+00
1.37E+00	7.57E+00
…	…

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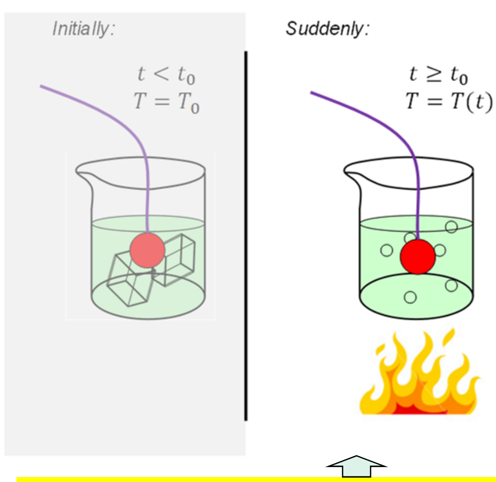

How do we set up the model for this problem?

Where does *h* come into it?

<u>Modeling exercise</u>: How does the temperature at the center of a sphere, subjected to **this history**, change with time?

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Microscopic Energy Balance

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right)=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right)+S$$

www.chem.mtu.edu/~fmorriso/cm310/energy2013.pdf

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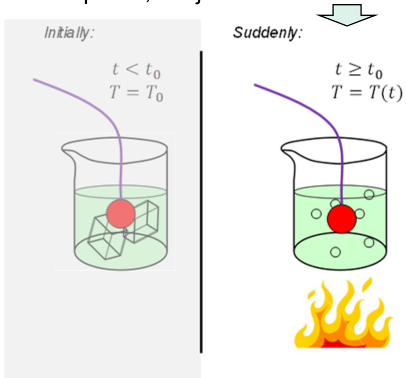
Modeling exercise: How does the temperature at the center of a sphere, subjected to **this history**, change with time?



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You try.

www.chem.mtu.edu/~fmorriso/cm310/energy2013.pdf

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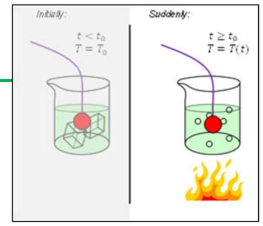
Unsteady State Heat Transfer to a Sphere



Microscopic energy balance in the sphere:

$$\frac{\partial T}{\partial t} = \underbrace{\frac{k}{\rho \hat{C}_p}}_{\equiv \alpha} \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \right)$$

- Unsteady
- Solid ($\underline{v} = 0$)
- θ, ϕ symmetry
- No current, no rxn

Boundary conditions:

$r = R,$ $\quad \frac{q_r}{A} = -k \frac{\partial T}{\partial r} = h(T(r) - T_{bulk})$ $\quad t > 0$

$r = 0,$ $\quad \frac{q_r}{A} = 0$ $\quad \forall t$

Initial condition:

$t = 0,$ $\quad T = T_{initial}$ $\quad \forall r$

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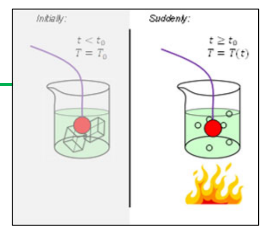
Unsteady State Heat Transfer to a Sphere



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(“∀” means “for all”)

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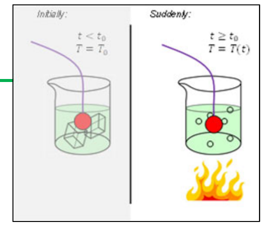

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- Unsteady
- Solid ($\underline{v} = 0$)

Now, Solve

Bo

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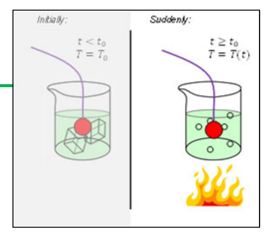
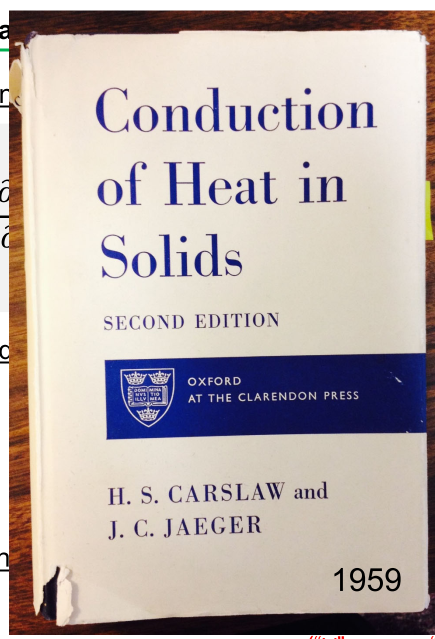
Unsteady Sta



Microscopic en



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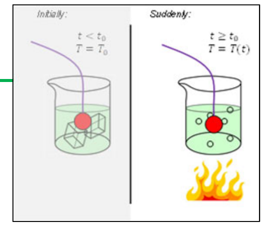

Solution:

$$\xi(r,t) \equiv \frac{T(r,t) - T_{bulk}}{T_{initial} - T_{bulk}}$$

$$\text{Bi} \equiv \frac{hR}{k} \qquad \text{Fo} \equiv \frac{\alpha t}{R^2}$$

(Bi=Biot number; Fo=Fourier number)

$$\xi = \frac{T - T_b}{T_i - T_b} = 2\text{Bi} \sum_{n=1}^{\infty} e^{-Fo(\lambda_n R)^2} \left(\frac{\sin r\lambda_n}{r\lambda_n}\right)\left(\frac{\sin R\lambda_n}{R\lambda_n}\right)\left(\frac{(R\lambda_n)^2 + (\text{Bi}-1)^2}{(R\lambda_n)^2 + \text{Bi}(\text{Bi}-1)}\right)$$

where the ***eigenvalues*** λ_n satisfy this equation:

$$\frac{R\lambda_n}{\tan R\lambda_n} + \text{Bi} - 1 = 0$$

Characteristic Equation

(Carslaw and Yeager, 1959, p237)

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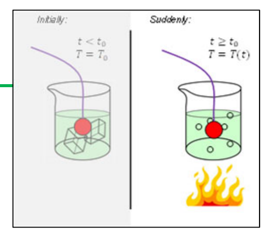
Unsteady State Heat Transfer to a Sphere



Solution:

$$\xi(r,t) \equiv \frac{T(r,t) - T_{bulk}}{T_{initial} - T_{bulk}}$$

Depends on material ($\alpha = k/\rho\hat{C}_p$), and heat transfer processes at surface (h)

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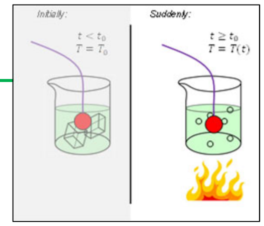

What does this look like?

$$\xi=\frac{T-T_b}{T_i-T_b}=2\text{Bi}\sum_{n=1}^{\infty}e^{-Fo(\lambda_n R)^2}\left(\frac{\sin r\lambda_n}{r\lambda_n}\right)\left(\frac{\sin R\lambda_n}{R\lambda_n}\right)\left(\frac{(R\lambda_n)^2+(\text{Bi}-1)^2}{(R\lambda_n)^2+\text{Bi}(\text{Bi}-1)}\right)$$

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Characteristic Equation

Let's plot it to find out. (Excel)

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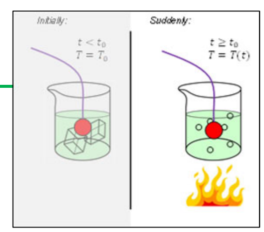
Let's plot it to find out: what are the variables?



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Exponential decay with Fo (scaled time)

$$\xi=\frac{T-T_b}{T_i-T_b}=2\text{Bi}\sum_{n=1}^{\infty}e^{-(Fo)(\lambda_n R)^2}\left(\begin{array}{c}\text{bunch of terms}\\ \text{that vary with Bi and }\lambda_n(\text{Bi})\end{array}\right)$$

$\lambda_n(\text{Bi})$ varies with Bi: $\frac{R\lambda_n}{\tan R\lambda_n}+\text{Bi}-1=0$

Characteristic Equation

If we choose a fixed Bi$=\frac{hR}{k}$, then only Fo $=\frac{\alpha t}{R^2}$ varies

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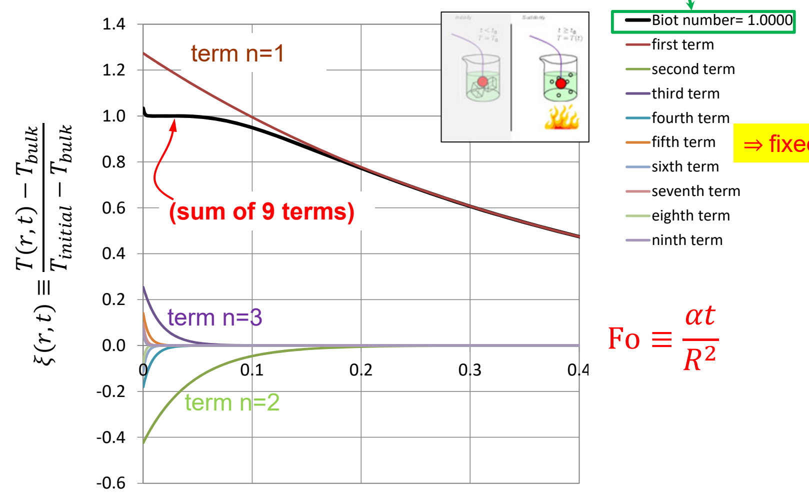
Unsteady State Heat Transfer to a Sphere

$$\xi = \frac{T - T_b}{T_i - T_b} = 2\text{Bi} \sum_{n=1}^{\infty} Term_n$$

If we choose a fixed Bi= $\frac{hR}{k}$, then only $\text{Fo} = \frac{\alpha t}{R^2}$ varies

For a fixed Bi

$$\text{Bi} \equiv \frac{hR}{k}$$



⇒ fixed h, R, k

$$\text{Fo} \equiv \frac{\alpha t}{R^2}$$

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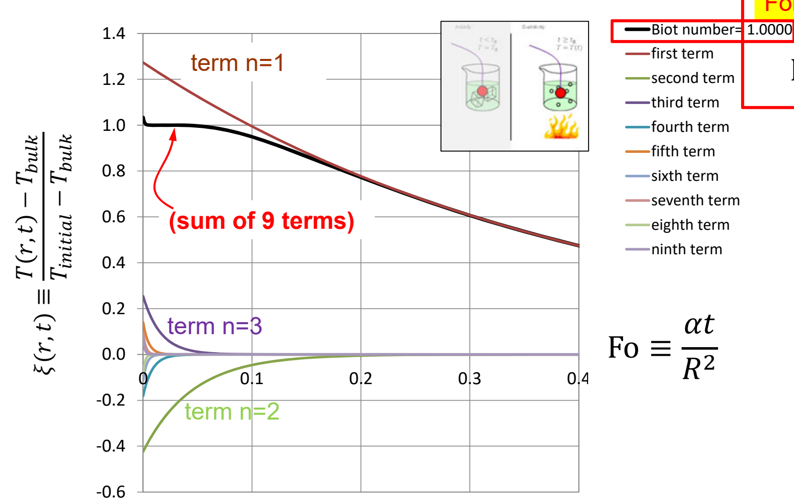
Unsteady State Heat Transfer to a Sphere

If we fix Bi, then only Fo varies:

- It is an infinite sum of terms.
- Each term corresponds to one eigenvalue, λ_n
- The first term $n = 1$ is the dominant one
- The $n > 1$ terms alternate in sign (positive and negative)
- Higher terms are "fixing" the short time behavior

For a fixed Bi

$$\text{Bi} \equiv \frac{hR}{k}$$



$$\text{Fo} \equiv \frac{\alpha t}{R^2}$$

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Unsteady State Heat Transfer to a Sphere

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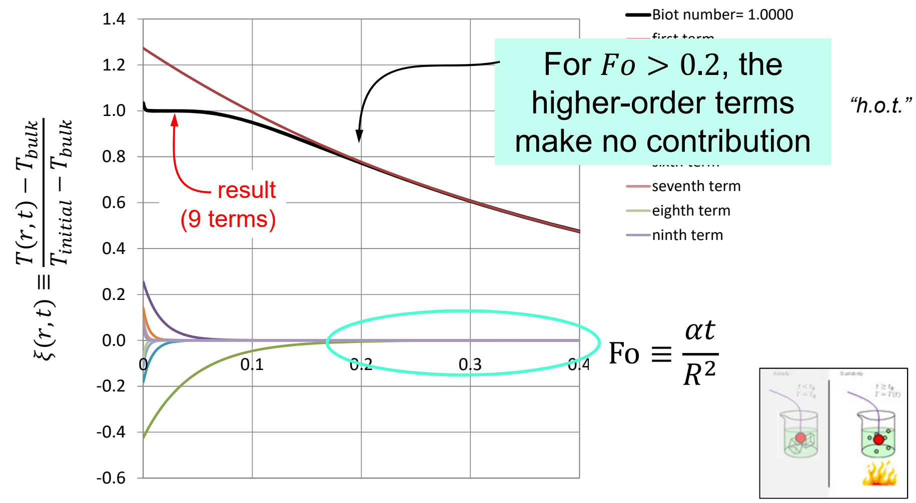

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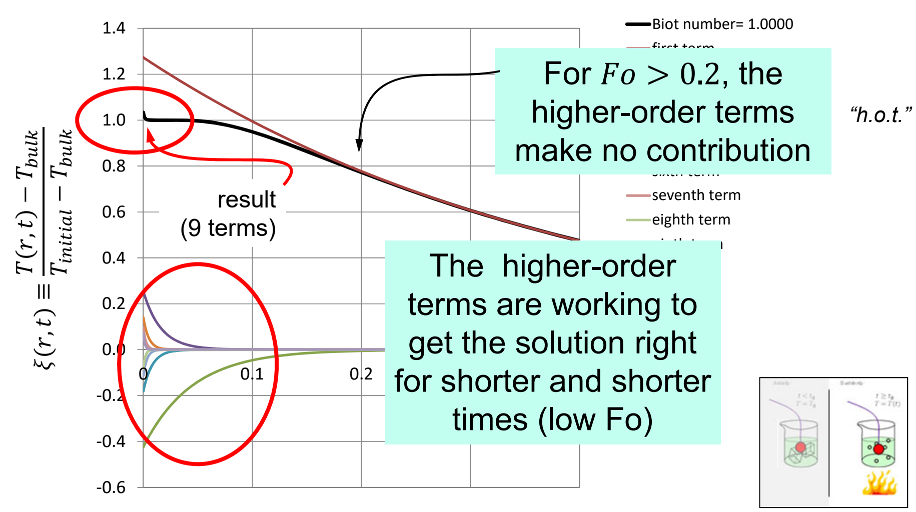

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Unsteady State Heat Transfer to a Sphere

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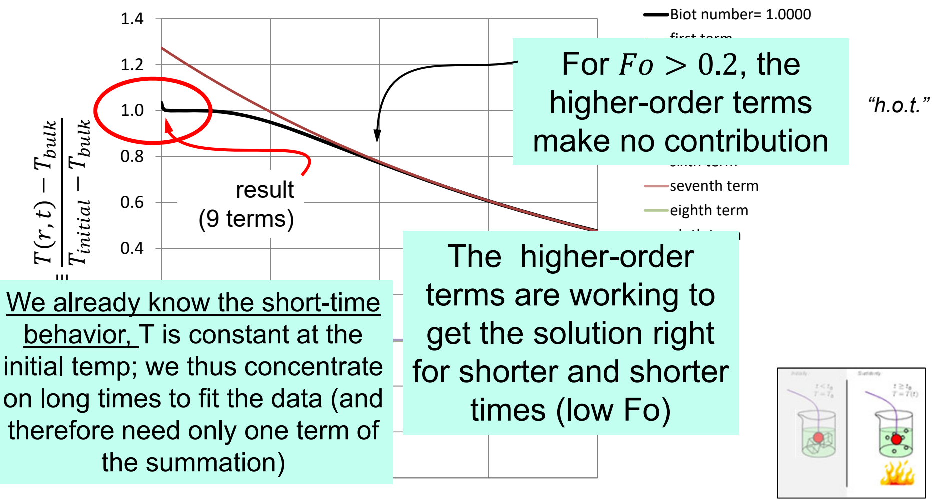

For $Fo > 0.2$, the higher-order terms make no contribution

The higher-order terms are working to get the solution right for shorter and shorter times (low Fo)

We already know the short-time behavior, T is constant at the initial temp; we thus concentrate on long times to fit the data (and therefore need only one term of the summation)

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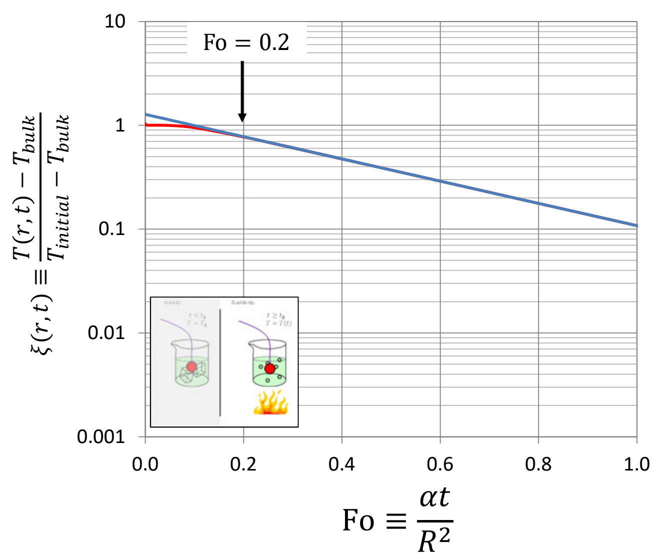
Unsteady State Heat Transfer to a Sphere

If we fix Bi, then only Fo varies:



- Plotted log-linear, the solution is linear for $Fo > 0.2$
- The slope depends on Biot number

$$\text{Bi} \equiv \frac{hR}{k}$$

$$\frac{1}{\text{Bi}} = \frac{k}{hR}$$

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Unsteady State Heat Transfer to a Sphere

If we fix Bi, then only Fo varies:

(summary)

For a fixed Bi: the results are only a function of Fo.

$$\text{Bi} \equiv \frac{hR}{k} \qquad \text{Fo} \equiv \frac{\alpha t}{R^2}$$

- Solution is an infinite sum of terms.
- Each term corresponds to one eigenvalue, λ_n
- The first term $n = 1$ is the dominant one
- The $n > 1$ terms alternate in sign (positive and negative)
- Higher terms are "fixing" the short time behavior
- At fixed Biot number, the time-dependence is an exponential decay (for $\text{Fo} > 0.2$)
- Our Biot number will be fixed; but we not know what it is until we fit our results to the model.

Question: How do various values of Biot number affect the heat transfer that occurs?

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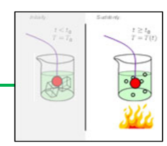
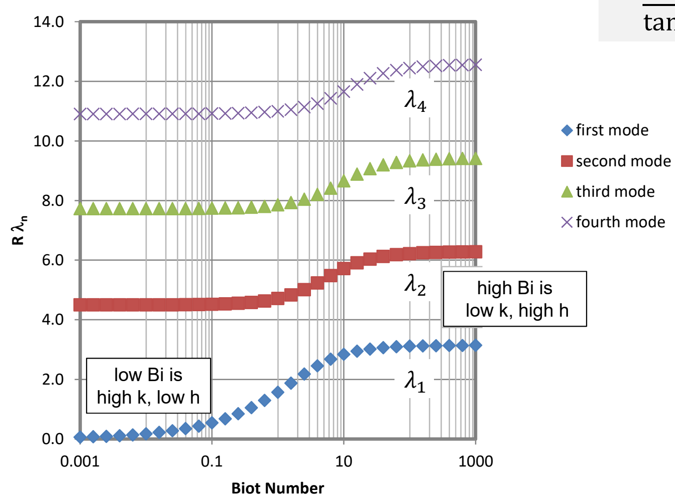
Unsteady State Heat Transfer to a Sphere



The roots of the characteristic equation λ_n vary with Bi

Characteristic Equation:

$$\frac{R\lambda_n}{\tan R\lambda_n} + \text{Bi} - 1 = 0$$



- The λ_n are the roots of the characteristic equation
- They depend on Biot number Bi

$$\text{Bi} \equiv \frac{hR}{k}$$

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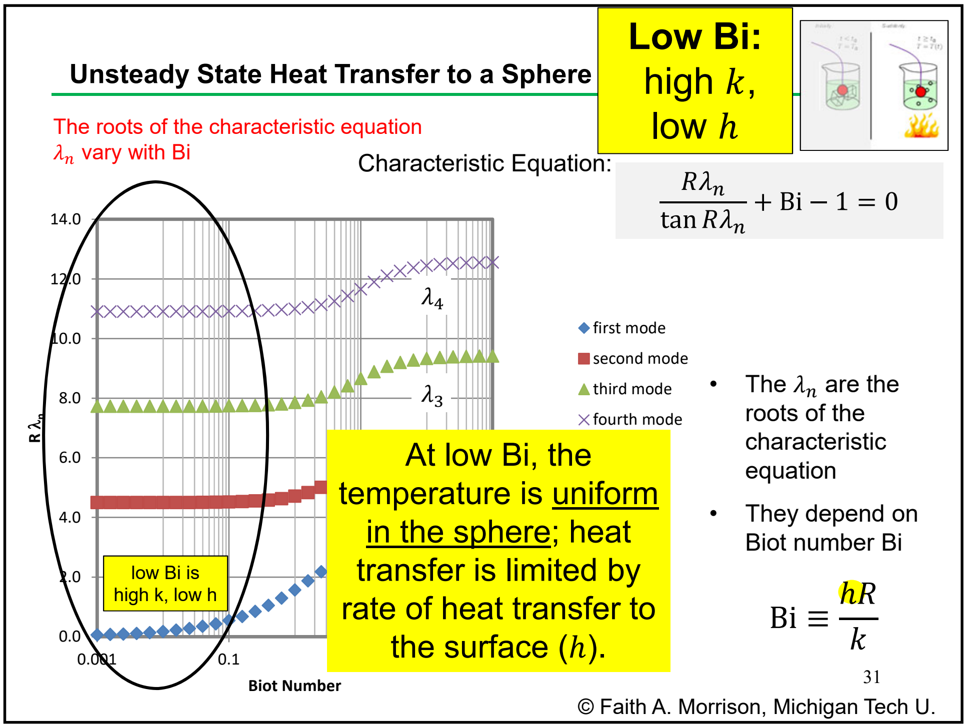
Unsteady State Heat Transfer to a Sphere
Low Bi: high k, low h
The roots of the characteristic equation λ_n vary with Bi
Characteristic Equation:
$$\frac{R\lambda_n}{\tan R\lambda_n} + \text{Bi} - 1 = 0$$
λ_4
λ_3
first mode
second mode
third mode
fourth mode
At low Bi, the temperature is uniform in the sphere; heat transfer is limited by rate of heat transfer to the surface (h).
low Bi is high k, low h
$R\lambda_n$
Biot Number
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0.0
0.001
0.1
The λ_n are the roots of the characteristic equation
They depend on Biot number Bi
$$\text{Bi} \equiv \frac{hR}{k}$$
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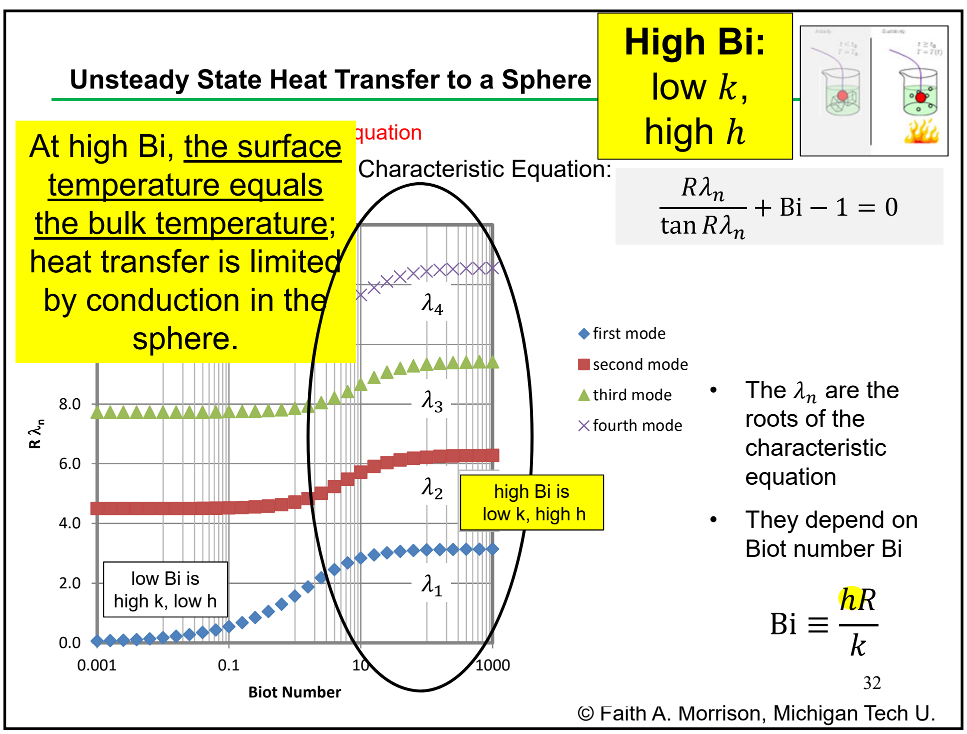
Unsteady State Heat Transfer to a Sphere
High Bi: low k, high h
At high Bi, the surface temperature equals the bulk temperature; heat transfer is limited by conduction in the sphere.
quation
Characteristic Equation:
$$\frac{R\lambda_n}{\tan R\lambda_n} + \text{Bi} - 1 = 0$$
λ_4
λ_3
λ_2
λ_1
high Bi is low k, high h
low Bi is high k, low h
first mode
second mode
third mode
fourth mode
$R\lambda_n$
Biot Number
8.0
6.0
4.0
2.0
0.0
0.001
0.1
10
1000
The λ_n are the roots of the characteristic equation
They depend on Biot number Bi
$$\text{Bi} \equiv \frac{hR}{k}$$
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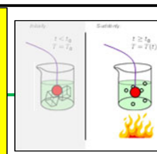
At moderate Bi, heat transfer is affected by <u>both</u> conduction in the sphere and the rate of heat transfer to the surface.

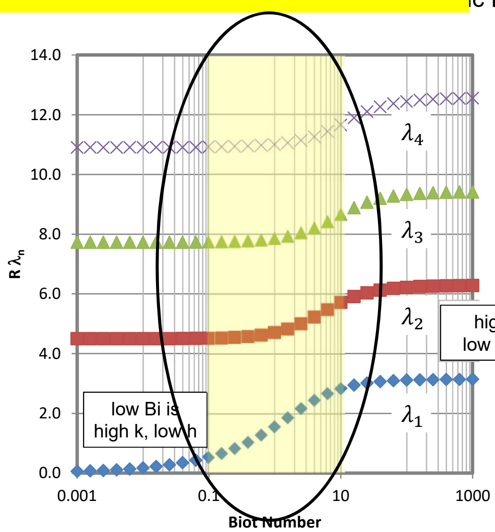
Moderate Bi: nether process dominates



...n Sp...

...ic Eq...

$$\frac{\quad}{\tan R\lambda_n} + \mathrm{Bi} - 1 = 0$$



- The λ_n are the roots of the characteristic equation
- They depend on Biot number Bi

$$\mathrm{Bi} \equiv \frac{hR}{k}$$

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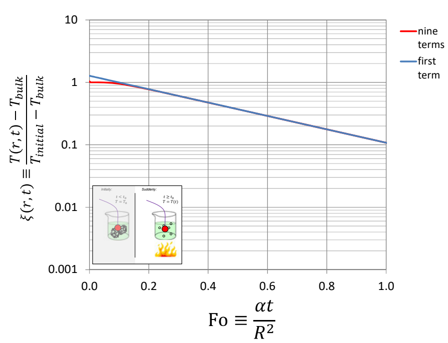
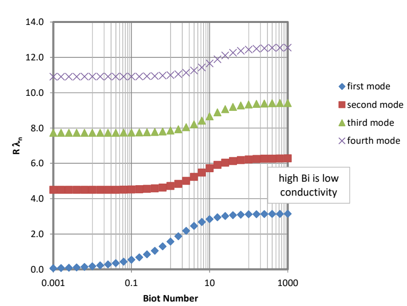
The key to analyzing our data is <u>determining the Biot number for the experiment</u>.

We determine the Biot number by fitting the model predictions to our data.

The value of Biot number that results in the closest fit between model and data tells us our value of h.

$$\mathrm{Bi} \equiv \frac{hR}{k}$$





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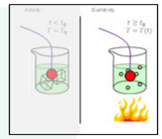
Summary:

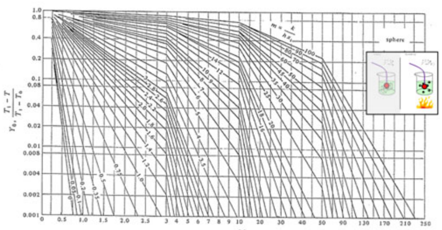
- The key to analyzing our data is determining the Biot number for the data.
- We determine the Biot number by fitting the model predictions to our data.
- The value of Biot number that results in the closest fit between model and data tells us our value of h.



In the literature, there are handy plots of the solutions to Unsteady State Heat Transfer to a Sphere (and other shapes)

Heisler charts

(Geankoplis; see also Wikipedia)



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From Geankpolis, 4th edition, page 374

Literature solutions to Unsteady State Heat Transfer to a Sphere

Heisler charts (Geankoplis; see also Wikipedia)

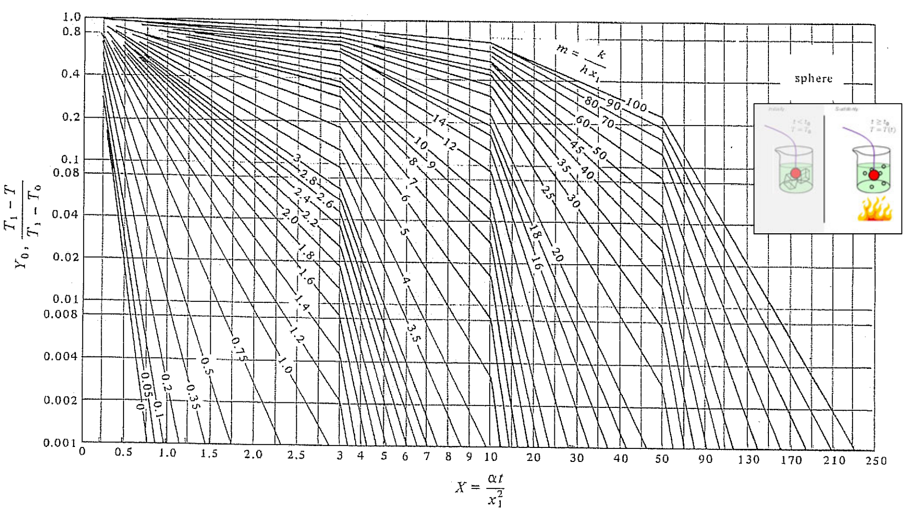

FIGURE 5.3-10. *Chart for determining the temperature at the center of a sphere for unsteady-state heat conduction. [From H. P. Heisler, Trans. A.S.M.E., 69, 227 (1947). With permission.]*

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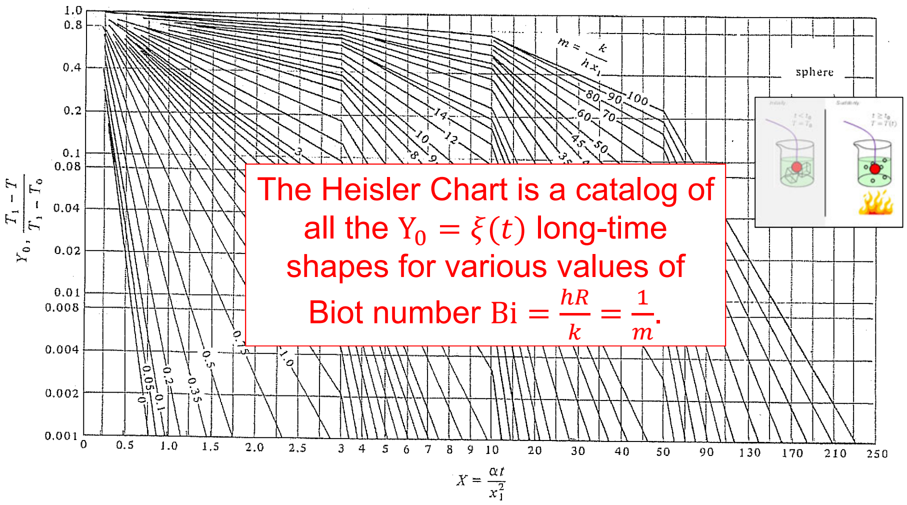
From Geankpolis, 4th edition, page 374; see also WRF, Figure F.6, p717

Literature solutions to Unsteady State Heat Transfer to a Sphere

Heisler charts (Geankoplis; see also Wikipedia)



The Heisler Chart is a catalog of all the $Y_0 = \xi(t)$ long-time shapes for various values of Biot number $\mathrm{Bi} = \frac{hR}{k} = \frac{1}{m}$.

FIGURE 5.3-10. *Chart for determining the temperature at the center of a sphere for unsteady-state heat conduction. [From H. P. Heisler, Trans. A.S.M.E., 69, 227 (1947). With permission.]*

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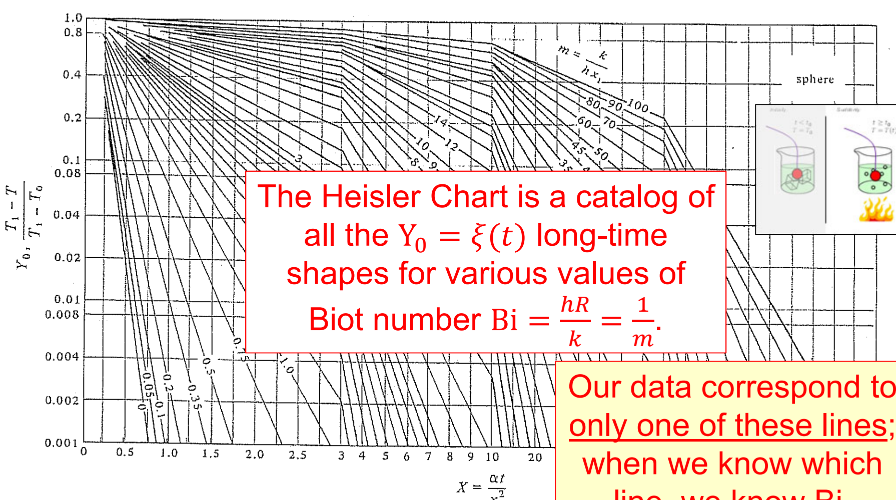
From Geankpolis, 4th edition, page 374; see also WRF, Figure F.6, p717

Literature solutions to Unsteady State Heat Transfer to a Sphere

Heisler charts (Geankoplis; see also Wikipedia)



The Heisler Chart is a catalog of all the $Y_0 = \xi(t)$ long-time shapes for various values of Biot number $\mathrm{Bi} = \frac{hR}{k} = \frac{1}{m}$.

Our data correspond to only one of these lines; when we know which line, we know Bi.

FIGURE 5.3-10. *Chart for determining the temperature at the center of a sphere for unsteady-state heat conduction. [From H. P. Heisler, Trans. A.S.M.E., 69, 227 (1947). With permission.]*

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From Geankpolis, 4th edition, page 374; see also WRF, Figure F.6, p717

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Literature solutions to Unsteady State Heat Transfer to a Sphere

Heisler charts (Geankoplis; see also Wikipedia)

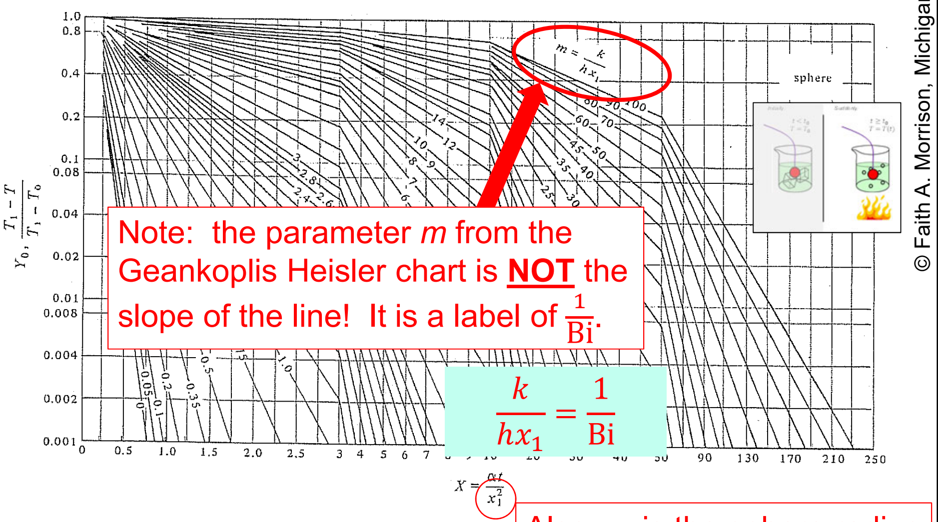

FIGURE 5.3-10. *Chart for determining the temperature at the ... heat conduction. [From H. P. Heisler, Trans. A.S.M.E., 69, 227 (1947). With permission.]*

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From Geankpolis, 4th edition, page 374; see also WRF, Figure F.6, p717

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Heisler charts (Geankoplis; see also Wikipedia)

Note also: think of it as 4 separate graphs

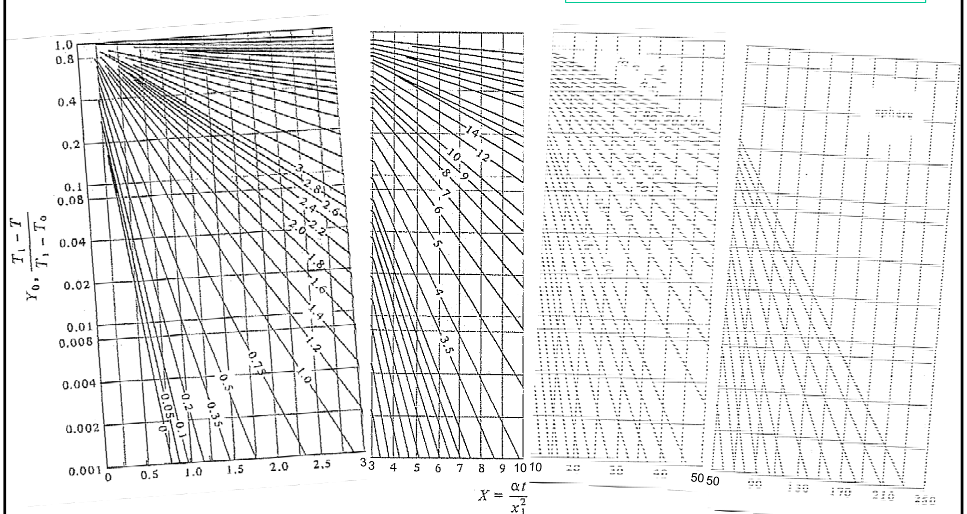

FIGURE 5.3-10. *Chart for determining the temperature at the center of a sphere for unsteady-state heat conduction. [From H. P. Heisler, Trans. A.S.M.E., 69, 227 (1947). With permission.]*

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From Geankpolis, 4th edition, page 374

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CM3215 Fundamentals of Chemical Engineering Laboratory

Assignment 8: Unsteady Heat Transfer to a Sphere

(team assignment)

•Using the time-dependent temperature versus time data measured by colleagues (supplied to you), calculate the heat transfer coefficient for the laboratory experiment (sphere plunged into a beaker of hot water).

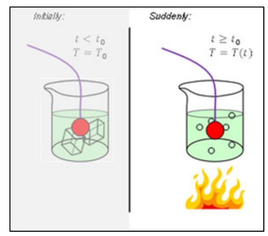
•Report the Biot number.

•Report the heat transfer coefficient, h.

•Indicate which physics is dominating the dynamics: heat transfer to the sphere (h), heat transfer within the sphere (k), or neither dominate.

•Describe your method for obtaining Bi and h (briefly; you may give steps as a list in your memo; it's not a report).

•Conduct and report an uncertainty analysis to determine error limits on h.



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Which physics dominates?

- At low Bi, the temperature is uniform in the sphere; heat transfer is limited by rate of heat transfer to the surface (h).
- At high Bi, the surface temperature equals the bulk temperature; heat transfer is limited by conduction in the sphere (k).
- At moderate Bi, heat transfer is affected by both conduction in the sphere and the rate of heat transfer to the surface (both h and k influence the heat transfer).
- The key to analyzing our data is determining the Biot number for the experiment.

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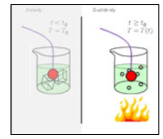

Pre-lab Assignment (due Tuesday in lab):

- In your lab notebook, outline how you plan to proceed; show the TA.
- Obtain $\alpha = k/\rho\hat{C}_p$ and k for brass ahead of time and record both in notebook (with reference)
- Submit your data to the TA for the archive:
  - ✓ DP meter calibration
  - ✓ Orifice meter calibration (two different units)
  - ✓ Rotameter calibration
  - ✓ Lossy pump characteristic curve

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