CM3215

Fundamentals of Chemical Engineering Laboratory

Statistics Reading Error

Extra: Sensing Limit

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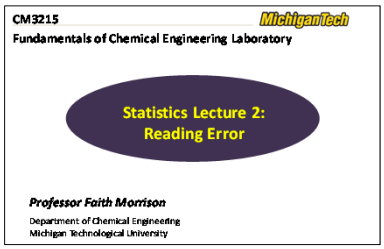
1. Quick start—Replicate error
2. **Reading Error – Extra!**
3. Calibration Error
4. Error Propagation
5. Least Squares Curve Fitting

Let's revisit:



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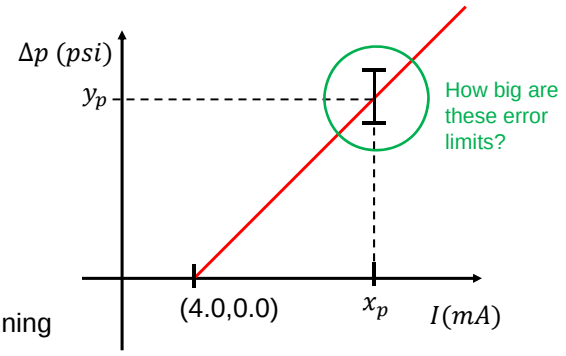
An Error question:

$$(\Delta p; psi) = (m)(I; mA) + b$$

What's the error on a value of Δp obtained from the calibration curve?



This is a case of determining a value of y_p from a fit

$$y_p = mx_p + b \pm ?$$

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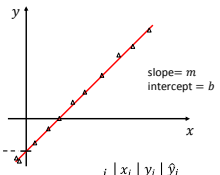
What's the error on a value of Δp obtained from the calibration curve?

From *Error Analysis Lecture 5* on LINEST:

Ordinary, *Least Squares*, Linear Regression

What are the error limits on a value of y obtained from the equation $y = \hat{m}x + \hat{b}$?



i	x_i	y_i	$\hat{y}_i$
1	x_1	y_1	$\hat{y}_1$
2	x_2	y_2	$\hat{y}_2$
⋮	⋮	⋮	⋮
n	x_n	y_n	$\hat{y}_n$

Answer:

at x_p, $y_p = (\hat{m}x_p + \hat{b}) \pm 2s_{y_p}$

$$s_{y_p}^2 = s_{y,x}^2\left(\frac{1}{n} + \frac{(x_p - \bar{x})^2}{SS_{xx}}\right)$$

(This is the final result of the algebra indicated on previous slide; see Appendix B of the handout.)

for $n - 2 \leq 6$, replace "2" with $t_{0.025,n-2}$

Use this for error limits on values obtained from the fit.

In Excel:
- $s_{y,x}$ = STEYX(y-range,x-range)
- SS_{xx} = DEVSQ(x-range)
- $\bar{x}$ = AVERAGE(x-range)

The error limits on a Δp obtained from the best-fit line:

$$\Delta p = \Delta p_{\substack{predicted\\ from\\ calibration\\ curve}} \pm 2s_{y_p}$$

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What's the error on a value of Δp obtained from the calibration curve?

From *Error Analysis Lecture 5* on LINEST:

Ordinary, *Least Squares*, Linear Regression

Confidence interval for values from the fit:

$$y_p = (\hat{m}x_p + \hat{b}) \pm 2s_{y_p}$$

$$s_{y_p}^2 = s_{y,x}^2\left(\frac{1}{n} + \frac{(x_p - \bar{x})^2}{SS_{xx}}\right)$$

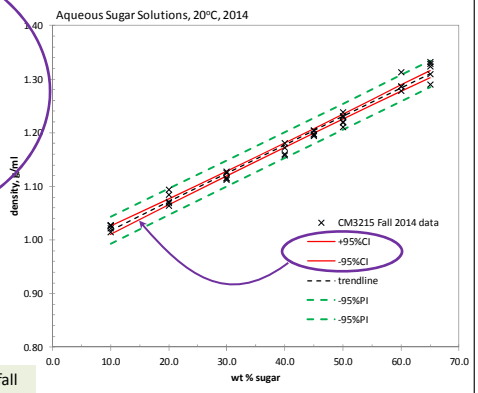
(for large n, the values of y at each x are well predicted (CI is narrow))

Prediction interval of data:

$$y_{\tilde{p}} = (\hat{m}x_p + \hat{b}) \pm 2s_{y_{\tilde{p}}}$$

$$s_{y_{\tilde{p}}}^2 = s_{y,x}^2\left(1 + \frac{1}{n} + \frac{(x_p - \bar{x})^2}{SS_{xx}}\right)$$

(Notice that ≈ 95% of the data points fall within the PI; that's what it means to be a PI. The next data point likely will fall here too.)



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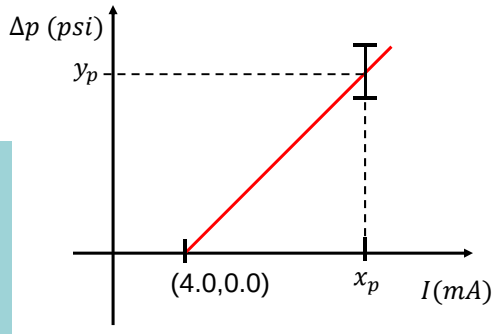
Another Error-Related question:

$$(\Delta p; psi) = (m)(I; mA) + b$$

What's the **lowest** accurate Δp the instrument is capable of measuring?



Can we measure
$\Delta p = p_1 - p_2 = 10psi?$
$1psi?$
$0.1psi?$
$0.01psi?$
$0.001psi?$
$0.0001psi?$

When will the **value** be sufficiently larger than the **noise** (error)?

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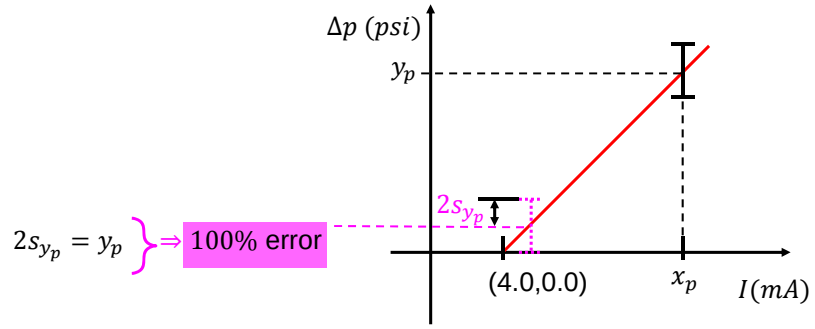
What's the lowest accurate Δp the instrument is capable of measuring?

What does it mean when the 95%CI on the fit "bottoms out"?

$$(\Delta p; psi) = (m)(I; mA) + b$$

$$\Delta p = \Delta p_{\substack{predicted \\ from \\ calibration \\ curve}} \pm 2s_{y_p}$$



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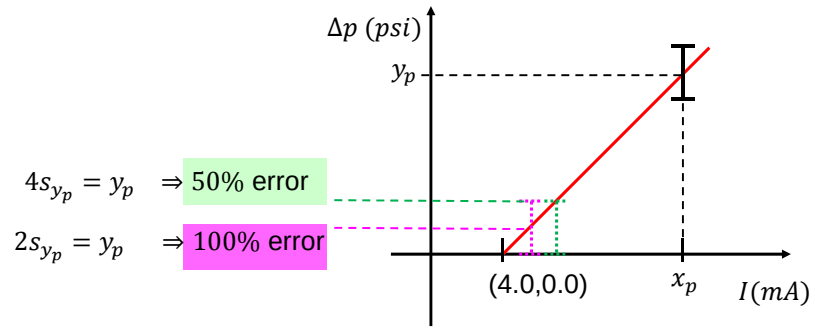
What's the lowest accurate Δp the instrument is capable of measuring?

$$(\Delta p; psi) = (m)(I; mA) + b$$

$$\Delta p = \Delta p_{\substack{predicted \\ from \\ calibration \\ curve}} \pm 2s_{y_p}$$



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What's the lowest accurate Δp the instrument is capable of measuring?

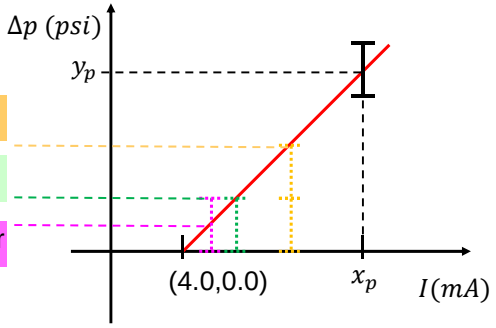
$$\Delta p = \Delta p_{\substack{predicted\\ from\\ calibration\\ curve}} \pm 2s_{y_p}$$

$$(\Delta p; psi) = (m)(I; mA) + b$$

$8s_{y_p} = y_p \Rightarrow$ 25% error

$4s_{y_p} = y_p \Rightarrow$ 50% error

$2s_{y_p} = y_p \Rightarrow$ 100% error



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What's the lowest accurate Δp the instrument is capable of measuring?

$$\Delta p = \Delta p_{\substack{predicted\\ from\\ calibration\\ curve}} \pm 2s_{y_p}$$

$$(\Delta p; psi) = (m)(I; mA) + b$$

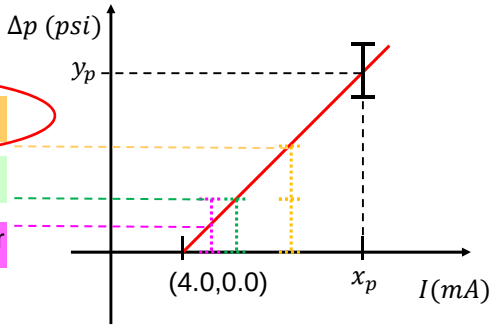
The choice is up to you.

Recommended rule of thumb

$8s_{y_p} = y_p \Rightarrow$ 25% error

$4s_{y_p} = y_p \Rightarrow$ 50% error

$2s_{y_p} = y_p \Rightarrow$ 100% error



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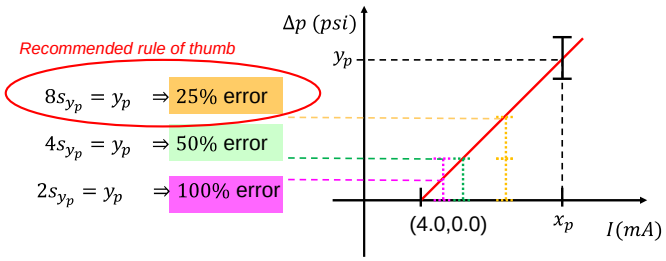
What's the lowest accurate Δp the instrument is capable of measuring?



Summary

- There is a **sensing limit** due to the "bottoming out" of the error limits
- The lowest number one can accurately report depends on one's tolerance for uncertainty
- 25% max relative error is a plausible rule of thumb ⇒ $\Delta p_{min} \approx 8s_{y_p}$

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