Homework 1

CM4650 Polymer Rheology

Spring 2018 *typos corrected 1/23/2018*

Due *Wednesday 24 January 2018, in class*

Note: The typos in the Understanding Rheology textbook are listed at this web page: *www.chem.mtu.edu/~fmorriso/cm4650/URerrata.html*

Please do not write on the back side of any page of your solution (to reduce paper waste, you may submit your homework on the back side of re-used paper, if you wish). Please write legibly and large. Thank you.

You may find this page helpful in this homework:
http://pages.mtu.edu/~fmorriso/cm4650/formula_sheet_for_exam1_2018.pdf

1. (20 points) Using Einstein notation, work out the following quantities (show your work). For your final answers, carry out all summations.
   a. $\underline{v} \cdot \underline{u}$
   b. $\underline{m} \cdot \underline{\underline{B}}$
   c. $\underline{s} + \underline{b}$
   d. $\underline{c} \cdot \underline{\underline{B}}^T$
   e. $\underline{\underline{\tau}} \cdot \underline{\underline{B}}$ (you may leave summation signs in this answer)

2. (10 points) What is the 23-component of this tensor (Use Einstein notation to work it out, then carry out all summations for the 23-component):
$$\underline{\underline{C}}^T \cdot \underline{\underline{B}}$$

3. (10 points) For the vectors given below, what are the following quantities equal to? Show your work in Einstein notation before substituting the numbers from below.
   a. $\underline{a} \cdot \underline{b}$ (inner product)
   b. $\underline{a}\,\underline{b}$ (indeterminate vector product)
$$\underline{a} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}_{xyz} \quad \underline{b} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}_{xyz}$$

4. (10 points) For $\underline{v} = \begin{pmatrix} 0 \\ U_0 \\ 0 \end{pmatrix}_{r\theta z}$, which is written in the cylindrical coordinate system, what is $\underline{v}$ in Cartesian coordinates? Note that U_0 is a constant. Your final answer should not contain the r, θ, z cylindrical coordinate variables or basis vectors but may contain the x, y, z Cartesian coordinate variables and must be written with respect to Cartesian basis vectors e_x, e_y, e_z. What is $\underline{v}$ at position $x = 1, y = 2, z = 1$? What is $\underline{v}$ at position $x = 0, y = 0, z = 1$? Comment on your answer.

Homework Rules

1. Homework must be legible or it will be returned to you to be rewritten. If a second homework is found to be illegible it will also be returned, but it will not be accepted for regrading.
2. You may consult with your classmates and friends for help on the homeworks, but you may not copy their solutions. You must hand in your own work. You may not show one another your work. Showing one another your work is not discussion/consultation.
3. If you find a solution to the homework in a text or in some other outside source, you may submit it, but you must:
   - understand the solution, and
   - cite the source, book, page, edition, year, etc. This includes all scoop.
4. No late homeworks will be accepted without prior consultation with me. I will grant extensions at my discretion.
5. All seven homework assignments are <u>weighted equally</u> except when announced.
6. Please only write on one side of the page for your homework. To minimize paper waste, you may use the back of scrap paper for your assignments. There is usually a big stack of high quality scrap paper available in the computer rooms.
7. Cheating on homeworks is a serious violation of University policy. Please see the student handbook for the procedures that govern incidents of academic fraud such as cheating on a homework assignment or on an exam. Punishment for cheating, which includes sharing of homeworks, ranges from receiving a zero grade for the assignment, an F for the course, and can even result in your expulsion.

5. (10 points) What are the magnitudes of the tensors below? The quantities A, ϕ_0, and β_0 are constants.

   a. $\underline{\underline{M}} = \begin{pmatrix} 1 & 0.5 & 1 \\ 0.5 & 2 & 1 \\ 1 & 1 & -1 \end{pmatrix}_{xyz}$ (answer is a number)

   b. $\underline{\underline{B}} = A\beta_0 \hat{e}_1\hat{e}_1 + \cos\phi_0\, \beta_0 x_1 x_2 \hat{e}_1\hat{e}_2 + 2\beta_0 x_2 \hat{e}_2\hat{e}_1 + A\hat{e}_2\hat{e}_2$ (answer is an equation)

6. (10 points) Text #2.47 (p58) Give your answer in terms of $\hat{n}, \underline{v}, \rho$, and A. Justify your answer.

7. (10 points) For the flows described in problems 3.16 and 3.18 (page 101) write the velocity and pressure boundary conditions in terms of the coordinate variables of the problem.