Homework 2
CM4650
Spring 2018

Part A: Problems 1-4 are Due *Wednesday 31 January 2018, in class*
Part B: Problems 5-7 are Due *Wednesday 7 February 2018, in class*

Please do not write on the back side of any page of your solution. Please write legibly and large.

You may find this page helpful in this homework:
http://pages.mtu.edu/~fmorriso/cm4650/formula_sheet_for_exam1_2018.pdf

Please make note of the discussion about the stress sign convention and the difference between the two symbols $\underline{\underline{\Pi}} = \underline{\underline{\tau}} + p\underline{\underline{I}}$ (*Understanding Rheology*) and $\underline{\underline{\tilde{\Pi}}} = \underline{\underline{\tilde{\tau}}} - p\underline{\underline{I}}$ (*An Introduction to Fluid Mechanics*).

Note that the *Understanding Rheology* textbook has some typos:
www.chem.mtu.edu/~fmorriso/cm4650/URerrata.html

Part A

1. (20 points) For the vectors given below, what are the following quantities equal to? Show your work in Einstein notation before substituting the specific vectors $\underline{a}$ and $\underline{b}$ from below.
    a. the gradient of $\underline{a}$, $\nabla \underline{a}$
    b. the divergence of $\underline{b}$, $\nabla \cdot \underline{b}$
    c. the Laplacian of $\underline{b}$, $\nabla \cdot \nabla \underline{b} = \nabla^2 \underline{b}$
    d. $\underline{a} - \underline{b}$

$$\underline{a} = \begin{pmatrix} 3x^2y^3 \\ 11x^4 - 2z \\ 2y^2 + 6x \end{pmatrix}_{xyz}$$

$$\underline{b} = (8z - 4yz)\hat{e}_x + 4x^3z\hat{e}_y - 8y\hat{e}_z$$

2. (10 points) What is the correct way to write the quantity $\alpha v_i A_{ic} \hat{e}_c$ (currently written in Einstein notation) when we write it in Gibbs (vector-tensor) notation? What is $\frac{\partial v_m}{\partial x_j}\hat{e}_j\hat{e}_m$ in Gibbs notation?

3. (10 points) The flow rate through a finite surface S can be written as:

$$Q = \iint_S [\hat{n} \cdot \underline{v}]_{surface}\, dS$$

where $\hat{n}$ is the unit normal to the surface dS and $\underline{v}$ is the velocity at infinitesimal surface dS. The solution for the velocity field in Poiseuille flow in a slit is given in the text (see example 3.5.2). Starting with the equation above, calculate the flow rate through the slit cross-section. Show your work. The answer is in equation 3.181.

4. (10 points) The fluid force $\underline{F}$ on a finite surface S can be written as (stress convention of our book):

$$\underline{F} = \iint_S \left[\hat{n} \cdot \left(-\underline{\underline{\Pi}}\right)\right]_{surface} dS$$

where $\hat{n}$ is the unit normal to the infinitesimal surface dS and $\underline{\underline{\Pi}}$ is the total stress tensor with the sign convention of our text. The solution for the velocity field in Poiseuille flow in a slit is given in the text (see example 3.5.2). Starting with the equation above, calculate the total vector force (three components) on the upper wall. Show your work.

Part B

5. (10 points) Sketch (by hand is all I require; you can use a computer if you want) the following vector velocity field (x, y, z in millimeters):

$$\underline{v}\left(\frac{mm}{s}\right) = \begin{pmatrix} -4x \\ -4y \\ 8z \end{pmatrix}_{xyz}$$

You may confine yourself to the first quadrant (x, y, z all positive) and the plane where $x = 0$.
Hint: You will need to choose some points, calculate $v = |\underline{v}|$ and the direction of $\underline{v}$ at those points, and then draw arrows of the appropriate lengths and directions at the points. Usually we center the vector at the points chosen.

6. (20 points) Text 3.16 (Combined pressure driven and drag flow in a slit). Do not use tables for the momentum balance; use Einstein notation as in the text, begin from Gibbs notation (do not use tables).
7. (20 points) Text 3.18 (Tangential annular flow; you may use tables, http://pages.mtu.edu/~fmorriso/cm4650/Operations_with_Del_cyl_sph.pdf)