Homework 3
CM4650
Spring 2018

Due: *Wednesday 28 February 2018, in class*

Please do not write on the backside of the pages. Please write legibly and large. Thank you.

1. (10 points) Text 2.20: Using Einstein notation, show that $\underline{\underline{A}} + \underline{\underline{A}}^T$ is symmetric and that $\underline{\underline{A}} - \underline{\underline{A}}^T$ is antisymmetric.

2. (10 points) Text 2.23: Calculate the magnitude of a given tensor (see text)

3. (10 points) Tensors (more precisely, second-order tensors) have three invariants, which are scalars that are independent of coordinate system. One set of three invariants, I, II, III, is defined in Chapter 2; another set of invariants I_1, I_2, I_3 is defined in Appendix B (page 453); the two sets are interrelated in equations C.81-C.83 (p 476). For the tensors given below, what are the values of the invariants? Calculate both sets from the definitions and verify that the interrelating equations on page 476 hold.

$$\underline{\underline{A}} = \begin{pmatrix} 3 & 10.2 & 0 \\ 10.2 & 0 & 0 \\ 0 & 0 & -3 \end{pmatrix}_{123}$$

$$\underline{\underline{B}} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -8 \end{pmatrix}_{123}$$

4. (20 points) What does the Fake-O® model predict for start-up of shear ($M_0 = 18{,}000\ poise; \dot{\gamma}_c = 0.67\ s^{-1}; n = .24, m = 12{,}000\ appropriate\ units$)? Give answers for all three shear start-up material functions. Plot your results (Excel, Matlab, other) as a function of time ($-10s \le t \le 10s$) for the following shear rates: $\dot{\gamma}_0 = 0.1, 0.2, 0.5, 1.0, 2.0, 5.0, 10s^{-1}$.

5. (20 points) In the lecture notes when we were evaluating the Fake-O® model, we found some shortcomings (did not predict nonzero normal stresses in shear flow) and we said we "need to try something else." On this list of "something else" was this idea:

$$\underline{\underline{\tau}} = A[\nabla \underline{v} \cdot (\nabla \underline{v})^T] + B\nabla \underline{v} + C(\nabla \underline{v})^T$$

Does this proposed model predict a symmetric stress tensor? Under what conditions? Discuss this situation.

6. (10 points) Text 5.1 What is a constitutive equation? What is a rheological material function? What is the difference and how are these two concepts/definitions relate? Please try to put the differences in your own words.

7. (20 points) Text 5.7 What is the elongational stress growth function $\bar{\eta}^+(t)$ predicted by the constitutive equation for an incompressible Newtonian fluid? Derive your answer from the starting definitions.