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Name:______________________________.

Exam 1

CM 4650 Polymer Rheology
13 February 2018

- ✓ Please be neat.
- ✓ Please write on only one side of each piece of paper in your solution.
- ✓ This exam is closed book, closed notes.
- ✓ No internet-capable devices are permitted.
- ✓ Submit only your own work.
- ✓ A calculator is permitted.

1. (20 points) Use Einstein notation to write this expression in terms of Cartesian components: $\underline{v} \cdot \nabla(\underline{u})$. Note that both $\underline{u}$ and $\underline{v}$ are variables (depend on position and time). If present in your final answer, expand any derivatives of products. Write your final answer in matrix notation with no summation signs (include neither implicit nor explicit summation signs).

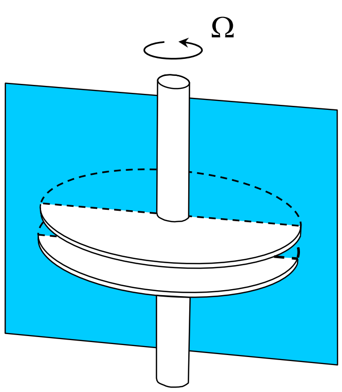
2. (20 points) For the following expressions written in Einstein notation, what is the equivalent expression in Gibbs notation (Gibbs notation is in terms of $\nabla, (\cdot), \underline{\underline{A}}, (:), \times, \underline{v}, \frac{\partial}{\partial t}, etc.$, i.e. anything you see in the Navier-Stokes)? All quantities are variables. Verify your answer with Einstein notation.

    a. $\frac{\partial A_{pk}}{\partial x_k} \hat{e}_p$
    b. $\frac{\partial h_m}{\partial t} \hat{e}_m$

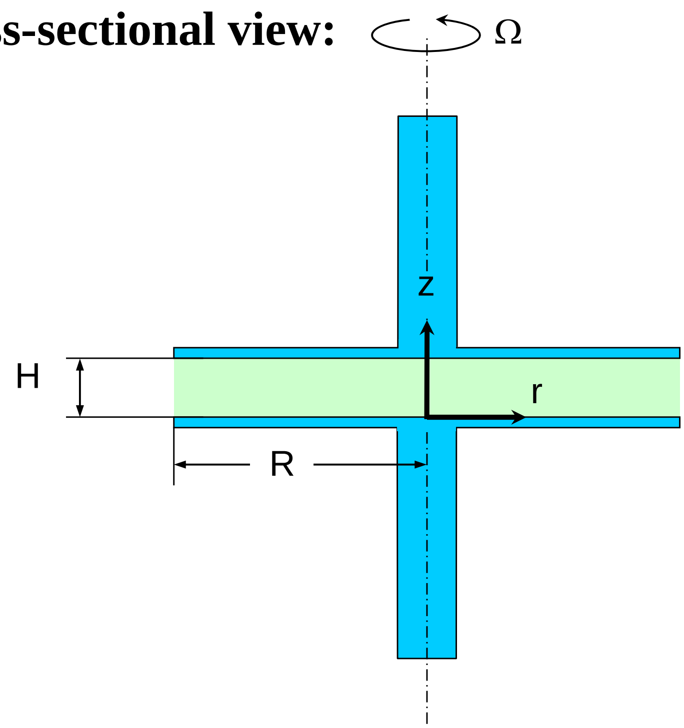
3. (20 points) For the flow described and depicted below, which is analyzed in cylindrical coordinates, what are the velocity boundary conditions for the z-direction? Please be specific and mathematical.

The torsional parallel-plate rheometer shown below measures rheological properties by subjecting a disk-shaped sample of fluid to a rotational flow (torsional flow; see figure). The fluid velocity field in the gap between the two circular plates is a function of both z and r. A motor rotates the top plate at a steady angular speed Ω (rad/s); the bottom plate is stationary. The fluid in the gap is a Newtonian, incompressible fluid of viscosity μ and density ρ.

3 dimensional view:



Cross-sectional view:



4. (20 points) In rheology as in fluid mechanics, we are often interested in quantities like the volumetric flow rate, the total force on a wall, or the total torque to move a part. Consider steady, pressure-driven flow of an incompressible, Newtonian fluid in a horizontal circular tube (tube length is L; total stress tensor shown below). For this flow and using the stress tensor given below, calculate the z-component of the total fluid force on the inner pipe surface.

   Note that the stress tensor is given in cylindrical coordinates (r, θ, z), R is the inner radius of the tube, and L, p_0, and p_L are constants

$$\underline{\underline{\Pi}} = \begin{pmatrix} \frac{(p_L - p_0)}{L} z + p_0 & 0 & \frac{(p_0 - p_L) r}{2L} \\ 0 & \frac{(p_L - p_0)}{L} z + p_0 & 0 \\ \frac{(p_0 - p_L) r}{2L} & 0 & \frac{(p_L - p_0)}{L} z + p_0 \end{pmatrix}_{r\theta z}$$

5. (20 points) An incompressible, Newtonian fluid (density ρ; viscosity μ) flows steadily in a film of thickness H down a long, wide tilted surface as shown below. The surface makes an angle α with respect to horizontal. The moving fluid is exposed to air on its top surface (atmospheric pressure). Out of the field of view the fluid is replenished at the top of the flow and flows away at the bottom of the flow. We are considering only the flow regions that are far from these more complex in-flow and outflow regions.

   In the coordinate system given below, calculate the velocity field in this flow as a function of position. Please show your work and indicate your assumptions. You may assume that the pressure does not vary with x_1. Express your final answer only in terms of quantities given in the problem or figure.