1.	/20
2.	/20
3.	/20
4.	/20
5.	/20

Name:

Exam 1

CM 4650 Polymer Rheology
18 February 2020

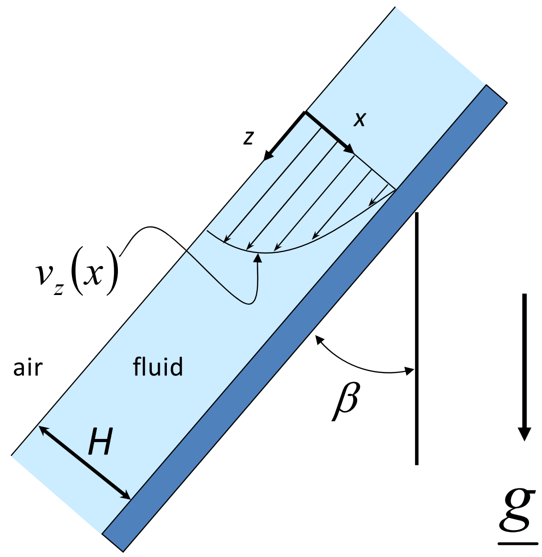
- ✓ Please be neat.
- ✓ Please write on only one side of each piece of paper in your solution.
- ✓ This exam is closed book, closed notes.
- ✓ No internet-capable devices are permitted.
- ✓ Submit only your own work.
- ✓ A calculator is permitted.
- ✓ Please be neat. Only neat answers will be granted partial credit.
- ✓ Please box your final answers.

1. (20 points)
    a. Use Einstein notation to write this expression in terms of Cartesian components: $(\nabla \underline{w})^{\mathrm{T}} \cdot \underline{u}$. Write the answer as a column vector.
    b. What is the "2" component of your answer? Write your answer to this part with no summation sign (that is, write out each term).

2. 20 points) How do we write the following vector, here written in Cartesian coordinates, in Gibbs notation? (Gibbs notation is in terms of $\nabla, (\cdot), \underline{\underline{A}}, (:), \times, \underline{v}, \frac{\partial}{\partial t}, etc.$) All quantities are variables. Verify your answer with Einstein notation.

$$\begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}_{123} = \begin{pmatrix} \frac{\partial^2 w_1}{\partial x_1^2} + \frac{\partial^2 w_1}{\partial x_2^2} + \frac{\partial^2 w_1}{\partial x_3^2} \\ \frac{\partial^2 w_2}{\partial x_1^2} + \frac{\partial^2 w_2}{\partial x_2^2} + \frac{\partial^2 w_2}{\partial x_3^2} \\ \frac{\partial^2 w_3}{\partial x_1^2} + \frac{\partial^2 w_3}{\partial x_2^2} + \frac{\partial^2 w_3}{\partial x_3^2} \end{pmatrix}_{123}$$

3. (20 points) Answer the questions for the flow sketched below. The figure shown is the steady, gravity-driven flow of an incompressible Newtonian fluid down a long plate of length L and width W, where only a cross-section at an arbitrary value of y is shown.



   a. What is the vector for gravity $\underline{g}$ written in the coordinate system shown in the figure?
   b. The fluid force on the wall may be calculated from the stress field $\underline{\underline{\Pi}}$ using the equation below.

$$\underline{F} = \iint_S (\hat{n} \cdot -\underline{\underline{\Pi}})\Big|_{surface} dS$$

   For the flow field shown, what are $\hat{n}$, dS, and the limits of the integrations needed to calculate the vector fluid force on the wall? You do not have to solve for the stress or velocity fields; you only need to provide the quantities requested.

4. (20 points) For the flow described and depicted in problem 3 above (steady flow of an incompressible fluid down an inclined plane), what are the velocity boundary conditions? Please be specific and mathematical, writing your answer in the coordinate system given there.

5. (20 points) A steady flow of an incompressible, Newtonian fluid is created between two very wide, long parallel plates as shown in the figure. The bottom plate is stationary and the top plate is pulled to the right at a speed V. There is an opposing pressure gradient applied to the fluid in the gap such that at the left edge the pressure is low (P_0) and downstream a distance L the pressure is high (P_L). Answer the questions below using the coordinate system given. Please show your work and indicate your assumptions. You may neglect gravity $\underline{g}$.
   a. What is the differential equation for the velocity?
   b. What are the boundary conditions on velocity? (express the boundary conditions mathematically)
   c. What is the steady state velocity profile? Express your final answer only in terms of quantities given in the problem or figure.