HW1
CM4650
Soln

1

(1)(a)

$$\underline{a} \cdot \underline{u} = a_p \hat{e}_p \cdot u_m \hat{e}_m$$

δ_{pm} "p becomes m"

$$= a_m u_m$$

$$= \boxed{a_1 u_1 + a_2 u_2 + a_3 u_3}$$

(b)

$$\underline{\underline{B}} \cdot \underline{M} = B_{jk} \hat{e}_j \hat{e}_k \cdot M_s \hat{e}_s$$

δ_{ks}
"k becomes s"

$$= B_{js} M_s \hat{e}_j \quad \text{(vector)}$$

Vector

$$\underline{\underline{B}} \cdot \underline{m} = \begin{pmatrix} B_{11} m_1 + B_{12} m_2 + B_{13} m_3 \\ B_{21} m_1 + B_{22} m_2 + B_{23} m_3 \\ B_{31} m_1 + B_{32} m_2 + B_{33} m_3 \end{pmatrix}_{123}$$

c) $\underline{S} + \underline{b} = S_m \hat{e}_m + b_j \hat{e}_j$

$m \to p \qquad j \to p$

$= S_p \hat{e}_p + b_p \hat{e}_p$

$= (S_p + b_p) \hat{e}_p$

$$\boxed{\underline{S} + \underline{b} = \begin{pmatrix} S_1 + b_1 \\ S_2 + b_2 \\ S_3 + b_3 \end{pmatrix}_{123}}$$

d) $\underline{P} \cdot \underline{\underline{B}}^T = P_j \hat{e}_j \cdot \left(B_{af} \hat{e}_a \hat{e}_f \right)^T$

$= P_j \underbrace{\hat{e}_j \cdot (B_{af} \hat{e}_f}_{\delta_{jf}} \hat{e}_a)$

δ_{jf} "j becomes f"

$= P_f B_{af} \hat{e}_a$ <u>vector</u>

$$\underline{P} \cdot \underline{\underline{B}}^T = \begin{pmatrix} P_1 B_{11} + P_2 B_{12} + P_3 B_{13} \\ P_1 B_{21} + P_2 B_{22} + P_3 B_{23} \\ P_1 B_{31} + P_2 B_{32} + P_3 B_{33} \end{pmatrix}_{123}$$

(vector)

e) $$\underline{\underline{\Delta}} \cdot \underline{\underline{B}} = \Delta_{ij} \hat{e}_i \hat{e}_j \cdot B_{pf} \hat{e}_p \hat{e}_f$$

δ_{jp}

"j becomes p"

$$= \Delta_{ip} B_{pf} \hat{e}_i \hat{e}_f$$

$$= \sum_{i=1}^{3} \sum_{f=1}^{3} \sum_{p=1}^{3} \Delta_{ip} B_{pf} \hat{e}_i \hat{e}_f$$

$$\underline{\underline{\Delta}} \cdot \underline{\underline{B}} = \begin{pmatrix} \sum_{p=1}^{3} \Delta_{1p} B_{p1} & \sum_{p=1}^{3} \Delta_{1p} B_{p2} & \sum_{p=1}^{3} \Delta_{1p} B_{p3} \\ \sum_{p=1}^{3} \Delta_{2p} B_{p1} & \sum_{p=1}^{3} \Delta_{2p} B_{p2} & \sum_{p=1}^{3} \Delta_{2p} B_{p3} \\ \sum_{p=1}^{3} \Delta_{3p} B_{p1} & \sum_{p=1}^{3} \Delta_{3p} B_{p2} & \sum_{p=1}^{3} \Delta_{3p} B_{p3} \end{pmatrix}_{123}$$

5

2) $\underline{\underline{C}} = \underline{\underline{M}} \cdot \underline{\underline{B}}^T = M_{as}\,\hat{e}_a\hat{e}_s \cdot \left(B_{km}\,\hat{e}_k\hat{e}_m\right)^T$

$$= M_{as}\,\hat{e}_a\,\underbrace{\hat{e}_s \cdot B_{km}\,\hat{e}_m}_{\delta_{sm}}\,\hat{e}_k$$

"s becomes m"

$$\underline{\underline{C}} = M_{am}\,B_{km}\,\hat{e}_a\hat{e}_k$$

"23 - component" $\Rightarrow$ $a=2$
$k=3$

$$C_{23} = \sum_{m=1}^{3} M_{2m}B_{3m}$$

$$\boxed{C_{23} = M_{21}B_{31} + M_{22}B_{32} + M_{23}B_{33}}$$

3) a) $\underline{a} \cdot \underline{b} = a_i \hat{e}_i \cdot b_f \hat{e}_f$

δ_{if} "i becomes f"

$= a_f b_f$

$= a_1 b_1 + a_2 b_2 + a_3 b_3$

$= (1)(-1) + (-0.5)(1) + (-2)(1)$

a) $= \boxed{-3.5}$

b) $\underline{\underline{W}} = \underline{a}\,\underline{b} = a_s \hat{e}_s \, b_m \hat{e}_m$

$= a_s b_m \hat{e}_s \hat{e}_m$

$$\underline{\underline{W}} = \begin{pmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{pmatrix}_{123}$$

$$\underline{\underline{w}} = \underline{a}\,\underline{b}$$

$$= \begin{pmatrix} (1)(-1) & (1)(1) & (1)(1) \\ (-0.5)(-1) & (-0.5)(1) & (-0.5)(1) \\ (-2)(-1) & (-2)(1) & (-2)(1) \end{pmatrix}_{123}$$

$$\boxed{\underline{\underline{w}} = \begin{pmatrix} -1 & 1 & 1 \\ 0.5 & -0.5 & -0.5 \\ 2 & -2 & -2 \end{pmatrix}_{123}}$$

4) $$\underline{v} = \begin{pmatrix} 0 \\ \frac{u_0}{r} \\ 0 \end{pmatrix}_{r\theta z} = \frac{u_0}{r}\,\hat{e}_\theta$$

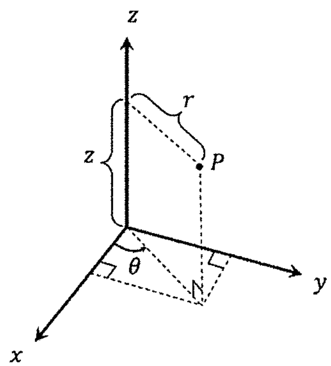
see lecture slide or book

$$= \frac{u_0}{\underset{\sqrt{x^2+y^2}}{r}}\left(-\underbrace{\sin\theta}_{\frac{y}{r}}\,\hat{e}_x + \underbrace{\cos\theta}_{\frac{x}{r}}\,\hat{e}_y \right)$$

$$\underline{v} = \begin{pmatrix} \frac{-u_0 y}{x^2+y^2} \\ \frac{u_0 x}{x^2+y^2} \\ 0 \end{pmatrix}_{xyz}$$

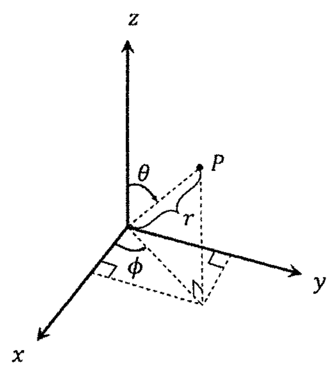
$$\underline{v}(1,2,1) = \begin{pmatrix} \frac{-2u_0}{5} \\ \frac{u_0}{5} \\ 0 \end{pmatrix}_{123} \qquad \underline{v}(0,0,1) = ?$$

(lecture notes) 9



Cylindrical Coordinates

System	Coordinates	Basis vectors
Cylindrical	$r = \sqrt{x^2 + y^2}$	$\hat{e}_r = \cos\theta\,\hat{e}_x + \sin\theta\,\hat{e}_y$
Cylindrical	$\theta = \tan^{-1}\left(\frac{y}{x}\right)$	$\hat{e}_\theta = (-\sin\theta)\hat{e}_x + \cos\theta\,\hat{e}_y$
Cylindrical	$z = z$	$\hat{e}_z = \hat{e}_z$
Cylindrical	$x = r\cos\theta$	$\hat{e}_x = \cos\theta\,\hat{e}_r + (-\sin\theta)\hat{e}_\theta$
Cylindrical	$y = r\sin\theta$	$\hat{e}_y = \sin\theta\,\hat{e}_r + \cos\theta\,\hat{e}_\theta$
Cylindrical	$z = z$	$\hat{e}_z = \hat{e}_z$



Note: my spherical θ comes from the z-axis.

Spherical Coordinates

System	Coordinates	Basis vectors
Spherical	$x = r\sin\theta\cos\phi$	$\hat{e}_x = (\sin\theta\cos\phi)\hat{e}_r + (\cos\theta\cos\phi)\hat{e}_\theta + (-\sin\phi)\,\hat{e}_\phi$
Spherical	$y = r\sin\theta\sin\phi$	$\hat{e}_y = (\sin\theta\sin\phi)\hat{e}_r + (\cos\theta\sin\phi)\hat{e}_\theta + \cos\phi\,\hat{e}_\phi$
Spherical	$z = r\cos\theta$	$\hat{e}_z = \cos\theta\;\hat{e}_r + (-\sin\theta)\hat{e}_\theta$
Spherical	$r = \sqrt{x^2 + y^2 + z^2}$	$\hat{e}_r = (\sin\theta\cos\phi)\hat{e}_x + (\sin\theta\sin\phi)\hat{e}_y + \cos\theta\hat{e}_z$
Spherical	$\theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right)$	$\hat{e}_\theta = (\cos\theta\cos\phi)\hat{e}_x + (\cos\theta\sin\phi)\hat{e}_y + (-\sin\theta)\hat{e}_z$
Spherical	$\phi = \tan^{-1}\left(\frac{y}{x}\right)$	$\hat{e}_\phi = (-\sin\phi)\hat{e}_x + \cos\phi\,\hat{e}_y$

can use l'Hopital's rule :

$$\underbrace{\lim_{x \to 0} \frac{f(x)}{g(x)}}_{\text{if this is undefined}} = \lim_{x \to 0} \frac{f'(x)}{g'(x)}$$

$$\lim_{\substack{x \to 0 \\ y \to 0}} \left(\frac{-u_0 y}{x^2 + y^2} \right) = \lim_{\substack{x \to 0 \\ y \to 0}} \frac{-u_0}{2x + 2y} = \text{undefined}$$

$\underline{v}(0, 0, 1)$ is undefined //

5) $\sqrt{\dfrac{\underline{\underline{M}} : \underline{\underline{M}}}{2}} = |\underline{\underline{M}}|$

calc $\underline{\underline{M}} : \underline{\underline{M}} =$

$$M_{ij} \overbrace{\underbrace{\hat{e}_i \hat{e}_j : M_{pk} \hat{e}_p \hat{e}_k}_{\delta_{ik}}}^{\delta_{jp}}$$

"p becomes j"
"k becomes i"

$$\underline{\underline{M}} : \underline{\underline{M}} = M_{ij} M_{ji}$$

$$= \cancel{(1)^2} + (0.5)(0.25)$$
$$+ \cancel{(1)(1)} + (0.25)(0.5)$$
$$+ 2^2 + \cancel{(1)(1)} + (1)(1)$$
$$+ \cancel{(1)(-1)} + (-1)^2$$

a

$$\boxed{|\underline{\underline{M}}| = \sqrt{\frac{6.25}{2}} = 1.76777}$$

5b)

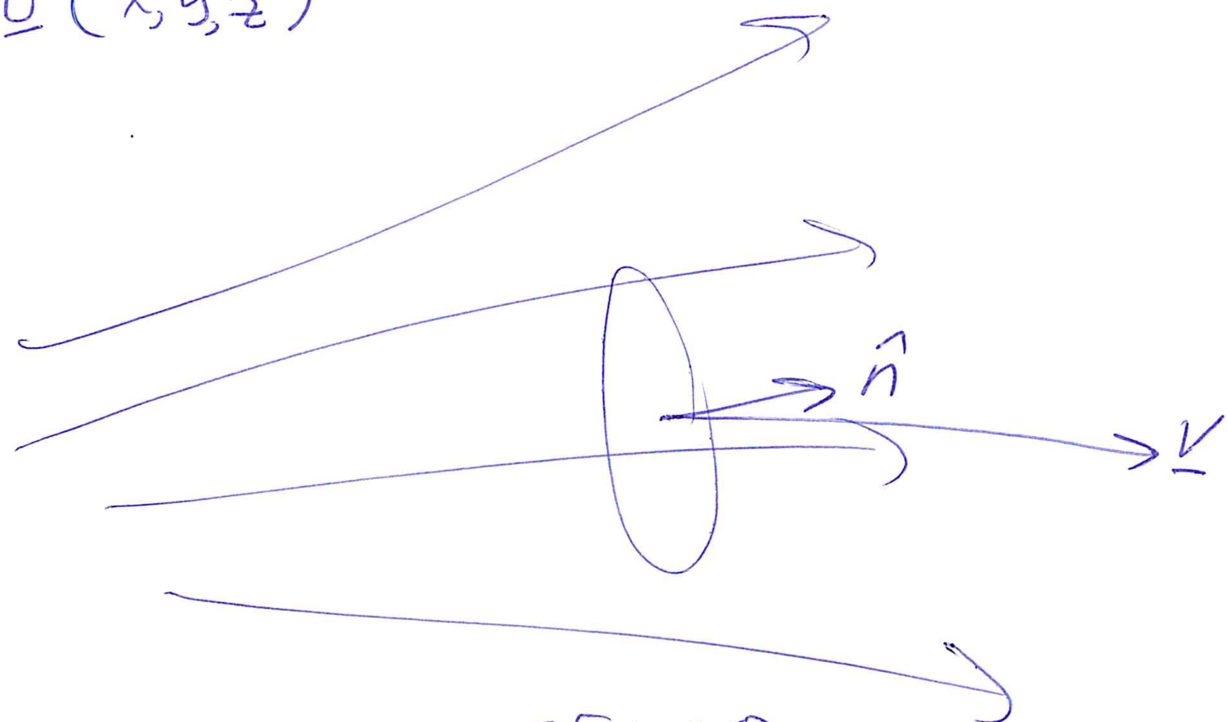
$$\underline{\underline{B}} = \begin{pmatrix} A\beta_0 & \cos\phi_0 \beta_0 X_1 X_2 & 0 \\ 2\beta_0 X_2 & A & 0 \\ 0 & 0 & u_0 \end{pmatrix}_{123}$$

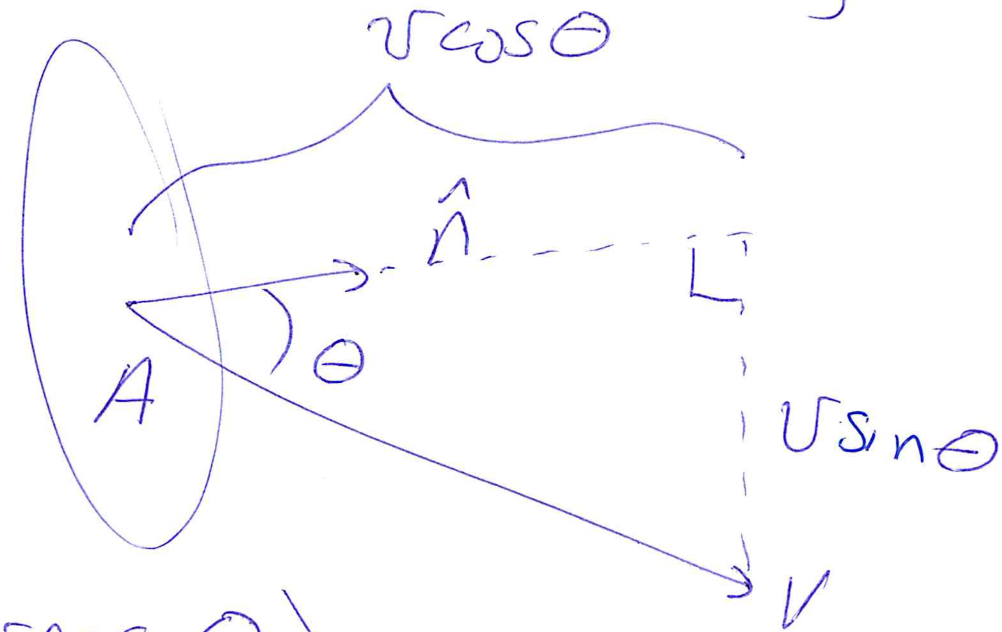
$$\underline{\underline{B}} : \underline{\underline{B}} = B_{ij} B_{ji}$$

$$\boxed{|\underline{\underline{B}}| = \sqrt{\frac{1}{2}\left((A\beta_0)^2 + 2\left(\cos\phi_0 \beta_0 X_1 X_2 \left[2\beta_0 X_2\right] + A^2 + u_0^2\right)\right.}}$$

6. Problem 2.47 p58 (B)

$\underline{v}(x,y,z)$





$$\underline{V} = \begin{pmatrix} v\cos\theta \\ v\sin\theta \end{pmatrix}_{\hat{n}\,\hat{t}}$$

normal

tangent coord system

The tangent component of $\underline{v}$ does not move fluid through the area A

(it only sloshes around on the other side).

$$\text{mass flow} = (\text{density})\left(\begin{matrix}\text{volumetric} \\ \text{flow rate}\end{matrix}\right)$$

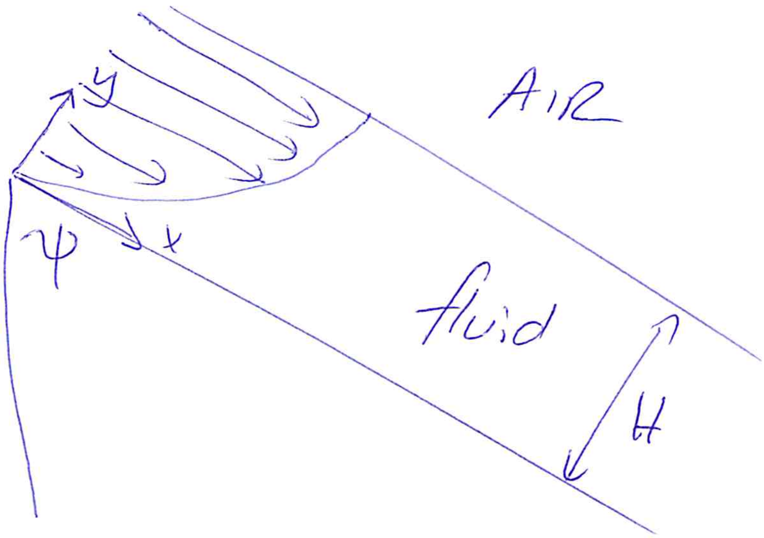
$$= \rho \left((\text{AREA}) \left(\begin{matrix}\text{Velocity} \\ \text{through}\end{matrix}\right)\right)$$

$$= \rho A v \cos\theta$$

$$= \boxed{\rho A \underline{v} \cdot \hat{n}}$$

7)

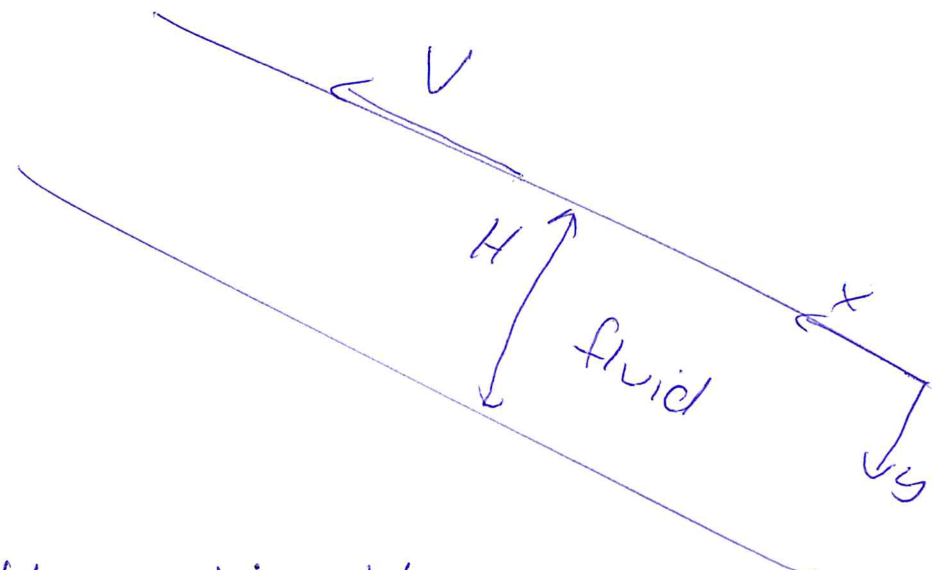
(a)
3.14



$y=0 \quad v_x = 0$

$y=H \quad \dfrac{dv_x}{dy} = 0$ (zero stress

(b)



$y=0 \quad v_x = V$

$y=H \quad v_x = 0$