Homework 3
CM4650
Spring 2020

Due: *Wednesday 26 February 2020, in class*

Please do not write on the backside of the pages. Please write legibly and large. Thank you.

1. (10 points) Text 5.1 What is a stress constitutive equation? What is a rheological material function? What is the difference and how are these two concepts/definitions related? Please put the differences in your own words (don't directly quote the book please).

2. (10 points) Text 2.19: Using Einstein notation, show that:

$$\left(\underline{\underline{A}} \cdot \underline{\underline{B}} \cdot \underline{\underline{C}}\right)^T = \underline{\underline{C}}^T \cdot \underline{\underline{B}}^T \cdot \underline{\underline{A}}^T$$

3. (10 points) Calculate the magnitude of the tensor $\underline{\underline{A}}$ given below:

$$\underline{\underline{A}} = 5\hat{e}_1\hat{e}_1 + 2\hat{e}_1\hat{e}_2 - \hat{e}_2\hat{e}_2 + 2\hat{e}_2\hat{e}_3 + \hat{e}_3\hat{e}_1 - 2\hat{e}_3\hat{e}_3$$

4. (10 points) Tensors (more precisely, second-order tensors) have three invariants, which are scalars that are independent of coordinate system. One set of three invariants, I, II, III, is defined in Chapter 2; another set of invariants I_1, I_2, I_3 is defined in Appendix B (page 453); the two sets are interrelated in equations C.81-C.83 (p 476). For the tensors given below, what are the values of the invariants? Calculate both sets from the definitions and verify that the interrelating equations on page 476 hold.

$$\underline{\underline{A}} = \begin{pmatrix} 5 & 8.2 & 0 \\ 8.2 & 0 & 0 \\ 0 & 0 & -5 \end{pmatrix}_{123}$$

$$\underline{\underline{B}} = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & -16 \end{pmatrix}_{123}$$

5. (20 points) What is the start-up of steady shearing material function $\eta^+(t, \dot{\gamma}_0)$ predicted by the proposed constitutive equation below? Derive your answer from the starting definitions on the "recipe card". Please sketch your answer for various values of $\dot{\gamma}_0$.

$$\underline{\underline{\tau}}(t, \dot{\gamma}_0) = \left(\frac{a}{\sqrt{\dot{\gamma}_0}}\right)\left(\nabla\underline{v} + (\nabla\underline{v})^T\right)$$

where $\dot{\gamma}_0$ is the parameter in the definition of the start-up material function. What are the units on a?