1.	/20
2.	/20
3.	/20
4.	/20
5.	/20

Name:____________________________.

Final Exam

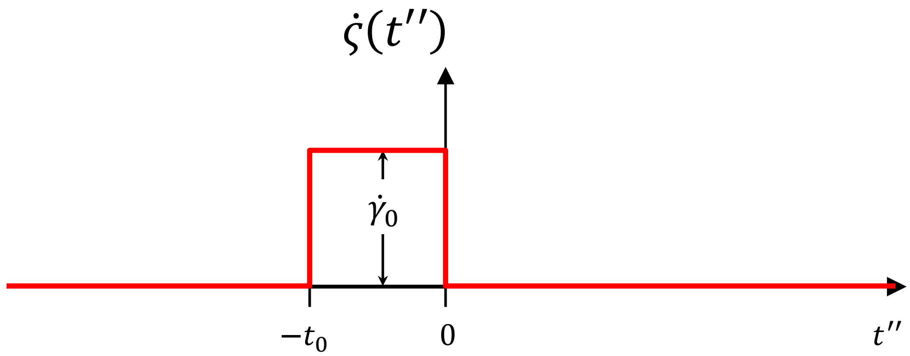
CM 4650 Polymer Rheology
30 April 2018

- ✓ Please be neat.
- ✓ Please write on only one side of each piece of paper in your solution.
- ✓ This exam is closed book, closed notes.
- ✓ No internet-capable devices are permitted.
- ✓ Submit only your own work.
- ✓ A calculator is permitted.

1. (20 points) Write the expression $\underline{a} \cdot \nabla \underline{b}$ in
   a. Einstein notation
   b. Einstein notation with all summation signs written in
   c. Matrix notation
2. (20 points) If you are unsure of your answer, provide a sentence or two or a sketch to indicate your thinking for partial credit.
   a. True/False: The Cox Merz rule (that is, $\eta(\dot{\gamma}) = \eta^*(\omega)|_{\omega=\dot{\gamma}}$) is true for all long-chain, linear polymers. η is the steady shear viscosity; η^* is the complex viscosity, measured in small-amplitude oscillatory shear, SAOS.
   b. True/False: Trouton's rule (that is, $\bar{\eta} = 3\eta_0$) is true for all long-chain linear polymers at all shear rates. $\bar{\eta}$ is the steady elongational viscosity; η_0 is the zero-shear viscosity of steady shear flow.
   c. What material behavior (measurements of material functions) can be used to tell if a linear polymer is entangled?
   d. What rheological material-function measurement can you make to indicate if a polymer sample is polydisperse?
3. (20 points) Beginning with the kinematics, calculate the steady shear material function Ψ_1 for the Lodge model. Ψ_1 is the steady shear flow first normal stress coefficient material function. Please show your work. When/if you use a calculator, please indicate that on your solution.

4. (20 points) I propose a new material function for shear flow based on a time-dependent flow that experiences a small amount of shear just before time $t = 0$. The kinematics $\dot{\varsigma}(t'')$ of my new shear material function are given below; t_0 is a positive, constant parameter with units of time and $\dot{\gamma}_0$ is a positive, constant shear-rate parameter with units of 1/time.

$$\dot{\varsigma}(t'') = \begin{cases} 0 & t'' < -t_0 \\ \dot{\gamma}_0 & -t_0 \le t'' < 0 \\ 0 & t'' \ge 0 \end{cases}$$



What is the shear strain $\gamma = \gamma(t', t)$ (the shear strain in Table 9.3 that appears in the finite-strain tensors) for this set of kinematics? As always, the time t varies and is always greater than zero; the time t'varies and may be less than or greater than zero and ranges from $-\infty$ up to t. Show explicitly how you obtain your answer. Please box your final answer.

5. (20 points) Calculate the velocity field $\underline{v}$ for steady, pressure-driven flow of a power-law, generalized Newtonian fluid flowing in a horizontal circular tube of radius R. You may neglect gravity, and the fluid is incompressible. Indicate your assumptions and show your work. Please be neat. The axial pressure gradient $\frac{\partial p}{\partial z}$ is a constant, $\frac{\partial p}{\partial z} = -\Delta p/L$, where L is the tube length. You may leave the two integration constants in your answer (no need to do the final algebra). You must write the boundary conditions correctly, and mathematically, to obtain full credit.