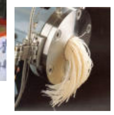
# On to . . . Polymer Rheology . . .

*We now know how to model Newtonian fluid motion,* $\underline{v}(\underline{x},t),\ p(\underline{x},t)$ :

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0$$

Continuity equation

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$$

Cauchy momentum equation

$$\underline{\underline{\tau}} = -\mu\left(\nabla \underline{v} + (\nabla \underline{v})^T\right)$$

Newtonian constitutive equation

1



Rheological Behavior of Fluids – Non-Newtonian

*How do we do the same for Non-Newtonian fluid motion?*

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0$$

Continuity equation

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$$

Cauchy Momentum Equation

$$\underline{\underline{\tau}} = f(\underline{x},t)$$

Non-Newtonian constitutive equation

2

Rheological Behavior of Fluids – Non-Newtonian

$$\underline{\underline{\tau}} = f(\underline{x}, t) \quad \textbf{?}$$

Where to begin?




Rheological Behavior of Fluids – Non-Newtonian

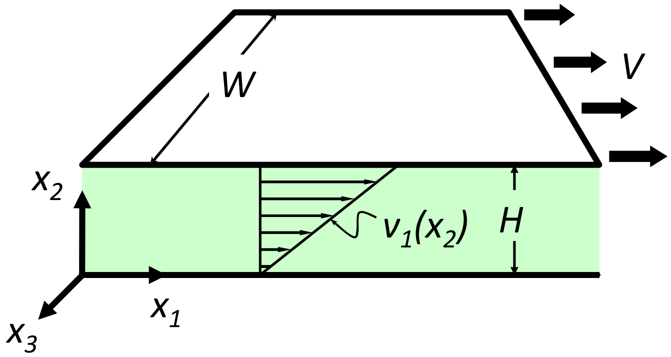
$$\underline{\underline{\tau}} = f(\underline{x}, t) \quad \textbf{?}$$

Let's begin here:

*How did Stokes/others come up with this* <u>*tensor*</u> *equation?*

$$\underline{\underline{\tau}} = -\mu\left(\nabla \underline{v} + (\nabla \underline{v})^T\right)$$

The approach: Beginning with the simplest experiments




$$\frac{F}{A} = \mu \frac{V}{H}$$

$$\tau_{21} = -\mu \frac{dv_1}{dx_2}$$

Deduce a scalar law for a particular flow.

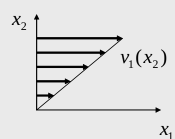
5



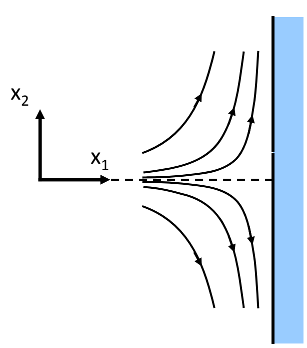
Seek something that works for <u>all</u> flows

Newtonian fluids: (shear flow only)

$$\tau_{21} = -\mu \frac{dv_1}{dx_2}$$




Newtonian fluids: (complex flows)



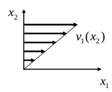

$$\underline{\underline{\tau}} = f(?) = f\left(\frac{\partial v_1}{\partial x_1}, \frac{\partial v_1}{\partial x_2}, \frac{\partial v_2}{\partial x_1}, \frac{\partial v_2}{\partial x_2}\right)$$

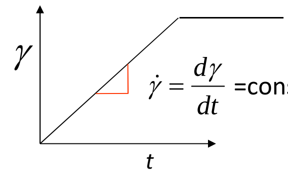
6

## Driven by observation

*1. Strain response to imposed shear stress*

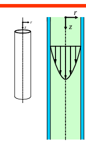
•shear rate is constant



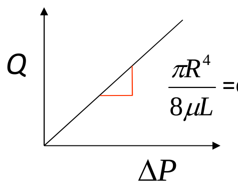

$$\dot{\gamma} = \frac{d\gamma}{dt} \text{ =constant}$$

*2. Pressure-driven flow in a tube (Poiseuille flow)*



•viscosity is constant

$$Q = \frac{\pi \Delta P R^4}{8\mu L}$$




$$\frac{\pi R^4}{8\mu L} \text{ =constant}$$

*3. Stress tensor in shear flow*

•only two components are nonzero

$$\underline{\underline{\tau}} = \begin{pmatrix} 0 & \tau_{12} & 0 \\ \tau_{21} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$

7



**From flow observations, Stokes/others:**

Deduced the constitutive equation.

(Aided by the ability to set aside fluids that had complex behavior.)

**Observations:**

•Shear stress is proportional to velocity gradient in shear
•No flow, no stress
•Q(Δp) is linear in pipe flow

(Aided also by exceptional mathematical knowledge and ability.)

8

## How is $\underline{\underline{\tau}}$ related to the velocity field?
Stokes, 1840s

Bird, Stewart, Lightfoot, *Transport Phenomena,* 2002, p18
Deen, *Analysis of Transport Phenomena,* 1998, p225
Aris, *Vectors, Tensors and the Basic Equations of Fluid Mechanics,* 1962, pp105-112

(Stokesian fluid)
If we assume:

- Stress is a linear combination of all possible velocity gradients;
  Motivated by Newton's law of viscosity and the need to accommodate other flow geometries; seems like a cautious guess.
- The expression should not contain time derivatives/integrals;
  Eliminates fluid memory from consideration; stress responds instantly to the local rate of strain.
- No viscous forces are present if the fluid is in a state of pure rotation
  Sensible, observed; implies a symmetric stress tensor
- The fluid is isotropic (has no preferred direction)
  Eliminates some materials, e.g. liquid crystals; implies that the principal axes of stress and strain coincide (see Aris).



## How is $\underline{\underline{\tau}}$ related to the velocity field?
Stokes, 1840s

Then:

$$\underline{\underline{\tau}} = -\mu\left(\nabla \underline{v} + (\nabla \underline{v})^T\right) + \left(\frac{2}{3}\mu - \kappa\right)(\nabla \cdot \underline{v})\underline{\underline{I}}$$

$\kappa$ is dilatational viscosity; is small; often neglected (e.g. in Comsol)

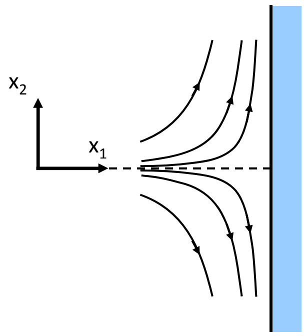
If, in addition, the fluid is incompressible:

$$\underline{\underline{\tau}} = -\mu\left(\nabla \underline{v} + (\nabla \underline{v})^T\right)$$

(facilitated by working with materials with simple molecular involvement, i.e. one scalar material parameter)

Check if it works in complex flow:

$$\underline{\underline{\tau}} = -\mu\left(\nabla \underline{v} + (\nabla \underline{v})^T\right)$$




$$\underline{\underline{\tau}} = f\left(\frac{\partial v_1}{\partial x_1}, \frac{\partial v_1}{\partial x_2}, \frac{\partial v_2}{\partial x_1}, \frac{\partial v_2}{\partial x_2}\right)$$

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & \tau_{12} & 0 \\ \tau_{21} & \tau_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$

$$\underline{\underline{\tau}} = \begin{pmatrix} -2\mu\frac{\partial v_1}{\partial x_1} & -2\mu\left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1}\right) & 0 \\ -2\mu\left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1}\right) & -2\mu\frac{\partial v_2}{\partial x_2} & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$

11



Rheological Behavior of Fluids - Newtonian

*Question: How did Stokes/others come up with this?*

$$\underline{\underline{\tau}} = -\mu\left(\nabla \underline{v} + (\nabla \underline{v})^T\right)$$

*Answer:*

- *Concentrate on simple flows*
- *Make careful observations*
- *Make cautious conjectures of general rules – see what the conjectures predict; compare with observations*
- *Keep what works; discard blind alleys*
- *(set aside more complex behavior for future studies. . . )*

12

For Newtonian fluids, the stress/deformation relationship is understood.

Ongoing challenges in Newtonian Fluid Mechanics modeling:

•*Navier-Stokes are difficult-to-solve nonlinear equations*

•*Even the most comprehensive models rely on many assumptions*

—Continuum
—Isothermal
—Neglect of small effects (fluctuations, some forces)
—Symmetry
—Chemical stability
—Chemical homogeneity
…

•*Computer solutions introduce limitations related to numerical algorithms*

For Non-Newtonian fluids,





We will follow the historical method of discovery,

BUT, for Non-Newtonian fluids:

What experiments should <u>we</u> use?

What calculations should <u>we</u> make?

What equations should <u>we</u> try?...

The material behavior is more complex and varied.

Is there any way to do this more systematically?

*To figure out what goes into these choices, let's try it ourselves.*

Exercise:

For the experiments in the film, write the problem statement (think, typical HW problem), for the calculation that needs to be performed to verify that the Newtonian constitutive equation predicts the observed behavior.

List simplifying assumptions.

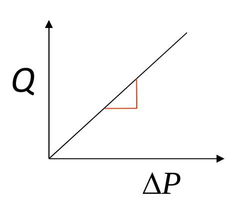
What data comparison will provide verification?

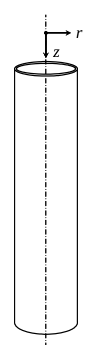
(e.g.: *For geometry shown, calculate the steady velocity field for an incompressible Newtonian fluid subject to . . .* )

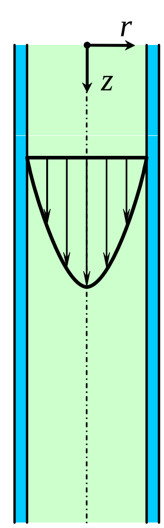



Rheological Behavior of Fluids - Newtonian

Flow rate in a tube is a linear function of driving pressure.



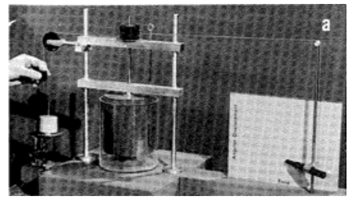



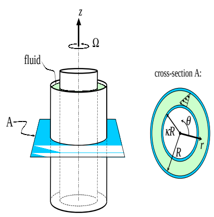



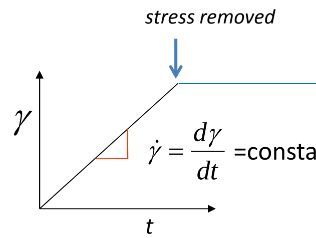
Rheological Behavior of Fluids - Newtonian

•shear rate (velocity gradient) is constant; strain (displacement) is linear

•flow ceases instantaneously when weight disengages






$$\dot{\gamma} = \frac{d\gamma}{dt} \text{ =constant}$$

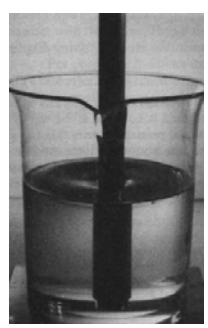



Rheological Behavior of Fluids - Newtonian

•Only shear stresses are generated, no normal stresses

•Interface dips in center (does not crawl up)

$$\underline{\underline{\tau}} = \begin{pmatrix} 0 & \tau_{12} & 0 \\ \tau_{21} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$



**Summary:**

•Matching flow models to observations requires complex modeling calculations, even when the constitutive equation is known

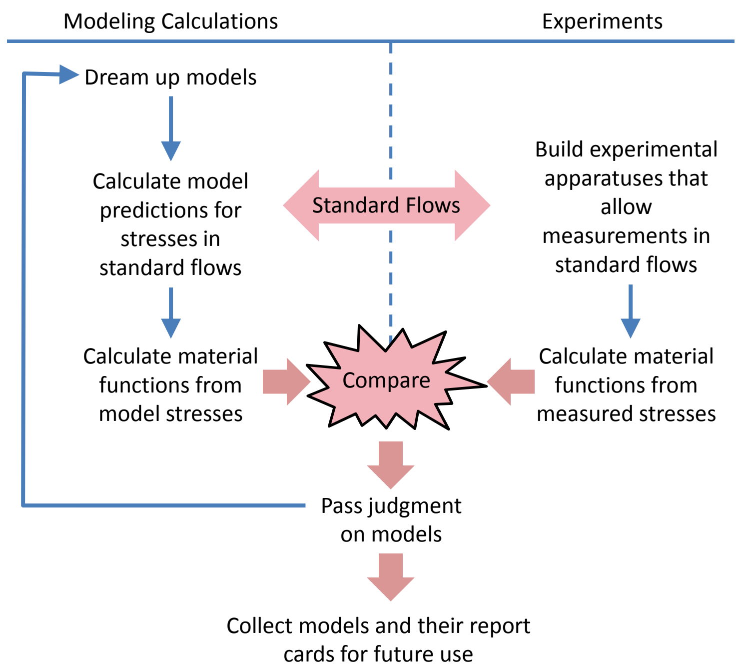
•The situation will be perhaps impossible, and certainly slow, when non-Newtonian constitutive equations are used

•How can we proceed with seeking models for non-Newtonian fluids?

19



**Tactic: Divide the Problem in half**




20

Standard flows – choose a velocity field (not an apparatus or a procedure)

- For model predictions, calculations are straightforward
- For experiments, design can be optimized for accuracy and fluid variety

Material functions – choose a common vocabulary of stress and kinematics to report results

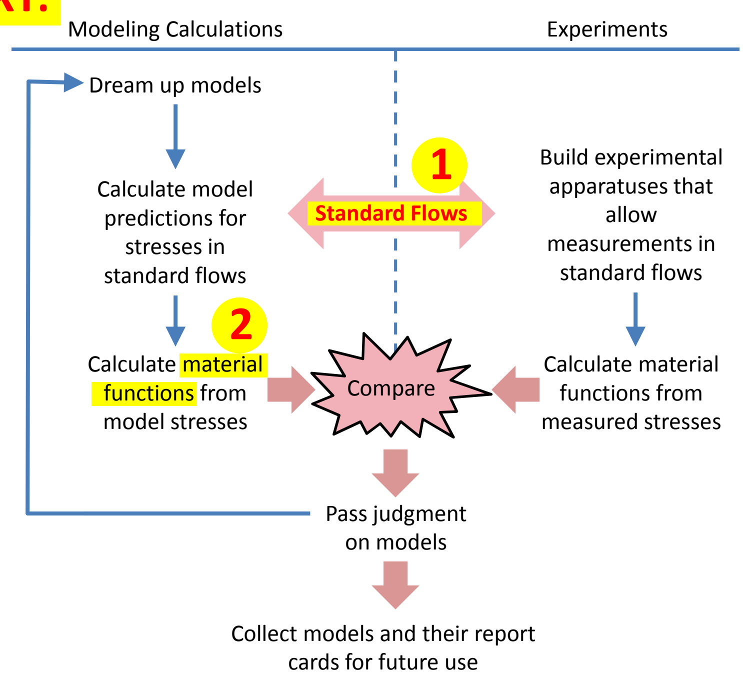
- Make it easier to compare model/experiment
- Record an "inventory" of fluid behavior (expertise)

21



**NEXT:**

**Tactic: Divide the Problem in half**




22

**Summary:**

•Matching flow models to observations requires complex modeling calculations, even when the constitutive equation is known

•The situation will be perhaps impossible when non-Newtonian constitutive equations are used

•Settle on standard flows and standardized material functions to make efforts more effective

•How can we proceed with seeking models for non-Newtonian fluids?

•Follow historical method of making cautious conjectures and comparing models to data
•Stick to standard flows/material functions to be most efficient

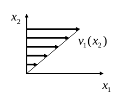


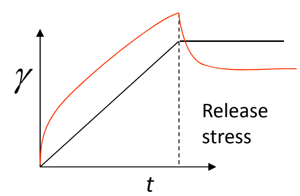
Rheological Behavior of Fluids – non-Newtonian

## Driven by observation

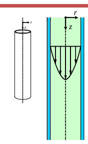
*1. Strain response to imposed shear stress*

•shear rate is variable

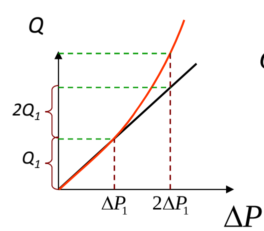



*2. Pressure-driven flow in a tube (Poiseuille flow)*



•viscosity is variable

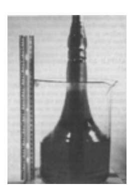



$$Q = f(\Delta P)$$

*3. Stress tensor in shear flow*

•all 9 components are nonzero

Normal stresses



$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}_{123}$$