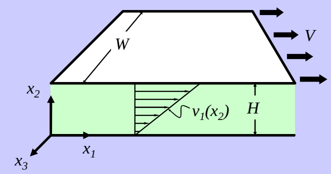
Chapter 3: Newtonian Fluid Mechanics

TWO GOALS

- Derive governing equations (mass and momentum balances
- Solve governing equations for velocity and stress fields

QUICK START

First, before we get deep into derivation, let's do a Navier-Stokes problem to get you started in the mechanics of this type of problem solving.



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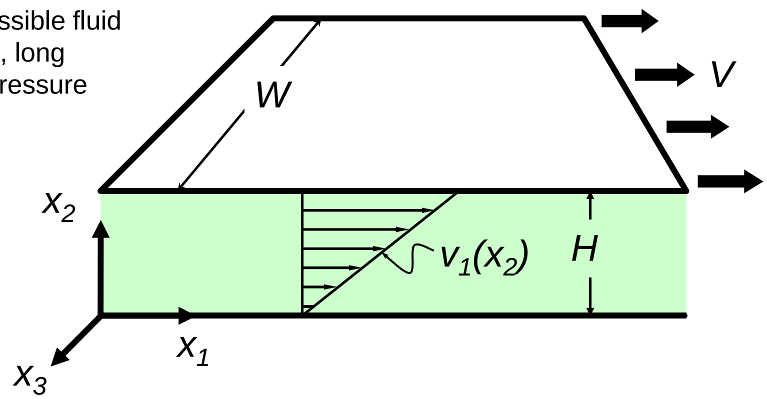
EXAMPLE: Drag flow between infinite parallel plates

$$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{123}$$

- Newtonian
- steady state
- incompressible fluid
- very wide, long
- uniform pressure



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Chapter 3: Newtonian Fluid Mechanics

TWO GOALS

- •Derive governing equations (mass and momentum balances
- •Solve governing equations for velocity and stress fields

Mass Balance

Consider an arbitrary control volume *V* enclosed by a surface *S*

$$\begin{pmatrix} \text{rate of increase} \\ \text{of mass in } CV \end{pmatrix} = \begin{pmatrix} \text{net flux of} \\ \text{mass into } CV \end{pmatrix}$$

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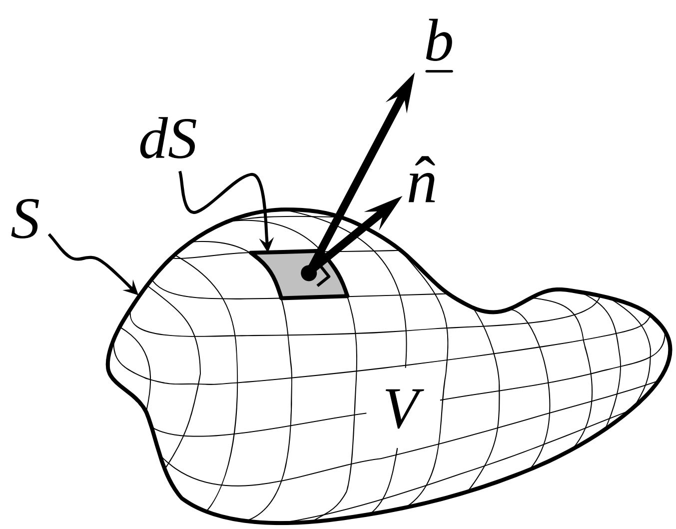
Mathematics Review

Polymer Rheology



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Mass Balance *(continued)*

Consider an arbitrary volume V enclosed by a surface S

$$\begin{pmatrix} \text{rate of increase} \\ \text{of mass in } V \end{pmatrix} = \frac{d}{dt}\left(\iiint_V \rho \, dV \right)$$

outwardly pointing unit normal

$$\begin{pmatrix} \text{net flux of} \\ \text{mass into } V \\ \text{through surface } S \end{pmatrix} = -\iint_S \rho \, \hat{n} \cdot \underline{v} \, dS$$

Mass Balance *(continued)*

$$\frac{d}{dt}\left(\iiint_V \rho \, dV \right) = -\iint_S \rho \, \hat{n} \cdot \underline{v} \, dS$$

Leibnitz rule

$$\iiint_V \frac{\partial \rho}{\partial t} dV = -\iint_S \hat{n} \cdot \left(\rho \underline{v} \right) dS$$

Gauss Divergence Theorem

$$= -\iiint_V \nabla \cdot \left(\rho \underline{v} \right) dV$$

$$\iiint_V \left(\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \underline{v} \right) \right) dV = 0$$

Mass Balance *(continued)*

$$\iiint_V \left(\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \underline{v} \right) \right) dV = 0$$

Since V is *arbitrary*,

Continuity equation:
microscopic mass balance

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \underline{v} \right) = 0}$$

Mass Balance *(continued)*

Continuity equation (general fluids)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \underline{v} \right) = 0$$

$$\frac{\partial \rho}{\partial t} + \rho \left(\nabla \cdot \underline{v} \right) + \underline{v} \cdot \nabla \rho = 0$$

$$\frac{D \rho}{Dt} + \rho \left(\nabla \cdot \underline{v} \right) = 0$$

For ρ=constant (incompressible fluids):

$$\nabla \cdot \underline{v} = 0$$

Momentum Balance

Consider an arbitrary control volume V enclosed by a surface S

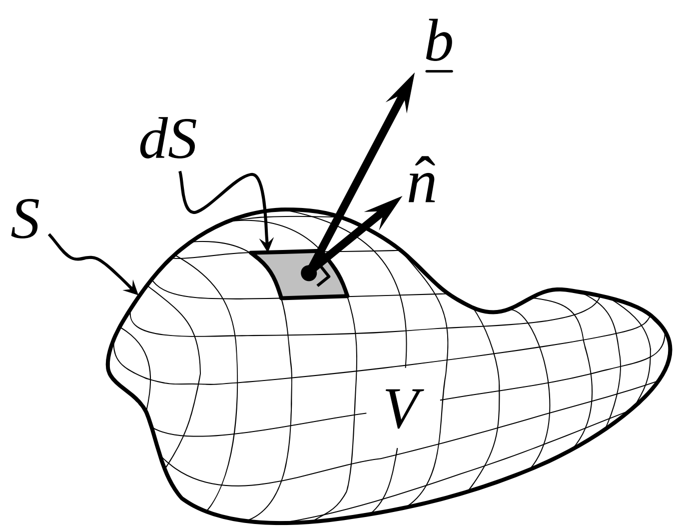
Momentum is conserved.

$$\begin{pmatrix} \textit{rate of increase} \\ \textit{of momentum in } CV \end{pmatrix} = \begin{pmatrix} \textit{net flux of} \\ \textit{momentum into } CV \end{pmatrix} + \begin{pmatrix} \textit{sum of} \\ \textit{forces on } CV \end{pmatrix}$$

resembles the rate term in the mass balance

resembles the flux term in the mass balance

Forces:
body (gravity)
molecular forces



Momentum Balance *(continued)*

Polymer Rheology

$$\begin{pmatrix} \text{rate of increase} \\ \text{of momentum in } V \end{pmatrix} = \frac{d}{dt}\left(\iiint_V \rho \underline{v}\, dV \right)$$

Leibnitz rule

$$= \iiint_V \frac{\partial}{\partial t}\left(\rho \underline{v}\right) dV$$

$$\begin{pmatrix} \text{net flux of} \\ \text{momentum into } V \end{pmatrix} = -\iint_S \hat{n}\cdot\left(\rho \underline{v}\,\underline{v}\right) dS$$

Gauss Divergence Theorem

$$= -\iiint_V \nabla\cdot\left(\rho \underline{v}\,\underline{v}\right) dV$$

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Momentum Balance *(continued)*

Polymer Rheology

Forces on V

Body Forces (non-contact)

$$\begin{pmatrix} \text{force on } V \\ \text{due to } \underline{g} \end{pmatrix} = \iiint_V \rho \underline{g}\; dV$$

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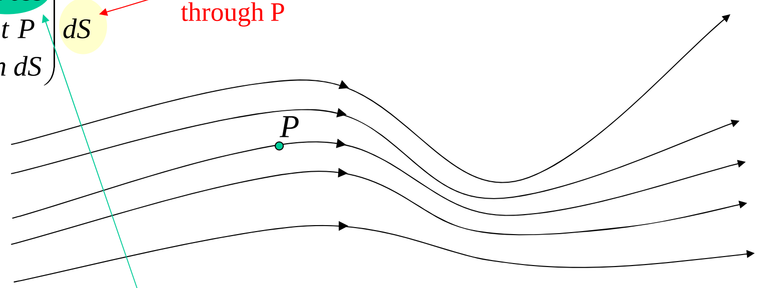
Molecular Forces (contact) – <u>this is the tough one</u>

$$\underline{f} = \left(\begin{array}{c} \textit{stress} \\ \textit{at } P \\ \textit{on } dS \end{array} \right) dS$$

choose a surface through P

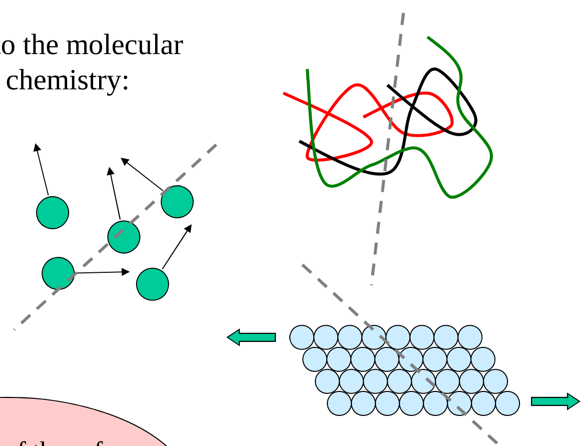
the force on that surface



We need an expression for the state of **stress** at an arbitrary point P in a flow.

Molecular Forces *(continued)*

Think back to the molecular picture from chemistry:



The specifics of these forces, connections, and interactions must be captured by the molecular forces term that we seek.

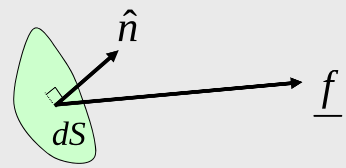
Molecular Forces *(continued)*

•We will concentrate on **expressing the molecular forces** mathematically;

•We leave to later the task of relating the resulting mathematical expression to experimental observations.

First, choose a surface:

•arbitrary shape
•small



$$\begin{pmatrix} stress \\ at\ P \\ on\ dS \end{pmatrix} dS = \underline{f}$$

What is f ?

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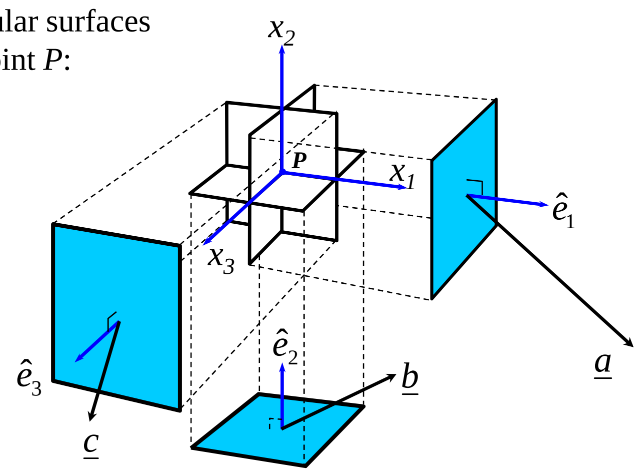
Consider the forces on three mutually perpendicular surfaces through point P:



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Molecular Forces *(continued)*

$\underline{a}$	is stress on a $\underbrace{\text{“1” surface}}_{\text{a surface with unit normal } \hat{e}_1}$ at P
$\underline{b}$	is stress on a “2” surface at P
$\underline{c}$	is stress on a “3” surface at P

We can write these vectors in a Cartesian coordinate system:

$$\begin{aligned}\underline{a} &= a_1\hat{e}_1 + a_2\hat{e}_2 + a_3\hat{e}_3 \\ &= \Pi_{11}\hat{e}_1 + \Pi_{12}\hat{e}_2 + \Pi_{13}\hat{e}_3\end{aligned}$$

stress on a “1” surface in the 1-direction

Molecular Forces *(continued)*

$$\begin{aligned}\underline{a} &= a_1\hat{e}_1 + a_2\hat{e}_2 + a_3\hat{e}_3 \\ &= \Pi_{11}\hat{e}_1 + \Pi_{12}\hat{e}_2 + \Pi_{13}\hat{e}_3 \\ \underline{b} &= b_1\hat{e}_1 + b_2\hat{e}_2 + b_3\hat{e}_3 \\ &= \Pi_{21}\hat{e}_1 + \Pi_{22}\hat{e}_2 + \Pi_{23}\hat{e}_3 \\ \underline{c} &= c_1\hat{e}_1 + c_2\hat{e}_2 + c_3\hat{e}_3 \\ &= \Pi_{31}\hat{e}_1 + \Pi_{32}\hat{e}_2 + \Pi_{33}\hat{e}_3\end{aligned}$$

$\underline{a}$	is stress on a “1” surface at P
$\underline{b}$	is stress on a “2” surface at P
$\underline{c}$	is stress on a “3” surface at P

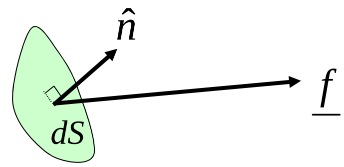
So far, this is nomenclature; next we relate these expressions to force on an arbitrary surface.

Π_{pk}

Stress on a “p” surface in the k-direction

Molecular Forces *(continued)*

How can we write $\underline{f}$ (the force on an arbitrary surface dS) in terms of the Π_{pk}?



$$\underline{f} = f_1\hat{e}_1 + f_2\hat{e}_2 + f_3\hat{e}_3$$

f_1 is force on dS in 1-direction

f_2 is force on dS in 2-direction

f_3 is force on dS in 3-direction

There are three Π_{pk} that relate to forces in the 1-direction: $\Pi_{11}, \Pi_{21}, \Pi_{31}$

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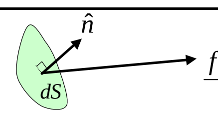
Molecular Forces *(continued)*



How can we write $\underline{f}$ (the force on an arbitrary surface dS) in terms of the quantities Π_{pk}?

$$\underline{f} = f_1\hat{e}_1 + f_2\hat{e}_2 + f_3\hat{e}_3$$

f_1 , the force on dS in 1-direction, can be broken into three parts associated with the three stress components: $\Pi_{11}, \Pi_{21}, \Pi_{31}$

first part:

$$(\Pi_{11})\overbrace{\begin{pmatrix}\textit{projection of}\\ dA\textit{ onto the}\\ 1-\textit{surface}\end{pmatrix}}^{\hat{n}\cdot\hat{e}_1\,dS} = \Pi_{11}\hat{n}\cdot\hat{e}_1\,dS$$

$$\left(\frac{force}{area}\right) \cdot (area)$$

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Molecular Forces *(continued)*

f_1, the force on dS in 1-direction, is composed of THREE parts:

first part: $(\Pi_{11})\begin{pmatrix} projection\ of \\ dA\ onto\ the \\ 1-surface \end{pmatrix} = \Pi_{11}\hat{n}\cdot\hat{e}_1\, dS$

second part: $(\Pi_{21})\begin{pmatrix} projection\ of \\ dA\ onto\ the \\ 2-surface \end{pmatrix} = \Pi_{21}\hat{n}\cdot\hat{e}_2\, dS$

third part: $(\Pi_{31})\begin{pmatrix} projection\ of \\ dA\ onto\ the \\ 3-surface \end{pmatrix} = \Pi_{31}\hat{n}\cdot\hat{e}_3\, dS$

stress on a 2 -surface in the 1-direction

the sum of these three = f_1

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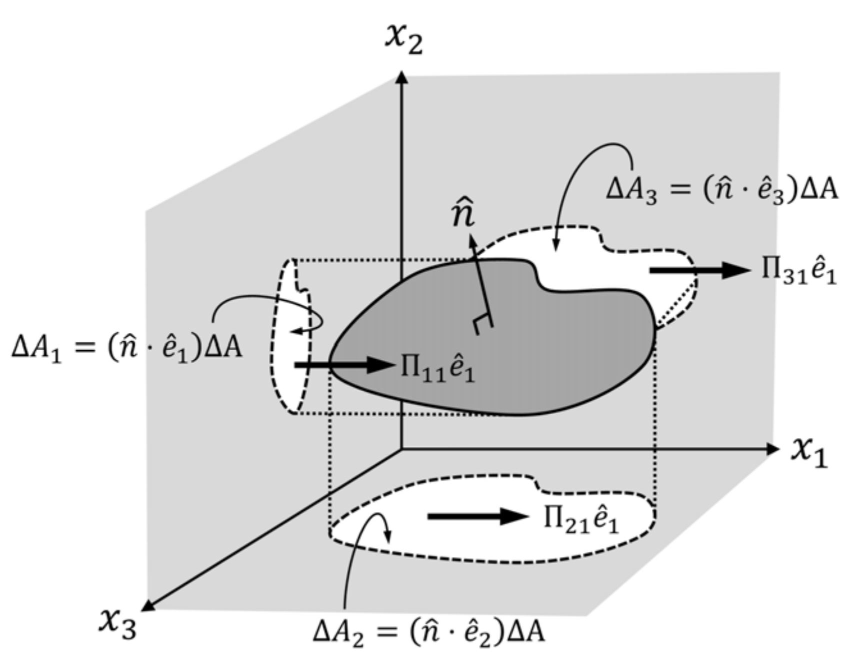

Molecular Forces *(continued)*

f_1, the force in the 1-direction on an arbitrary surface dS is composed of THREE parts.

$$f_1 = \underset{\text{stress}}{\Pi_{11}} \hat{n}\cdot\hat{e}_1\, dS + \Pi_{21}\underbrace{\hat{n}\cdot\hat{e}_2\, dS}_{\text{appropriate area}} + \Pi_{31}\hat{n}\cdot\hat{e}_3\, dS$$

Using the distributive law:

$$\boxed{f_1 = \hat{n}\cdot\left(\Pi_{11}\hat{e}_1 + \Pi_{21}\hat{e}_2 + \Pi_{31}\hat{e}_3\right) dS}$$

Force in the 1-direction on an arbitrary surface dS

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Molecular Forces *(continued)*

The same logic applies in the 2-direction and the 3-direction

$$\boxed{\begin{aligned} f_1 &= \hat{n}\cdot\left(\Pi_{11}\hat{e}_1 + \Pi_{21}\hat{e}_2 + \Pi_{31}\hat{e}_3\right) dS \\ f_2 &= \hat{n}\cdot\left(\Pi_{12}\hat{e}_1 + \Pi_{22}\hat{e}_2 + \Pi_{32}\hat{e}_3\right) dS \\ f_3 &= \hat{n}\cdot\left(\Pi_{13}\hat{e}_1 + \Pi_{23}\hat{e}_2 + \Pi_{33}\hat{e}_3\right) dS \end{aligned}}$$

Assembling the force vector:

$$\begin{aligned} \underline{f} &= f_1\hat{e}_1 + f_2\hat{e}_2 + f_3\hat{e}_3 \\ &= dS\ \hat{n}\cdot\left(\Pi_{11}\hat{e}_1 + \Pi_{21}\hat{e}_2 + \Pi_{31}\hat{e}_3\right)\hat{e}_1 \\ &\quad + dS\ \hat{n}\cdot\left(\Pi_{12}\hat{e}_1 + \Pi_{22}\hat{e}_2 + \Pi_{32}\hat{e}_3\right)\hat{e}_2 \\ &\quad + dS\ \hat{n}\cdot\left(\Pi_{13}\hat{e}_1 + \Pi_{23}\hat{e}_2 + \Pi_{33}\hat{e}_3\right)\hat{e}_3 \end{aligned}$$

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Molecular Forces *(continued)*

Assembling the force vector:

$$\underline{f} = f_1\hat{e}_1 + f_2\hat{e}_2 + f_3\hat{e}_3$$

$$\begin{aligned} &= dS\ \hat{n}\cdot\left(\Pi_{11}\hat{e}_1 + \Pi_{21}\hat{e}_2 + \Pi_{31}\hat{e}_3\right)\hat{e}_1 \\ &\quad + dS\ \hat{n}\cdot\left(\Pi_{12}\hat{e}_1 + \Pi_{22}\hat{e}_2 + \Pi_{32}\hat{e}_3\right)\hat{e}_2 \\ &\quad + dS\ \hat{n}\cdot\left(\Pi_{13}\hat{e}_1 + \Pi_{23}\hat{e}_2 + \Pi_{33}\hat{e}_3\right)\hat{e}_3 \end{aligned}$$

$$= dS\ \hat{n}\cdot\underbrace{\left[\Pi_{11}\hat{e}_1\hat{e}_1 + \Pi_{21}\hat{e}_2\hat{e}_1 + \Pi_{31}\hat{e}_3\hat{e}_1 + \Pi_{12}\hat{e}_1\hat{e}_2 + \Pi_{22}\hat{e}_2\hat{e}_2 + \Pi_{32}\hat{e}_3\hat{e}_2 + \Pi_{13}\hat{e}_1\hat{e}_3 + \Pi_{23}\hat{e}_2\hat{e}_3 + \Pi_{33}\hat{e}_3\hat{e}_3\right]}$$

linear combination of
dyadic products = **tensor**

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Molecular Forces *(continued)*

Assembling the force vector:

$$\begin{aligned} \underline{f} &= dS\ \hat{n}\cdot\left[\Pi_{11}\hat{e}_1\hat{e}_1 + \Pi_{21}\hat{e}_2\hat{e}_1 + \Pi_{31}\hat{e}_3\hat{e}_1 \right.\\ &\qquad + \Pi_{12}\hat{e}_1\hat{e}_2 + \Pi_{22}\hat{e}_2\hat{e}_2 + \Pi_{32}\hat{e}_3\hat{e}_2 \\ &\qquad \left. + \Pi_{13}\hat{e}_1\hat{e}_3 + \Pi_{23}\hat{e}_2\hat{e}_3 + \Pi_{33}\hat{e}_3\hat{e}_3\right] \\ &= dS\ \hat{n}\cdot\sum_{p=1}^{3}\sum_{m=1}^{3}\Pi_{pm}\hat{e}_p\hat{e}_m \\ &= dS\ \hat{n}\cdot\Pi_{pm}\hat{e}_p\hat{e}_m \end{aligned}$$

$$\boxed{\underline{f} = dS\ \hat{n}\cdot\underline{\underline{\Pi}}}$$

Total stress tensor
(molecular stresses)

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Momentum Balance *(continued)*

Polymer Rheology

$$\begin{pmatrix} \textit{rate of increase} \\ \textit{of momentum in } V \end{pmatrix} = \begin{pmatrix} \textit{net flux of} \\ \textit{momentum into } V \end{pmatrix} + \begin{pmatrix} \textit{sum of} \\ \textit{forces on } V \end{pmatrix}$$

$$\iiint_V \frac{\partial}{\partial t}(\rho \underline{v})\, dV = -\iiint_V \nabla \cdot (\rho \underline{v}\underline{v})\, dV + \iiint_V \rho \underline{g}\, dV + \begin{matrix} \textit{molecular} \\ \textit{forces} \end{matrix}$$

$$\boxed{\begin{aligned} \begin{matrix} \textit{molecular} \\ \textit{forces} \end{matrix} &= \iint_S \begin{pmatrix} \textit{molecular} \\ \textit{forces on} \\ dS \end{pmatrix} \\ &= \iint_S \hat{n} \cdot \left(-\underline{\underline{\Pi}}\right) dS \\ &= \iiint_V \nabla \cdot \left(-\underline{\underline{\Pi}}\right) dV \end{aligned}}$$

We use a stress sign convention that requires a negative sign here.

Gauss Divergence Theorem

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Momentum Balance *(continued)*

Polymer Rheology

$$\begin{pmatrix} \textit{rate of increase} \\ \textit{of momentum in } V \end{pmatrix} = \begin{pmatrix} \textit{net flux of} \\ \textit{momentum into } V \end{pmatrix} + \begin{pmatrix} \textit{sum of} \\ \textit{forces on } V \end{pmatrix}$$

$$\iiint_V \frac{\partial}{\partial t}(\rho \underline{v})\, dV = -\iiint_V \nabla \cdot (\rho \underline{v}\underline{v})\, dV + \iiint_V \rho \underline{g}\, dV + \begin{matrix} \textit{molecular} \\ \textit{forces} \end{matrix}$$

$$\boxed{\begin{aligned} \begin{matrix} \textit{molecular} \\ \textit{forces} \end{matrix} &= \iint_S \begin{pmatrix} \textit{molecular} \\ \textit{forces on} \\ dS \end{pmatrix} \\ &= \iint_S \hat{n} \cdot \left(-\underline{\underline{\Pi}}\right) dS \\ &= \iiint_V \nabla \cdot \left(-\underline{\underline{\Pi}}\right) dV \end{aligned}}$$

UR/Bird choice: positive compression (pressure is positive)

Gauss Divergence Theorem

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Momentum Balance *(continued)*

$$\underline{F}_{\substack{on\\surface}} = \iint_S \hat{n}\cdot\left(-\underline{\underline{\Pi}}\right) dS = \iint_S \hat{n}\cdot\left(\underline{\underline{\tilde{\Pi}}}\right) dS$$

$$\Pi_{yx} \qquad \tilde{\Pi}_{yx}$$

UR/Bird choice: fluid at lesser *y* exerts force on fluid at greater *y*

(IFM/Mechanics choice: (opposite)

Momentum Balance *(continued)*

Final Assembly:

$$\begin{pmatrix}\text{rate of increase}\\ \text{of momentum in } V\end{pmatrix} = \begin{pmatrix}\text{net flux of}\\ \text{momentum into } V\end{pmatrix} + \begin{pmatrix}\text{sum of}\\ \text{forces on } V\end{pmatrix}$$

$$\iiint_V \frac{\partial}{\partial t}\left(\rho\underline{v}\right)dV = -\iiint_V \nabla\cdot\left(\rho\underline{v}\,\underline{v}\right)dV + \iiint_V \rho\underline{g}\;dV - \iiint_V \nabla\cdot\underline{\underline{\Pi}}\;dV$$

$$\iiint_V \left[\frac{\partial\rho\underline{v}}{\partial t} + \nabla\cdot\left(\rho\underline{v}\,\underline{v}\right) - \rho\underline{g} + \nabla\cdot\underline{\underline{\Pi}}\right]dV = 0$$

Because *V* is *arbitrary*, we may conclude:

$$\boxed{\frac{\partial\rho\underline{v}}{\partial t} + \nabla\cdot\left(\rho\underline{v}\,\underline{v}\right) - \rho\underline{g} + \nabla\cdot\underline{\underline{\Pi}} = 0}$$

Microscopic momentum balance

Momentum Balance *(continued)*

Polymer Rheology

Microscopic momentum balance

$$\boxed{\frac{\partial \rho \underline{v}}{\partial t} + \nabla \cdot \left(\rho \underline{v}\,\underline{v}\right) - \rho \underline{g} + \nabla \cdot \underline{\underline{\Pi}} = 0}$$

After some rearrangement:

Equation of Motion

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla \cdot \underline{\underline{\Pi}} + \rho \underline{g}$$

$$\rho \frac{D\underline{v}}{Dt} = -\nabla \cdot \underline{\underline{\Pi}} + \rho \underline{g}$$

Now, what to do with $\underline{\underline{\Pi}}$?

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Momentum Balance *(continued)*

Polymer Rheology

Now, what to do with $\underline{\underline{\Pi}}$?

Pressure is part of it.

Pressure

definition: An isotropic force/area of molecular origin. Pressure is the same on any surface drawn through a point and acts normally to the chosen surface.

$$pressure = p\,\underline{\underline{I}} = p\,\hat{e}_1\hat{e}_1 + p\,\hat{e}_2\hat{e}_2 + p\,\hat{e}_3\hat{e}_3 = \begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix}_{123}$$

Test: what is the force on a surface with unit normal $\hat{n}$?

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Momentum Balance *(continued)* Polymer Rheology

back to our question,

Now, what to do with $\underline{\underline{\Pi}}$?

Pressure is part of it.

There are other, nonisotropic stresses

Extra Molecular Stresses

definition: The extra stresses are the molecular stresses that are not isotropic

$$\underbrace{\underline{\underline{\tau}}}_{\text{Extra stress tensor,}} \equiv \underline{\underline{\Pi}} - p\,\underline{\underline{I}}$$

i.e. everything complicated in molecular deformation

Now, what to do with $\underline{\underline{\tau}}$?

This becomes the central question of rheological study

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Momentum Balance *(continued)* Polymer Rheology

Stress sign convention affects any expressions with $\underline{\underline{\Pi}}, \underline{\underline{\tilde{\Pi}}}$ or $\underline{\underline{\tau}}, \underline{\underline{\tilde{\tau}}}$

$$\underline{\underline{\Pi}} \equiv \underline{\underline{\tau}} + p\,\underline{\underline{I}}$$

UR/Bird choice: fluid at lesser *y* exerts force on fluid at greater *y*

$$\underline{\underline{\tilde{\Pi}}} \equiv \underline{\underline{\tilde{\tau}}} - p\,\underline{\underline{I}}$$

(IFM/Mechanics choice: (opposite)

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Constitutive equations for Stress

- are tensor equations
- relate the velocity field to the stresses generated by molecular forces
- are based on observations (empirical) or are based on molecular models (theoretical)
- are typically found by trial-and-error
- are justified by how well they work for a system of interest
- are observed to be symmetric

$$\underline{\underline{\tau}} = f(\nabla \underline{v}, \ material\ properties)$$

Observation: the stress tensor is symmetric

Microscopic momentum balance

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla\cdot\underline{\underline{\Pi}} + \rho\underline{g}$$

Equation of Motion

In terms of the extra stress tensor:

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p - \nabla\cdot\underline{\underline{\tau}} + \rho\underline{g}$$

Equation of Motion

Cauchy Momentum Equation

Components in three coordinate systems (our sign convention):
http://www.chem.mtu.edu/~fmorriso/cm310/Navier2007.pdf

Newtonian Constitutive equation

$$\boxed{\underline{\underline{\tau}} = -\mu\left(\nabla \underline{v} + \left(\nabla \underline{v}\right)^T\right)}$$

- for incompressible fluids (see text for compressible fluids)
- is empirical
- may be justified for some systems with molecular modeling calculations

Note: $\underline{\underline{\tilde{\tau}}} = +\mu\left(\nabla \underline{v} + \left(\nabla \underline{v}\right)^T\right)$

How is the Newtonian Constitutive equation related to Newton's Law of Viscosity?

$$\boxed{\underline{\underline{\tau}} = -\mu\left(\nabla \underline{v} + \left(\nabla \underline{v}\right)^T\right)}$$

- incompressible fluids

$$\boxed{\tau_{21} = -\mu \frac{\partial v_1}{\partial x_2}}$$

- incompressible fluids
- rectilinear flow (straight lines)
- no variation in x_3-direction

Momentum Balance *(continued)* Polymer Rheology

Back to the momentum balance . . .

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p - \nabla\cdot\underline{\underline{\tau}} + \rho\underline{g}$$

Equation of Motion

$$\underline{\underline{\tau}} = -\mu\left(\nabla\underline{v} + (\nabla\underline{v})^T\right)$$

We can incorporate the Newtonian constitutive equation into the momentum balance to obtain a momentum-balance equation that is specific to incompressible, Newtonian fluids

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Momentum Balance *(continued)* Polymer Rheology

Navier-Stokes Equation

$$\boxed{\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \mu\nabla^2\underline{v} + \rho\underline{g}}$$

•incompressible fluids
•Newtonian fluids

Note: The Navier-Stokes is unaffected by the stress sign convention.

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Momentum Balance *(continued)* Polymer Rheology

Next?

Navier-Stokes Equation

$$\boxed{\rho\left(\frac{\partial \underline{v}}{\partial t}+\underline{v}\cdot\nabla\underline{v}\right)=-\nabla p+\mu\nabla^2\underline{v}+\rho\underline{g}}$$

Newtonian Problem Solving

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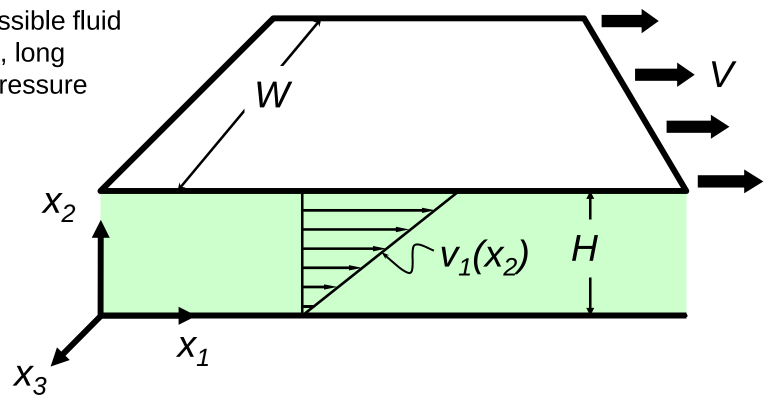
from **QUICK START**

EXAMPLE: Drag flow between infinite parallel plates

$$\underline{v}=\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{123}$$

- Newtonian
- steady state
- incompressible fluid
- very wide, long
- uniform pressure



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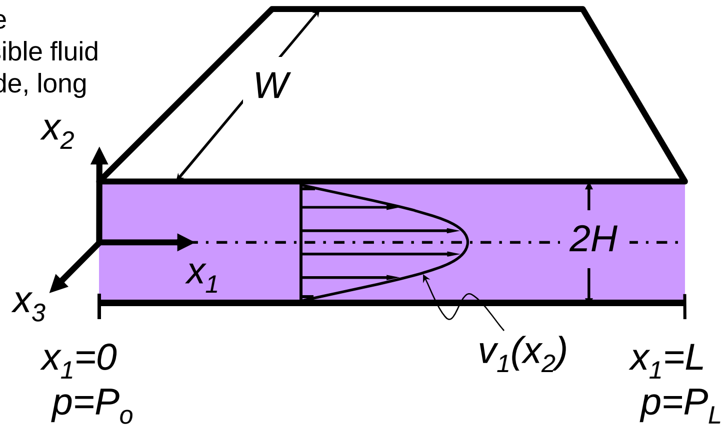
EXAMPLE: Poiseuille flow between infinite parallel plates

- Newtonian
- steady state
- Incompressible fluid
- infinitely wide, long



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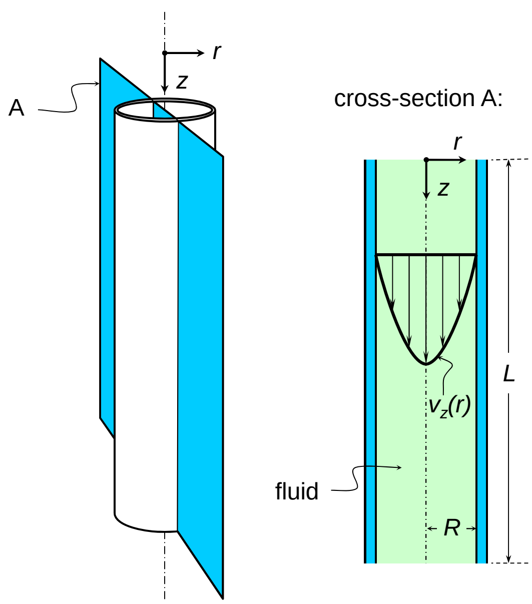
EXAMPLE: Poiseuille flow in a tube

- Newtonian
- Steady state
- incompressible fluid
- long tube



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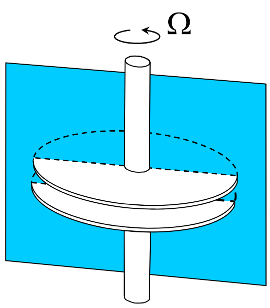
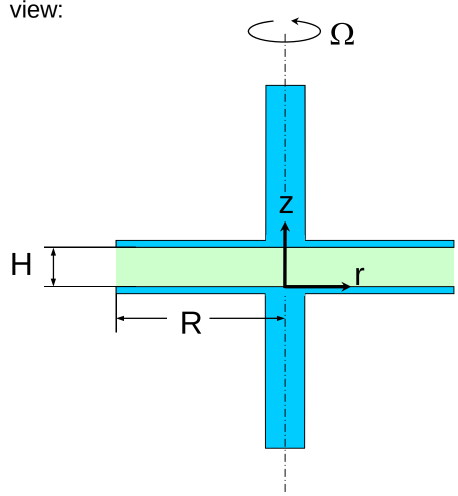
EXAMPLE: Torsional flow between parallel plates

- Newtonian
- Steady state
- incompressible fluid
- $v_\theta = zf(r)$



cross-sectional view:



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