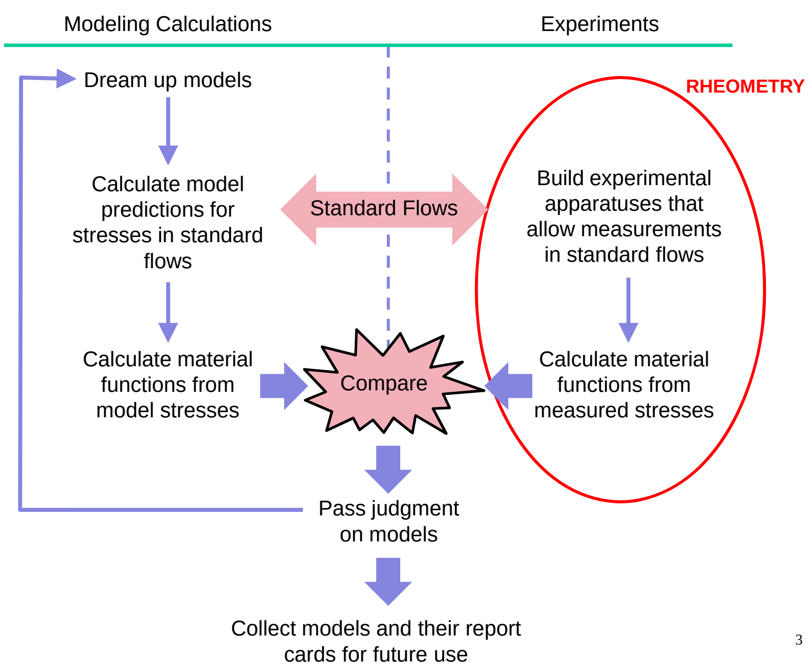
Rheometry (Chapter 10)

measurement

All the comparisons we have discussed require that we somehow measure the material functions on actual fluids.

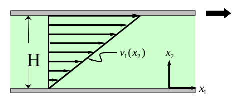
Tactic: Divide the Problem in half



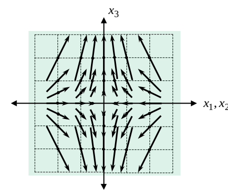
Standard Flows Summary

Choose velocity field: Symmetry of flow alone implies:

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \qquad \underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & \tau_{12} & 0 \\ \tau_{21} & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix}_{123}$$



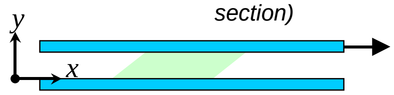
$$\underline{v} = \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)(1+b)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)(1-b)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123} \qquad \underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & 0 & 0 \\ 0 & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix}_{123}$$



To measure the stresses we need for material functions, we must **produce** the defined flows

Simple Shear flow (Drag)

(z-plane section)



$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

Challenges:
- Sample loading
- Maintain parallelism
- Producing linear motion
- Stress measurement (Edge effects)
- Signal strength

From the McGill website (2006): Hee Eon Park, first-year postdoc in Chemical Engineering, works on a high-pressure sliding plate rheometer, the only instrument of its kind in the world.

Interlaken Technology

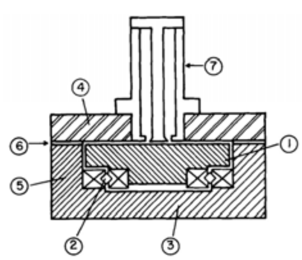


Fig. 2. Cross section of the sliding plate rheometer, showing the moving plate [1], linear bearing [2], back plate, [3], stationary plate [4], side supports [5], and shims [6]. (Details, e.g. such as assembly bolts, etc. not shown.)

J. M. Dealy and S. S. Soong "A Parallel Plate Melt Rheometer Incorporating a Shear Stress Transducer,"J. Rheol. 28, 355 (1984)

Although we stipulated simple, homogeneous shear flow be produced throughout the flow domain, can we, perhaps, relax that requirement?

$$\underline{v} \equiv \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$$

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Viscometric flow: motions that are locally equivalent to steady simple shearing motion at every particle

- •globally steady with respect to some frame of reference
- •streamlines that are straight, circular, or helical
- •each flow can be visualized as the relative motion of a sheaf of material surfaces (slip surfaces)
- •each slip surface moves without changing shape during the motion
- •every particle lies on a material surface that moves without stretching (inextensible slip surfaces)

Viscometric Flows:
1. Steady tube flow
2. Steady tangential annular flow
3. Steady torsional flow (parallel plate flow)
4. Steady cone-and-plate flow (small cone angle)
5. Steady helical flow

Wan-Lee Yin, Allen C. Pipkin, "Kinematics of viscometric flow," *Archive for Rational Mechanics and Analysis,* 37(2) 111-135, 1970
R. B. Bird, R. Armstrong, O. Hassager, *Dynamics of Polymeric Liquids*, 2nd edition, Wiley (1986), section 3.7.

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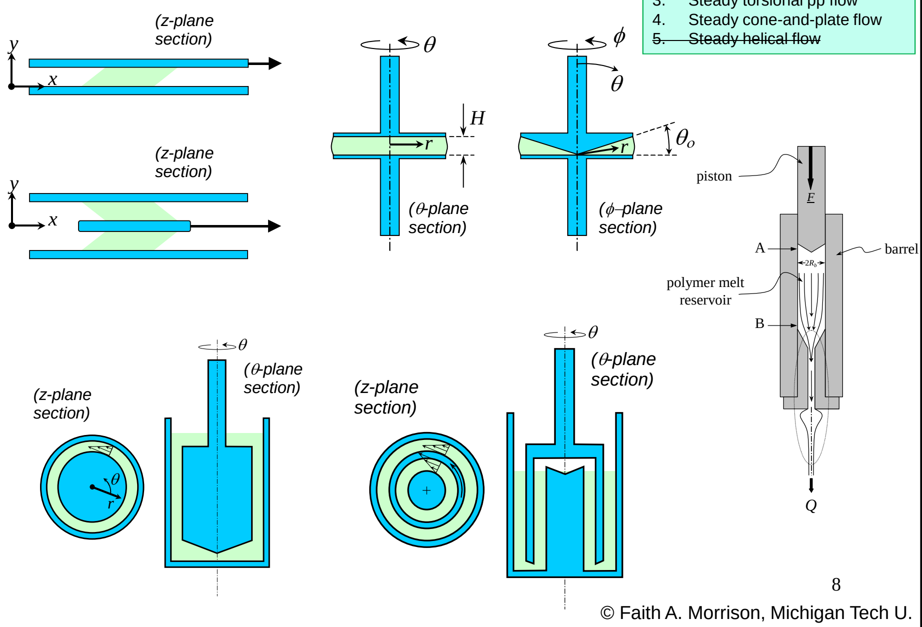
Experimental Shear Geometries (viscometric flows)

Viscometric Flows:
1. Steady tube flow
2. Steady tangential annular flow
3. Steady torsional pp flow
4. Steady cone-and-plate flow
5. ~~Steady helical flow~~



Types of Shear Rheometry

Mechanical:

•Mechanically produce **linear drag flow**;
Measure (shear strain transducer):
Shear stress on a surface

1. planar Couette

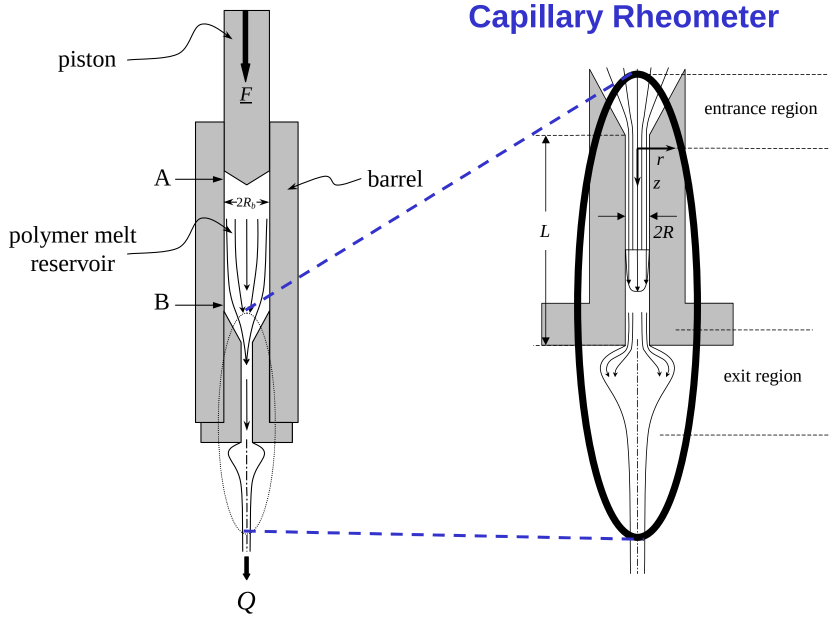
•Mechanically produce **torsional drag flow**;
Measure: (strain-gauge; force rebalance)
Torque to rotate surfaces
Back out material functions

1. cone and plate;
2. parallel plate;
3. circular Couette

•Produce **pressure-driven flow** through conduit
Measure:
Pressure drop/flow rate
Back out material functions

1. capillary flow
2. slit flow

Capillary Rheometer



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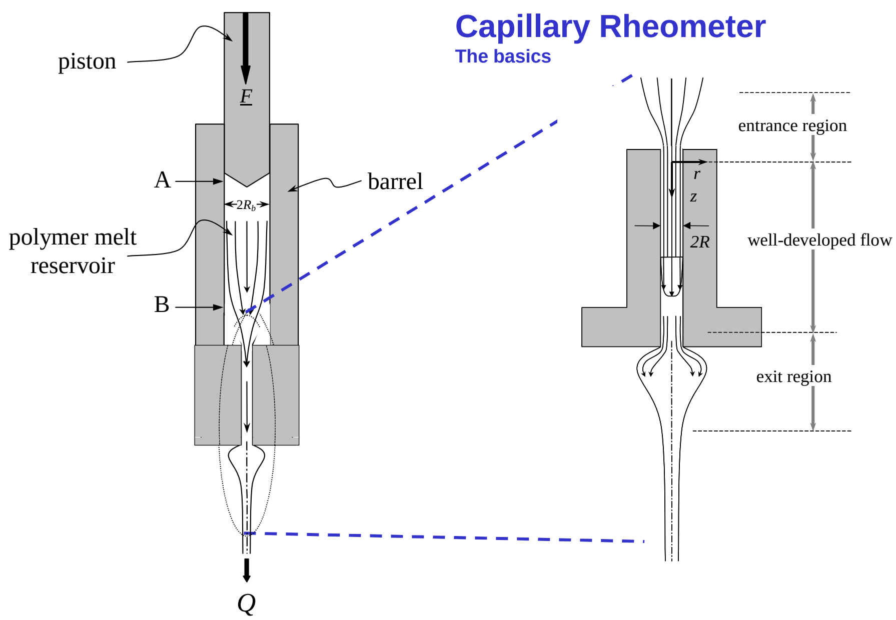
Capillary Rheometer

The basics



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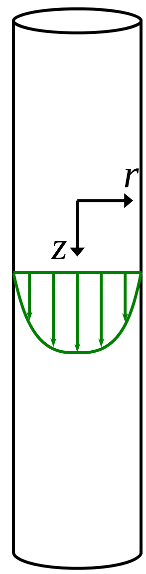
Exercise:

- What is the shear stress in capillary flow, for a fluid with unknown constitutive equation?
- What is the shear rate in capillary flow?



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To calculate shear rate, shear stress, look at EOM:

$$\eta = \frac{\tau_R}{\dot{\gamma}_R}$$

$$P \equiv p - \rho g z$$

$$\rho\left(\underbrace{\cancel{\frac{\partial \underline{v}}{\partial t}}}_{\text{steady state}} + \underbrace{\cancel{\underline{v} \cdot \nabla \underline{v}}}_{\text{unidirectional}}\right) = -\nabla P - \nabla \cdot \underline{\underline{\tau}}$$

Assume:
- Incompressible fluid
- no θ-dependence
- long tube
- symmetric stress tensor
- Isothermal
- Viscosity indep of pressure

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{r\theta z} = \begin{pmatrix} -\frac{\partial P}{\partial r} \\ 0 \\ -\frac{\partial P}{\partial z} \end{pmatrix}_{r\theta z} - \begin{pmatrix} \frac{1}{r}\frac{\partial r\tau_{rr}}{\partial r} - \frac{\tau_{\theta\theta}}{r} \\ \frac{1}{r^2}\frac{\partial r^2 \tau_{r\theta}}{\partial r} \\ \frac{1}{r}\frac{\partial r \tau_{rz}}{\partial r} \end{pmatrix}_{r\theta z}$$

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Shear stress in capillary flow:

$$\boxed{\tau_{rz} = \frac{(P_0 - P_L)r}{2L} = \tau_R \frac{r}{R}} \qquad \left(\frac{\partial P}{\partial z} = \text{constant}\right)$$

(varies with position, i.e. inhomogeneous flow)

What was the shear stress in drag flow?

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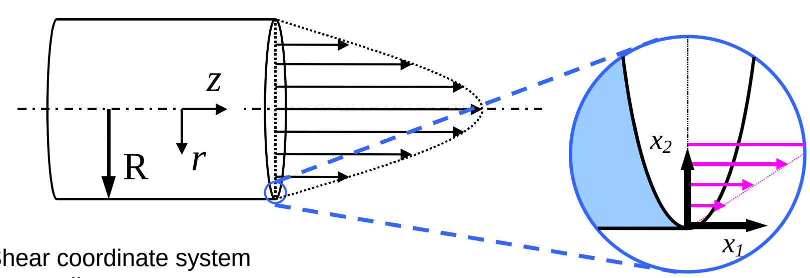
Viscosity from capillary flow – inhomogeneous shear flow



Shear coordinate system near wall:

$$\hat{e}_1 = \hat{e}_z$$
$$\hat{e}_2 = -\hat{e}_r$$
$$\hat{e}_3 = -\hat{e}_\theta$$

$$\tau_{21} = -\tau_{rz}\big|_{r=R} \equiv -\tau_R$$

$$\dot{\gamma}_0 = \frac{\partial v_z}{\partial(-r)} = -\frac{\partial v_z}{\partial r} = \dot{\gamma}\big|_{r=R} \equiv \dot{\gamma}_R$$

$$\boxed{\eta = \frac{-\tau_{21}}{\dot{\gamma}_0} = \frac{\tau_R}{\dot{\gamma}_R}}$$

wall shear stress

wall shear rate

It is not the same shear rate everywhere, but if we focus on the wall we can still get $\eta(\gamma_R)$

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Viscosity from Wall Stress/Shear rate

Note: we are assuming no-slip at the wall

Wall shear stress in capillary flow:

$$\boxed{\tau_{rz}\Big|_{r=R} = \frac{(P_0 - P_L)r}{2L}\Bigg|_{r=R} = \frac{\Delta P R}{2L}}$$

$$\left(\frac{\partial P}{\partial z} = \text{constant}\right)$$

What is shear rate at the wall in capillary flow?

$$\dot{\gamma} = \frac{\partial v_z}{\partial(-r)} = -\frac{\partial v_z}{\partial r} = \dot{\gamma}\Big|_{r=R} \equiv \dot{\gamma}_R$$

If $v_z(r)$ is known, this is easy to calculate.

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Velocity fields, Flow in a Capillary

Newtonian fluid:

$$v_z(r) = \frac{2Q}{\pi R^2}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Power-law GNF fluid:

$$v_z(r) = R^{\frac{1}{n}+1}\left(\frac{P_0 - P_L}{2mL}\right)^{\frac{1}{n}}\left(\frac{1}{1/n+1}\right)\left[1 - \left(\frac{r}{R}\right)^{\frac{1}{n}+1}\right]$$

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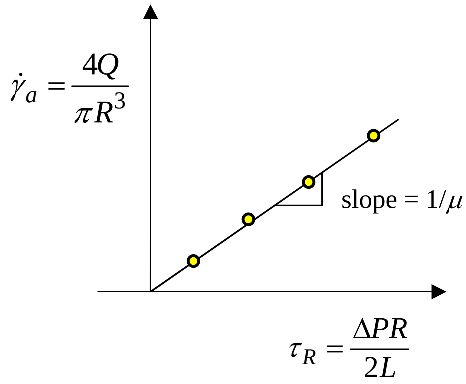
Wall shear-rate for a Newtonian fluid

Hagen-Poiseuille:

$$Q = \frac{\pi \Delta P R^4}{8 \mu L}$$

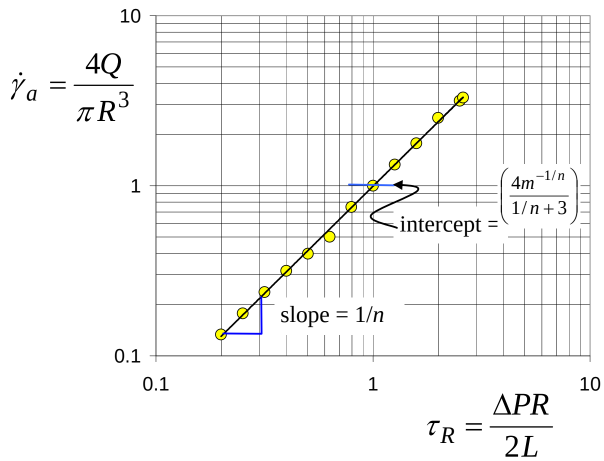
$$\frac{4Q}{\pi R^3} = \frac{1}{\mu}\frac{\Delta P R}{2L}$$



Wall shear-rate for a Power-law GNF

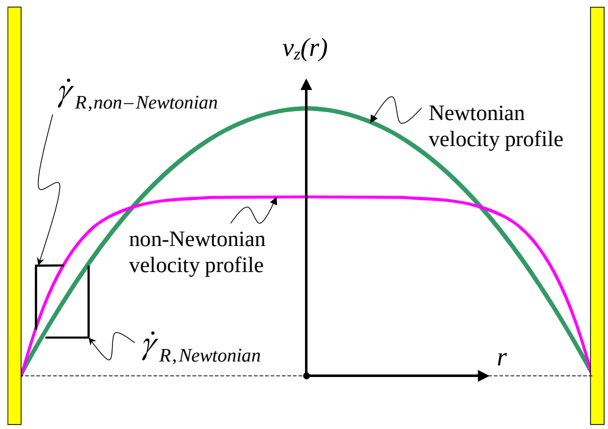
PL-GNF flow rate:

$$Q = \left(\frac{\Delta P R}{2L}\right)^{\frac{1}{n}} \frac{1}{m^{1/n}} \frac{n \pi R^3}{1+3n}$$



For an unknown, non-Newtonian fluid, we need to take special steps to determine the wall shear rate

The wall shear rate is generally greater than for a Newtonian fluid.



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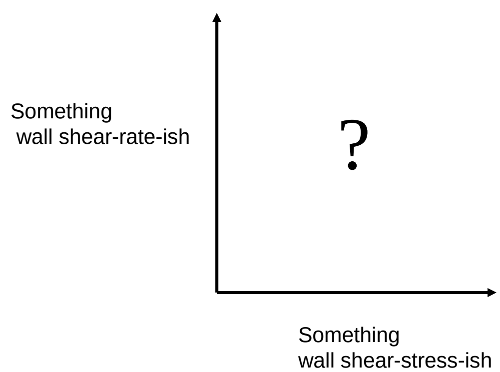
For a General non-Newtonian fluid

$$Q = ?$$



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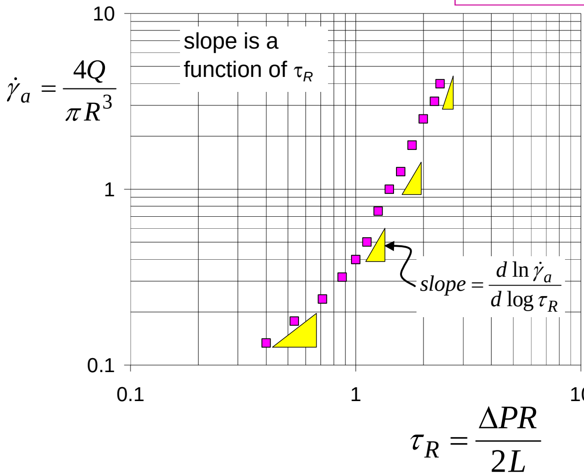
Weissenberg-Rabinowitsch correction

$$\dot{\gamma}_R(\tau_R) = \frac{4Q}{\pi R^3}\left[\frac{1}{4}\left(3 + \frac{d\ln\dot{\gamma}_a}{d\ln\tau_R}\right)\right]$$



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Capillary flow

Assumptions:

- •Steady……………………….•No intermittent flow allowed
- •θ symmetry……………………•No spiraling flow allowed
- •Unidirectional ………………….•Check end effects
- •Incompressible……………….•Avoid high absolute pressures
- •Constant pressure gradient…•Check end effects
- •No slip……………………….•Check wall slip

Methods have been devised to account for
•Slip
•End effects

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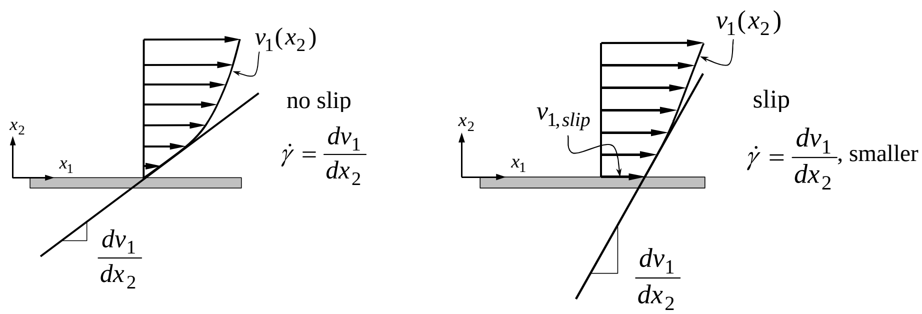
Slip at the wall - Mooney analysis

Slip at the wall reduces the shear rate near the wall.



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Slip at the wall - Mooney analysis

Slip at the wall reduces the shear rate near the wall.

$$v_{z,true} = v_{z,measured} - v_{z,slip}$$

$$v_{z,av} = \frac{Q}{\pi R^2}$$

$$\frac{4v_{z,av}}{R} = \frac{4Q}{\pi R^3} = \dot{\gamma}_a$$

$$\dot{\gamma}_{a,slip-corrected} \equiv \frac{4v_{z,av}}{R} - \frac{4v_{z,slip}}{R}$$

$$\frac{4v_{z,av}}{R} = \underbrace{4v_{z,slip}}_{\text{slope}}\left(\frac{1}{R}\right) + \dot{\gamma}_{a,slip-corrected}$$

intercept

$$\frac{4v_{z,av}}{R} = \frac{4Q_{measured}}{\pi R^3}$$

At constant wall shear stress, take data in capillaries of various R

The Mooney correction is a correction to the apparent shear rate

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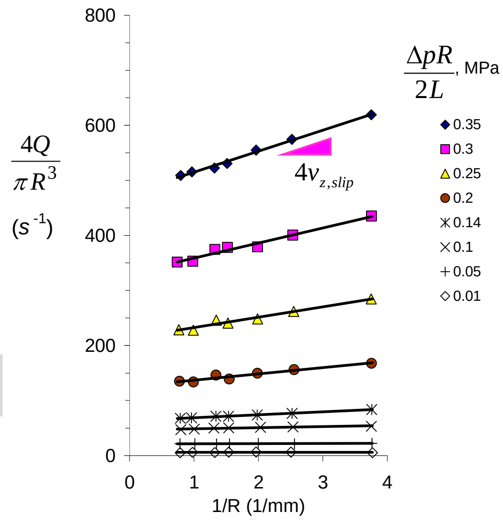
Slip at the wall - Mooney analysis



Note: each line is at constant wall shear stress (data may need to be interpolated to meet this requirement).

Turns out there is slip sometimes!

Figure 10.10, p. 396 Ramamurthy, LLDPE

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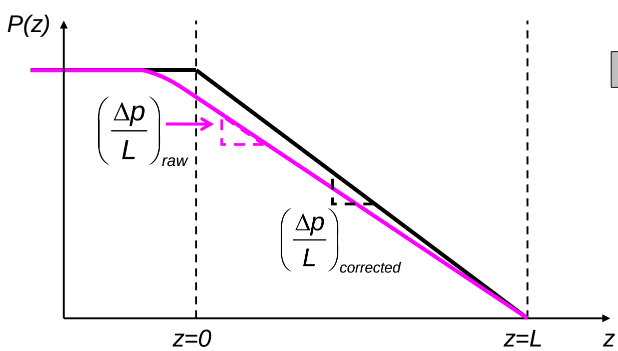
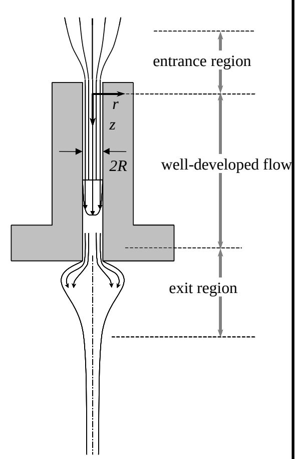
Entrance and exit effects - Bagley correction

The pressure gradient is not accurately represented by the raw pressure drop: $\frac{P_{reservoir} - P_{atm}}{L}$





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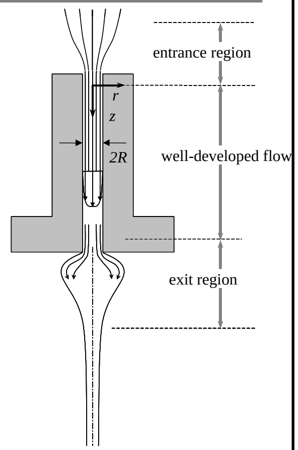
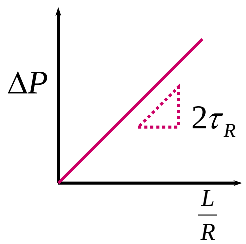
Entrance and exit effects - Bagley correction

$$\tau_R = \frac{\Delta P R}{2L} \Rightarrow \quad \Delta P = \left(2\tau_R\right)\frac{L}{R}$$

Constant at constant Q

Run for different capillaries





This is the result when the end effects are negligible.

The Bagley correction is a correction to the wall shear stress

Bagley Plot

$$\Delta P_{\substack{end\\effects}} = f(Q) = f(\dot{\gamma}_a)$$

Turns out that end effects are visible sometimes!

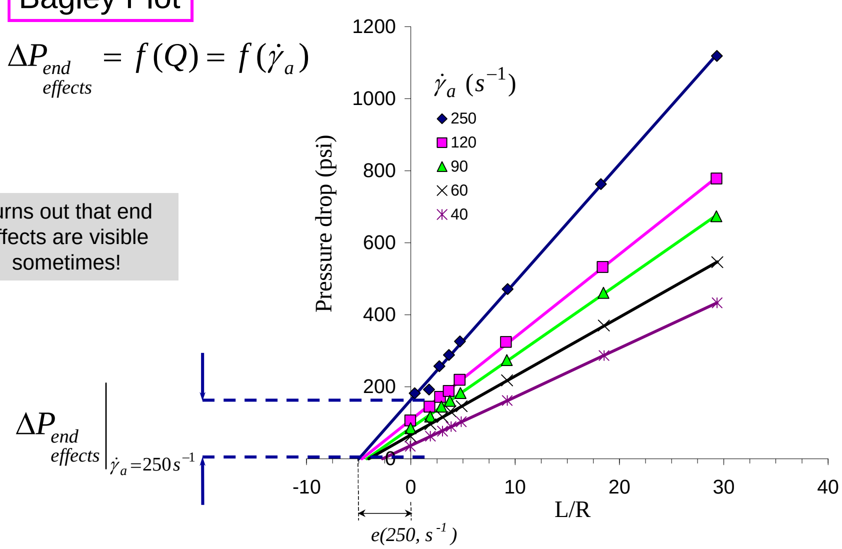
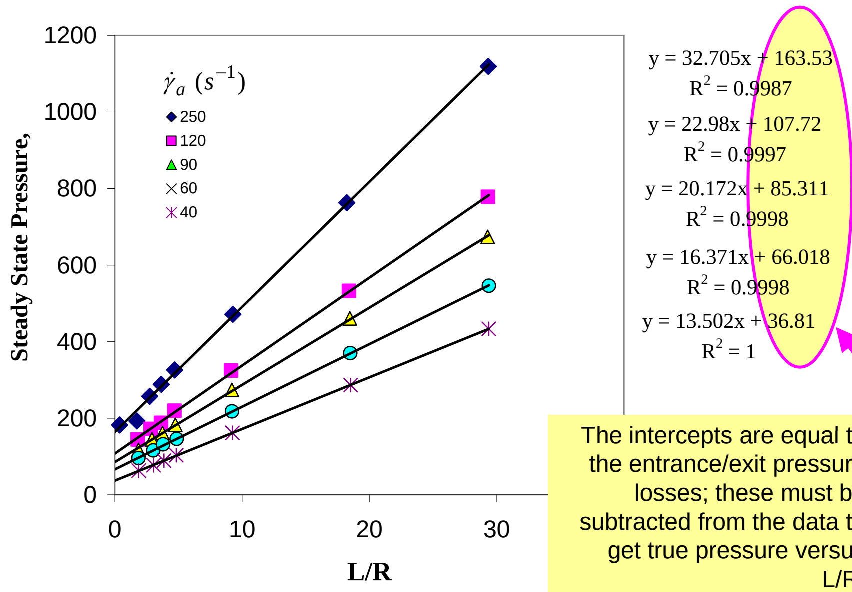


Figure 10.8, p. 394 Bagley, PE



The intercepts are equal to the entrance/exit pressure losses; these must be subtracted from the data to get true pressure versus L/R.

Figure 10.8, p. 394 Bagley, PE

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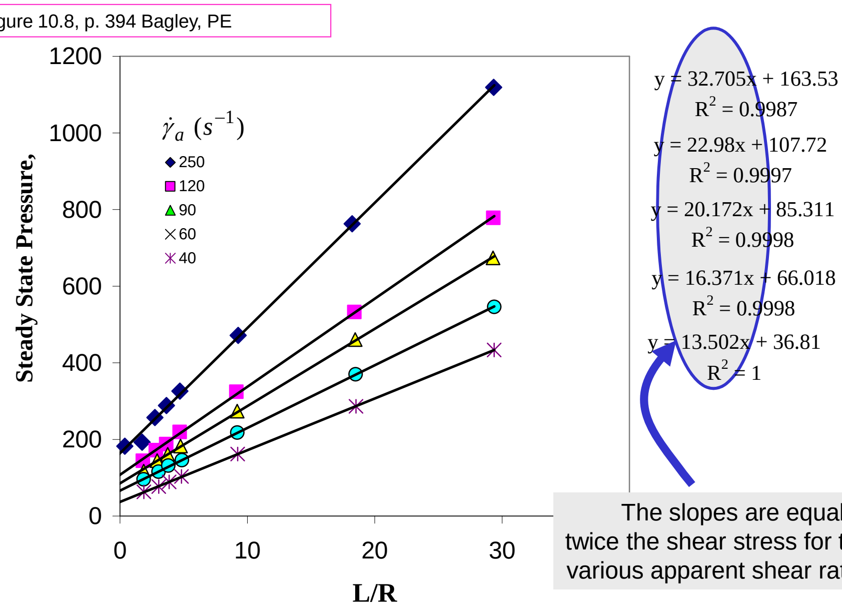
Figure 10.8, p. 394 Bagley, PE



The slopes are equal to twice the shear stress for the various apparent shear rates

$$\tau_{rz} = \frac{\Delta p R}{2L} \quad \Rightarrow \quad \Delta p = 2\tau_{rz}\left(\frac{L}{R}\right)$$

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The data so far:

$\dot{\gamma}_a$ gammdotA (1/s)	ΔP_{ent} deltPent psi	slope psi	τ_R sh stress psi	τ_R sh stress Pa
250	163.53	32.705	16.3525	1.1275E+05
120	107.72	22.98	11.49	7.9220E+04
90	85.311	20.172	10.086	6.9540E+04
60	66.018	16.371	8.1855	5.6437E+04
40	36.81	13.502	6.751	4.6546E+04

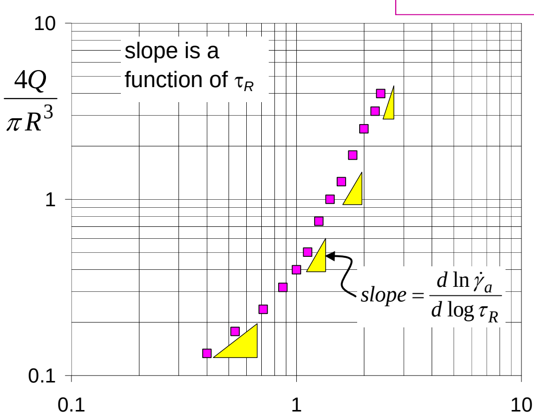
Now, turn apparent shear rate into wall shear rate (correct for non-parabolic velocity profile).

Figure 10.8, p. 394 Bagley, PE

Weissenberg-Rabinowitsch correction

$$\dot{\gamma}_R(\tau_R) = \frac{4Q}{\pi R^3}\left[\frac{1}{4}\left(3 + \frac{d\ln\dot{\gamma}_a}{d\ln\tau_R}\right)\right]$$

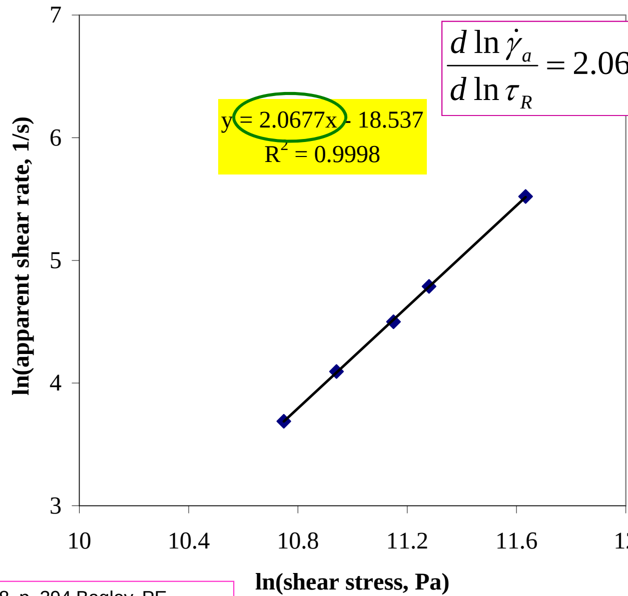
$$\dot{\gamma}_a = \frac{4Q}{\pi R^3}$$



$$\tau_R = \frac{\Delta P R}{2L}$$

Sometimes the WR correction varies from point-to-point; sometimes it is a constant that applies to all data points (PL region).

Weissenberg-Rabinowitsch correction



$$\frac{d\ln\dot{\gamma}_a}{d\ln\tau_R} = 2.0677$$

This slope is usually >1

Figure 10.8, p. 394 Bagley, PE

The data corrected for entrance/exit and non-parabolic velocity profile:

$$\eta = \frac{\tau_R}{\dot{\gamma}_R}$$

$\dot{\gamma}_a$	ΔP_{ent}	ΔP_{ent}	τ_R				$\dot{\gamma}_R$	η
gammdotA (1/s)	deltPent psi	deltPent Pa	sh stress Pa	ln(sh st)	ln(gda)	WR correction	gam-dotR 1/s	viscosity Pa s
250	163.53	1.1275E+06	1.1275E+05	11.63289389	5.521460918	2.0677	316.73125	3.5597E+02
120	107.72	7.4270E+05	7.9220E+04	11.2799902	4.787491743	2.0677	152.031	5.2108E+02
90	85.311	5.8820E+05	6.9540E+04	11.14966143	4.49980967	2.0677	114.02325	6.0988E+02
60	66.018	4.5518E+05	5.6437E+04	10.9408774	4.094344562	2.0677	76.0155	7.4244E+02
40	36.81	2.5380E+05	4.6546E+04	10.74820375	3.688879454	2.0677	50.677	9.1849E+02

Now, plot viscosity versus wall-shear-rate

Figure 10.8, p. 394 Bagley, PE

Viscosity of polyethylene from Bagley's data

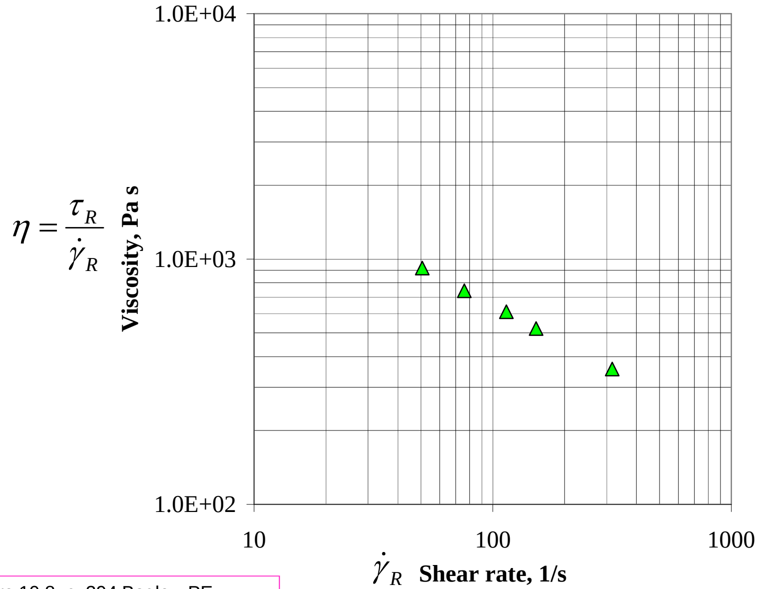


Figure 10.8, p. 394 Bagley, PE

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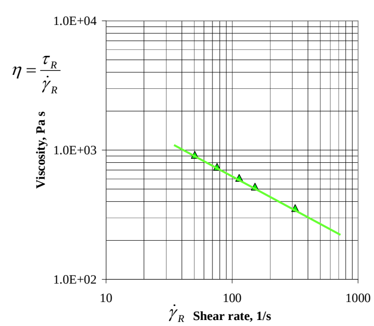
Viscosity from Capillary Experiments, Summary:

1. Take data of pressure-drop versus flow rate for capillaries of various lengths; perform Bagley correction on Δp (entrance pressure losses)
2. If possible, also take data for capillaries of different radii; perform Mooney correction on Q (slip)
3. Perform the Weissenberg-Rabinowitsch correction (obtain correct wall shear rate)
4. Plot true viscosity versus true wall shear rate
5. Calculate power-law *m*, *n* from fit to final data (if appropriate)

raw data: $\Delta P(Q)$

final data: $\eta = \tau_R / \dot{\gamma}_R$



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