Plm 3.15 (continued)

Flow rate:

$$Q = \int_{-W/2}^{W/2}\int_{-H}^{H} v_1(x_2)\,dx_2\,dx_3$$

$$= 4\int_0^{W/2}\int_0^{H} v_1(x_2)\,dx_2\,dx_3$$

$$= 2W\int_0^{H} v_1(x_2)\,dx_2$$

$$v_1 = \underbrace{\left(\frac{P_L - P_0}{L} + \rho g \sin\alpha\right)\frac{H^2}{2\mu}}_{A}\left(\left(\frac{x_2}{H}\right)^2 - 1\right)$$

$$Q = 2WH\int_0^{H} A\left[\left(\frac{x_2}{H}\right)^2 - 1\right]\frac{dx_2}{H}$$

$$= 2WHA\left(\left(\frac{x_2}{H}\right)^3\frac{1}{3} - \frac{x_2}{H}\right)_0^H$$

$$= 2WHA\left[\underbrace{\frac{1}{3} - 1}_{-\frac{2}{3}}\right]$$

$$Q = -\frac{\cancel{4}^{\,2}}{3} WH \frac{H^2}{\cancel{2}\mu}\left(\frac{P_L - P_0}{L} + \rho g \sin\alpha\right)$$

$$\boxed{Q = -\frac{2WH^3}{3\mu}\left(\frac{P_L - P_0}{L} + \rho g \sin\alpha\right)}$$

Check: for horizontal plates, $\sin\alpha = 0$

$$Q = -\frac{2WH^3}{3\mu}\left(\frac{P_L - P_0}{L}\right)$$

✓ $$Q = \frac{2WH^3(P_0 - P_L)}{3\mu L}$$ eqn 3.181 //