Exam 1 Formulas

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Rate of deformation tensor: $\underline{\underline{\dot{\gamma}}} = \nabla \underline{v} + (\nabla \underline{v})^T$

Rate of deformation: $\dot{\gamma} = \left|\underline{\underline{\dot{\gamma}}}\right|$

Tensor magnitude: $A = \left|\underline{\underline{A}}\right| = +\sqrt{\frac{\underline{\underline{A}}:\underline{\underline{A}}}{2}}$

Total stress tensor: $\underline{\underline{\Pi}} = p\underline{\underline{I}} + \underline{\underline{\tau}}$
(Bird, UR sign convention on stress)

Navier-Stokes Equation: $\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \mu\nabla^2\underline{v} + \rho\underline{g}$

Cauchy Momentum Equation: $\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p - \nabla\cdot\underline{\underline{\tau}} + \rho\underline{g}$
(Bird, UR sign convention on stress)

Continuity Equation: $\frac{\partial \rho}{\partial t} = -\nabla\cdot(\rho\underline{v})$

Newtonian, incompressible constitutive equation: $\underline{\underline{\tau}} = -\mu(\nabla\underline{v} + (\nabla\underline{v})^T)$
(Bird sign convention on stress)

Fluid force $\underline{F}_{on}$ on a surface **S**:
(Bird, UR sign convention on stress)

$$\underline{F}_{on} = \iint_S \left[\hat{n}\cdot -\underline{\underline{\Pi}}\right]\Big|_{surface} dS$$

Flow rate Q through a surface **S**:

$$Q = \iint_S [\hat{n}\cdot\underline{v}]_{surface} dS$$

Fluid torque $\underline{T}_{on}$ on a surface **S**: ($\underline{\tilde{R}}$ is the lever arm vector from the axis of rotation to the point of application of the force)
(Bird, UR sign convention on stress)

$$\underline{T}_{on} = \iint_S \left[\underline{\tilde{R}} \times (\hat{n}\cdot -\underline{\underline{\Pi}})\right]_{surface} dS$$

Table of Integrals

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \quad n \text{ is a constant}$$

$$\int \frac{1}{u} du = \ln u + C$$

$$\int (\ln u) du = u \ln u - u + C$$

$$\int_a^b u\, dv = uv\Big|_a^b - \int_a^b v\, du$$

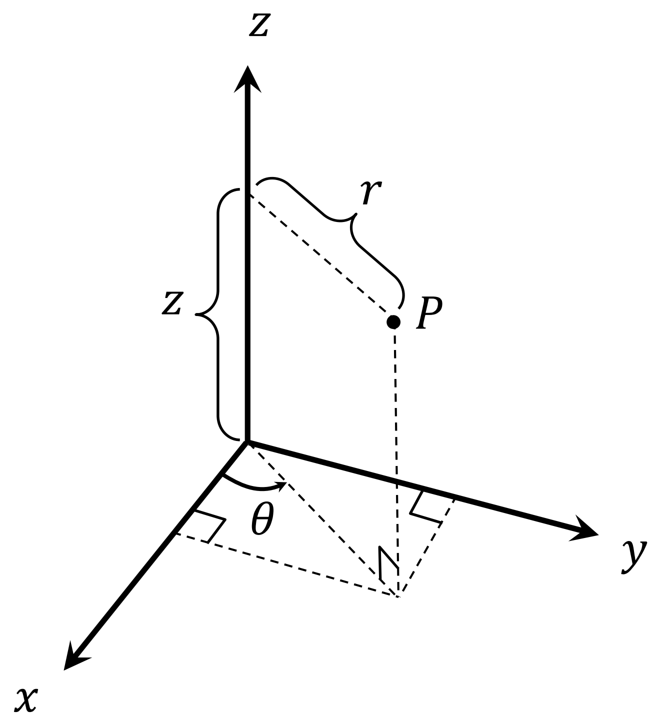
Miscellaneous

$$\underline{\underline{A}} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}_{123}$$

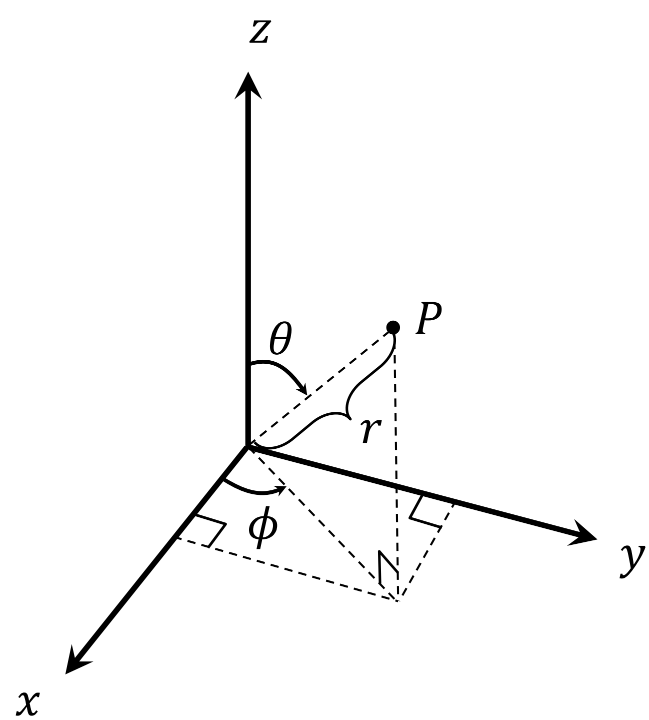
$$\frac{d}{ds}(uw) = u\frac{dw}{ds} + w\frac{du}{ds}$$

$$\underline{u} \times \underline{w} = \det \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = \begin{pmatrix} u_2 w_3 - u_3 w_2 \\ -(u_1 w_3 - u_3 w_1) \\ u_1 w_2 - u_2 w_1 \end{pmatrix}_{123}$$

Cylindrical Coordinate System: Note that the $\boldsymbol{\theta}$-coordinate swings around the $\boldsymbol{z}$-axis and the $\boldsymbol{r}$-coordinate is perpendicular to the z-axis.



Spherical Coordinate System: Note that the $\boldsymbol{\theta}$-coordinate swings <u>down</u> from the $\boldsymbol{z}$-axis and the $\boldsymbol{r}$-coordinate emits radially from the origin to the point; these are different from their definitions in the cylindrical system above.



Cylindrical Coordinates

System	Coordinates	Basis vectors
Cylindrical	$\boldsymbol{r = \sqrt{x^2 + y^2}}$	$\boldsymbol{\hat{e}_r = \cos\theta\,\hat{e}_x + \sin\theta\,\hat{e}_y}$
Cylindrical	$\boldsymbol{\theta = \tan^{-1}\left(\frac{y}{x}\right)}$	$\boldsymbol{\hat{e}_\theta = (-\sin\theta)\hat{e}_x + \cos\theta\,\hat{e}_y}$
Cylindrical	$\boldsymbol{z = z}$	$\boldsymbol{\hat{e}_z = \hat{e}_z}$
Cylindrical	$\boldsymbol{x = r\cos\theta}$	$\boldsymbol{\hat{e}_x = \cos\theta\,\hat{e}_r + (-\sin\theta)\hat{e}_\theta}$
Cylindrical	$\boldsymbol{y = r\sin\theta}$	$\boldsymbol{\hat{e}_y = \sin\theta\,\hat{e}_r + \cos\theta\,\hat{e}_\theta}$
Cylindrical	$\boldsymbol{z = z}$	$\boldsymbol{\hat{e}_z = \hat{e}_z}$

Spherical Coordinates

System	Coordinates	Basis vectors
Spherical	$\boldsymbol{x = r\sin\theta\cos\phi}$	$\boldsymbol{\hat{e}_x = (\sin\theta\cos\phi)\hat{e}_r + (\cos\theta\cos\phi)\hat{e}_\theta + (-\sin\phi\,)\,\hat{e}_\phi}$
Spherical	$\boldsymbol{y = r\sin\theta\sin\phi}$	$\boldsymbol{\hat{e}_y = (\sin\theta\sin\phi)\hat{e}_r + (\cos\theta\sin\phi)\hat{e}_\theta + \cos\phi\,\hat{e}_\phi}$
Spherical	$\boldsymbol{z = r\cos\theta}$	$\boldsymbol{\hat{e}_z = \cos\theta\;\hat{e}_r + (-\sin\theta)\hat{e}_\theta}$
Spherical	$\boldsymbol{r = \sqrt{x^2 + y^2 + z^2}}$	$\boldsymbol{\hat{e}_r = (\sin\theta\cos\phi)\hat{e}_x + (\sin\theta\sin\phi)\hat{e}_y + \cos\theta\hat{e}_z}$
Spherical	$\boldsymbol{\theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right)}$	$\boldsymbol{\hat{e}_\theta = (\cos\theta\cos\phi)\hat{e}_x + (\cos\theta\sin\phi)\hat{e}_y + (-\sin\theta)\hat{e}_z}$
Spherical	$\boldsymbol{\phi = \tan^{-1}\left(\frac{y}{x}\right)}$	$\boldsymbol{\hat{e}_\phi = (-\sin\phi)\hat{e}_x + \cos\phi\,\hat{e}_y}$

Coordinate system	surface differential dS
Cartesian (top, $\hat{n} = \hat{e}_z$)	$dS = dxdy$
Cartesian (side a, $\hat{n} = \hat{e}_y$)	$dS = dxdz$
Cartesian (side b, $\hat{n} = \hat{e}_x$)	$dS = dydz$
cylindrical (top, $\hat{n} = \hat{e}_z$)	$dS = rdrd\theta$
cylindrical (side, $\hat{n} = \hat{e}_r$)	$dS = Rd\theta dz$
spherical, ($\hat{n} = \hat{e}_r$)	$dS = R^2 \sin\theta d\theta d\phi$

Coordinate system	volume differential dV
Cartesian	$dV = dxdydz$
cylindrical	$dV = rdrd\theta dz$
spherical	$dV = r^2 \sin\theta drd\theta d\phi$