Exam 2 Formulas

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Rate of deformation tensor: $\underline{\underline{\dot{\boldsymbol{\gamma}}}} = \nabla \underline{\boldsymbol{v}} + (\nabla \underline{\boldsymbol{v}})^{\boldsymbol{T}}$

Rate of deformation: $\dot{\boldsymbol{\gamma}} = \left|\underline{\underline{\dot{\boldsymbol{\gamma}}}}\right|$

Tensor magnitude: $\boldsymbol{A} = \left|\underline{\underline{\boldsymbol{A}}}\right| = \left|\sqrt{\frac{\underline{\underline{\boldsymbol{A}}}:\underline{\underline{\boldsymbol{A}}}}{\boldsymbol{2}}}\right|$

Total stress tensor: $\underline{\underline{\boldsymbol{\Pi}}} = \boldsymbol{p}\underline{\underline{\boldsymbol{I}}} + \underline{\underline{\boldsymbol{\tau}}}$
(Bird, UR sign convention on stress)

Shear strain: $\gamma_{21}(t_a, t_b) = \int_{t_a}^{t_b} \dot{\gamma}_{21}(t'')dt''$

(Bird, UR sign convention on stress)

Newtonian, incompressible fluid: $\underline{\underline{\boldsymbol{\tau}}} = -\boldsymbol{\mu}(\nabla \underline{\boldsymbol{v}} + (\nabla \underline{\boldsymbol{v}})^{\boldsymbol{T}})$

Generalized Newtonian fluid (GNF): $\underline{\underline{\boldsymbol{\tau}}} = -\boldsymbol{\eta}(\dot{\boldsymbol{\gamma}})\underline{\underline{\dot{\boldsymbol{\gamma}}}}$

Power-law GNF model: $\boldsymbol{\eta}(\dot{\boldsymbol{\gamma}}) = \boldsymbol{m}\dot{\boldsymbol{\gamma}}^{\boldsymbol{n-1}}$
(Note that m and n are parameters of the model and are constants)

Carreau-Yasuda GNF model: $\boldsymbol{\eta}(\dot{\boldsymbol{\gamma}}) = \boldsymbol{\eta}_\infty + (\boldsymbol{\eta}_{\boldsymbol{0}} - \boldsymbol{\eta}_\infty)[\boldsymbol{1} + (\dot{\boldsymbol{\gamma}}\boldsymbol{\lambda})^{\boldsymbol{a}}]^{\frac{\boldsymbol{n-1}}{\boldsymbol{a}}}$
(Note that a, λ, n, η_0, and η_∞ are parameters of the model and are constants)

Navier-Stokes Equation: $\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p + \mu \nabla^2 \underline{v} + \rho \underline{g}$

Cauchy Momentum Equation: $\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$
(Bird, UR sign convention on stress)

Continuity Equation: $\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \underline{v})$

Fluid force $\underline{\boldsymbol{F}}_{\boldsymbol{on}}$ on a surface **S**:
(Bird, UR sign convention on stress)

$$\underline{\boldsymbol{F}}_{\boldsymbol{on}} = \iint_{\boldsymbol{S}} [\hat{\boldsymbol{n}} \cdot -\underline{\underline{\boldsymbol{\Pi}}}]\big|_{\boldsymbol{surface}} \boldsymbol{dS}$$

Flow rate $\boldsymbol{Q}$ through a surface **S**:

$$\boldsymbol{Q} = \iint_{\boldsymbol{S}} [\hat{\boldsymbol{n}} \cdot \underline{\boldsymbol{v}}]_{\boldsymbol{surface}} \boldsymbol{dS}$$

Fluid torque $\underline{\boldsymbol{T}}_{\boldsymbol{on}}$ on a surface **S**:

($\underline{\tilde{\boldsymbol{R}}}$ is the lever arm vector from the axis of rotation to the point of application of the force)
(Bird, UR sign convention on stress)

$$\underline{\boldsymbol{T}}_{\boldsymbol{on}} = \iint_{\boldsymbol{S}} \left[\underline{\tilde{\boldsymbol{R}}} \times (\hat{\boldsymbol{n}} \cdot -\underline{\underline{\boldsymbol{\Pi}}})\right]_{\boldsymbol{surface}} \boldsymbol{dS}$$

Elongational flow: $\underline{v} = \begin{pmatrix} -\frac{\dot{\varepsilon}(t)}{2} x_1 \\ -\frac{\dot{\varepsilon}(t)}{2} x_2 \\ \dot{\varepsilon}(t) x_3 \end{pmatrix}_{123}$

Shear flow: $\underline{v} = \begin{pmatrix} \dot{\varsigma}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$

Steady shearing kinematics: $\dot{\varsigma}(t) = \dot{\gamma}_0$ for all values of time t

Start-up of steady shearing kinematics: $\dot{\varsigma}(t) = \begin{cases} 0 & t < 0 \\ \dot{\gamma}_0 & t \geq 0 \end{cases}$

Cessation of steady shearing kinematics: $\dot{\varsigma}(t) = \begin{cases} \dot{\gamma}_0 & t < 0 \\ 0 & t \geq 0 \end{cases}$

Step shear strain: $\dot{\varsigma}(t) = \begin{cases} 0 & t < 0 \\ \dot{\gamma}_0 & 0 \leq t < \varepsilon \\ 0 & t \geq \varepsilon \end{cases}$

Small-amplitude oscillatory shear: $\dot{\varsigma}(t) = \gamma_0 \omega \cos \omega t$

Steady elongational kinematics: $\dot{\varepsilon}(t) = \dot{\varepsilon}_0$ for all values of time t

Start-up of steady elongation kinematics: $\dot{\varepsilon}(t) = \begin{cases} 0 & t < 0 \\ \dot{\varepsilon}_0 & t \geq 0 \end{cases}$

Shear viscosity: $\eta = \frac{-(\tau_{21})}{\dot{\gamma}_0}$

Shear normal stress coefficients: $\Psi_1 = \frac{-(\tau_{11} - \tau_{22})}{\dot{\gamma}_0^2}, \Psi_2 = \frac{-(\tau_{22} - \tau_{33})}{\dot{\gamma}_0^2}$

Steady elongational viscosity: $\bar{\eta} = \eta_e = \frac{-(\tau_{33} - \tau_{11})}{\dot{\varepsilon}_0}$

Step strain: $G = \frac{-(\tau_{21})}{\gamma_0}$

Small-amplitude oscillatory shear: $-\tau_{21} = G' \sin \omega t + G'' \cos \omega t$

$$G' = \frac{-(\tau_0)}{\gamma_0} \cos \delta \qquad G'' = \frac{-(\tau_0)}{\gamma_0} \sin \delta$$

Table of Integrals

$$\int \boldsymbol{u^{\alpha} du} = \frac{\boldsymbol{u^{\alpha+1}}}{\boldsymbol{\alpha+1}} + \boldsymbol{C} \qquad \boldsymbol{\alpha} \text{ is a constant}$$

$$\int \frac{\mathbf{1}}{\boldsymbol{u}} \boldsymbol{du} = \mathbf{ln}\, \boldsymbol{u} + \boldsymbol{C}$$

$$\int (\mathbf{ln}\, \boldsymbol{u}) \boldsymbol{du} = \boldsymbol{u} \,\mathbf{ln}\, \boldsymbol{u} - \boldsymbol{u} + \boldsymbol{C}$$

$$\int_{a}^{b} \boldsymbol{u\, dv} = \boldsymbol{uv}\Big|_{a}^{b} - \int_{a}^{b} \boldsymbol{v\, du}$$

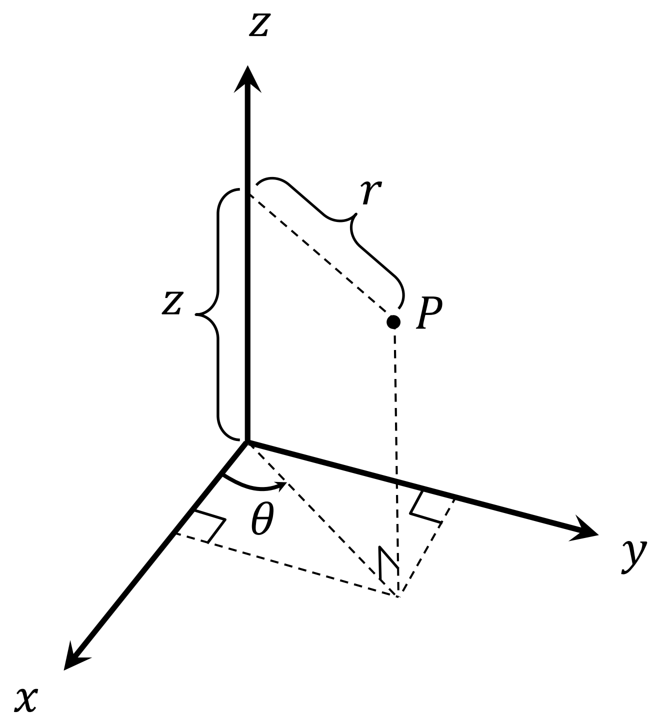
Miscellaneous

$$\underline{\underline{\boldsymbol{A}}} = \begin{pmatrix} \boldsymbol{A_{11}} & \boldsymbol{A_{12}} & \boldsymbol{A_{13}} \\ \boldsymbol{A_{21}} & \boldsymbol{A_{22}} & \boldsymbol{A_{23}} \\ \boldsymbol{A_{31}} & \boldsymbol{A_{32}} & \boldsymbol{A_{33}} \end{pmatrix}_{\mathbf{123}}$$

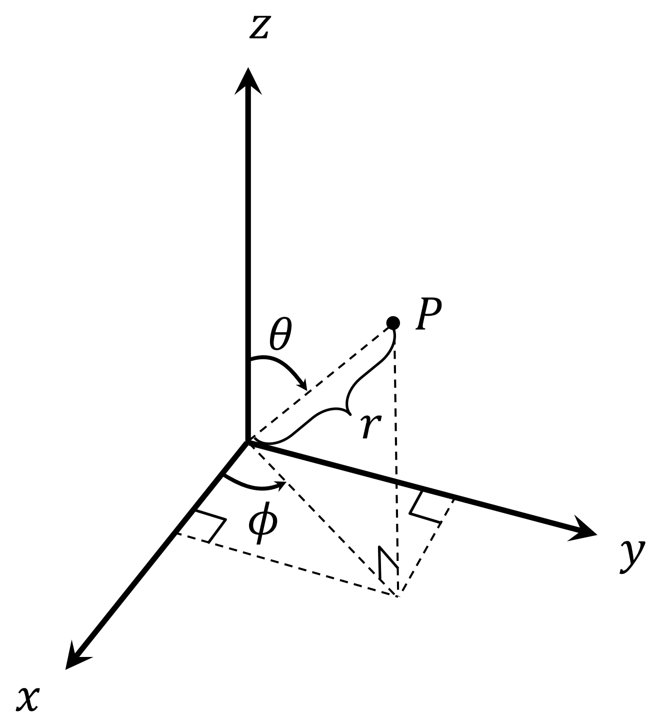
$$\frac{\boldsymbol{d}}{\boldsymbol{ds}}(\boldsymbol{uw}) = \boldsymbol{u}\frac{\boldsymbol{dw}}{\boldsymbol{ds}} + \boldsymbol{w}\frac{\boldsymbol{du}}{\boldsymbol{ds}}$$

$$\underline{\boldsymbol{u}} \times \underline{\boldsymbol{w}} = \mathbf{det}\begin{vmatrix} \hat{\boldsymbol{e}}_{\mathbf{1}} & \hat{\boldsymbol{e}}_{\mathbf{2}} & \hat{\boldsymbol{e}}_{\mathbf{3}} \\ \boldsymbol{u_1} & \boldsymbol{u_2} & \boldsymbol{u_3} \\ \boldsymbol{w_1} & \boldsymbol{w_2} & \boldsymbol{w_3} \end{vmatrix} = \begin{pmatrix} \boldsymbol{u_2 w_3 - u_3 w_2} \\ -(\boldsymbol{u_1 w_3 - u_3 w_1}) \\ \boldsymbol{u_1 w_2 - u_2 w_1} \end{pmatrix}_{\mathbf{123}}$$

Cylindrical Coordinate System: Note that the $\boldsymbol{\theta}$-coordinate swings around the $\boldsymbol{z}$-axis and the $\boldsymbol{r}$-coordinate is perpendicular to the z-axis.



Spherical Coordinate System: Note that the $\boldsymbol{\theta}$-coordinate swings down from the $\boldsymbol{z}$-axis and the $\boldsymbol{r}$-coordinate emits radially from the origin to the point; these are different from their definitions in the cylindrical system above.



Cylindrical Coordinates

System	Coordinates	Basis vectors
Cylindrical	$r = \sqrt{x^2 + y^2}$	$\hat{e}_r = \cos\theta\,\hat{e}_x + \sin\theta\,\hat{e}_y$
Cylindrical	$\theta = \tan^{-1}\left(\frac{y}{x}\right)$	$\hat{e}_\theta = (-\sin\theta)\hat{e}_x + \cos\theta\,\hat{e}_y$
Cylindrical	$z = z$	$\hat{e}_z = \hat{e}_z$
Cylindrical	$x = r\cos\theta$	$\hat{e}_x = \cos\theta\,\hat{e}_r + (-\sin\theta)\hat{e}_\theta$
Cylindrical	$y = r\sin\theta$	$\hat{e}_y = \sin\theta\,\hat{e}_r + \cos\theta\,\hat{e}_\theta$
Cylindrical	$z = z$	$\hat{e}_z = \hat{e}_z$

Spherical Coordinates

System	Coordinates	Basis vectors
Spherical	$x = r\sin\theta\cos\phi$	$\hat{e}_x = (\sin\theta\cos\phi)\hat{e}_r + (\cos\theta\cos\phi)\hat{e}_\theta + (-\sin\phi)\,\hat{e}_\phi$
Spherical	$y = r\sin\theta\sin\phi$	$\hat{e}_y = (\sin\theta\sin\phi)\hat{e}_r + (\cos\theta\sin\phi)\hat{e}_\theta + \cos\phi\,\hat{e}_\phi$
Spherical	$z = r\cos\theta$	$\hat{e}_z = \cos\theta\;\hat{e}_r + (-\sin\theta)\hat{e}_\theta$
Spherical	$r = \sqrt{x^2 + y^2 + z^2}$	$\hat{e}_r = (\sin\theta\cos\phi)\hat{e}_x + (\sin\theta\sin\phi)\hat{e}_y + \cos\theta\hat{e}_z$
Spherical	$\theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right)$	$\hat{e}_\theta = (\cos\theta\cos\phi)\hat{e}_x + (\cos\theta\sin\phi)\hat{e}_y + (-\sin\theta)\hat{e}_z$
Spherical	$\phi = \tan^{-1}\left(\frac{y}{x}\right)$	$\hat{e}_\phi = (-\sin\phi)\hat{e}_x + \cos\phi\,\hat{e}_y$

Coordinate system	surface differential dS
Cartesian (top, $\hat{n} = \hat{e}_z$)	$dS = dxdy$
Cartesian (side a, $\hat{n} = \hat{e}_y$)	$dS = dxdz$
Cartesian (side b, $\hat{n} = \hat{e}_x$)	$dS = dydz$
cylindrical (top, $\hat{n} = \hat{e}_z$)	$dS = rdrd\theta$
cylindrical (side, $\hat{n} = \hat{e}_r$)	$dS = Rd\theta dz$
spherical, ($\hat{n} = \hat{e}_r$)	$dS = R^2 \sin\theta d\theta d\phi$

Coordinate system	volume differential dV
Cartesian	$dV = dxdydz$
cylindrical	$dV = rdrd\theta dz$
spherical	$dV = r^2 \sin\theta drd\theta d\phi$