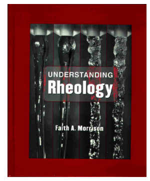
Polymer Rheology

Rhe-

$\rho\varepsilon\iota$ – *Greek* for flow



CM4650 Polymer Rheology
Michigan Tech

What is rheology anyway?

Rheology = the study of deformation and flow.

"What is Rheology Anyway?" Faith A. Morrison, *The Industrial Physicist*, 10(2) 29-31, April/May 2004.

What is rheology anyway?

To the layperson, rheology is

•Mayonnaise does not flow even under stress for a long time; honey always flows

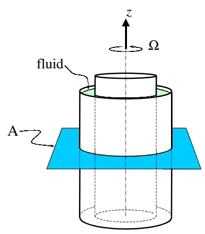
•Silly Putty bounces (is elastic) but also flows (is viscous)

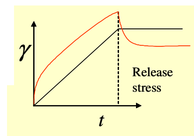
•Dilute flour-water solutions are easy to work with but doughs can be quite temperamental

•Corn starch and water can display strange behavior – poke it slowly and it deforms easily around your finger; punch it rapidly and your fist bounces off of the surface

What is rheology anyway?

To the scientist, engineer, or technician, rheology is





- Yield stresses
- Viscoelastic effects
- Memory effects
- Shear thickening and shear thinning

For both the layperson and the technical person, rheology is a set of problems or observations related to how the stress in a material or force applied to a material is related to deformation (change of shape) of the material.

What is rheology anyway?

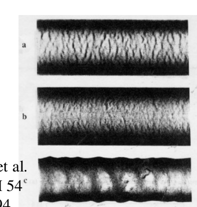
Rheology affects:

- End use (food texture, product pour, motor-oil function)

- Processing (design, costs, production rates)

www.corrugatorman.com/pic/akron%20extruder.JPG

www.math.utwente.nl/mpcm/aamp/examples.html



Pomar et al. JNNFM 54 143 1994

- Product quality (surface distortions, anisotropy, strength, structure development)

Goal of the scientist, engineer, or technician:

•**Understand** the kinds of flow and deformation **effects** exhibited by complex systems

•**Apply qualitative** rheological **knowledge** to diagnostic, design, or optimization problems

•In diagnostic, design, or optimization problems, **use or devise quantitative** analytical **tools** that correctly capture rheological effects

How do we reach these goals?

How?

By observing the behavior of different systems

•**Understand** the kinds of flow and deformation **effects** exhibited by complex systems

•**Apply qualitative** rheological **knowledge** to diagnostic, design, or optimization problems

By learning which quantitative models apply in what circum-stances

•In diagnostic, design, or optimization problems, **Use or devise quantitative** analytical **tools** that correctly capture rheological effects

By making calculations with models in appropriate situations

Learning Rheology (bibliography)

Descriptive Rheology

Barnes, H., J. Hutton, and K. Walters, *An Introduction to Rheology* (Elsevier, 1989)

Quantitative Rheology

Morrison, Faith, *Understanding Rheology* (Oxford, 2001)
Bird, R., R. Armstrong, and O. Hassager, *Dynamics of Polymeric Liquids, Volume 1* (Wiley, 1987)

Industrial Rheology

Dealy, John and Kurt Wissbrun, *Melt Rheology and Its Role in Plastics Processing* (Van Nostrand Reinhold, 1990)

Polymer Behavior

Larson, Ron, *The Structure and Rheology of Complex Fluids* (Oxford, 1999)
Ferry, John, *Viscoelastic Properties of Polymers* (Wiley, 1980)

Suspension Behavior

Larson, Ron, *The Structure and Rheology of Complex Fluids* (Oxford, 1999)
Macosko, Chris, *Rheology: Principles, Measurements, and Applications* (VCH Publishers, 1994)

The Physics Behind Rheology:

1. Conservation laws
   - mass
   - momentum
   - energy

Cauchy Momentum Equation

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p - \nabla\cdot\underline{\underline{\tau}} + \rho\underline{g}$$

2. Mathematics
   - differential equations
   - vectors
   - tensors

3. Constitutive law = law that relates **stress** to **deformation** for a particular fluid

Newtonian fluids:
(fluid mechanics)

$$\tau_{21} = -\underbrace{\mu}_{\text{material parameter}} \underbrace{\frac{dv_1}{dx_2}}_{\text{deformation}}$$

Newton's Law of Viscosity

- This is an empirical law (measured or observed)
- May be derived theoretically for some systems

Non-Newtonian fluids:
(rheology)

Need a new law or new laws

- These laws will also either be empirical or will be derived theoretically

Newtonian fluids:
(shear flow only)

$$\tau_{21} = -\mu \frac{dv_1}{dx_2}$$

Constitutive Equation

Non-Newtonian fluids:
(all flows)

stress tensor

$$\underline{\underline{\tau}} = -f\left(\underline{\underline{\dot{\gamma}}}\right)$$

Rate-of-deformation tensor

non-linear function
(in time and position)

Introduction to Non-Newtonian Behavior

Rheological Behavior of Fluids, National Committee on Fluid Mechanics Films, 1964

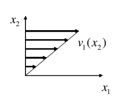
Velocity gradient tensor $\dot{\underline{\underline{\gamma}}}$

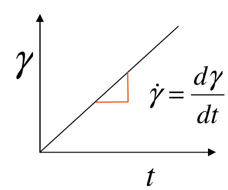
Type of fluid	Momentum balance	Stress –Deformation relationship (constitutive equation)
Inviscid (zero viscosity, μ=0)	Euler equation (Navier-Stokes with zero viscosity)	Stress is isotropic
Newtonian (finite. constant viscosity, μ)	Navier-Stokes (Cauchy momentum equation with Newtonian constitutive equation)	Stress is a function of the **instantaneous** velocity gradient
Non-Newtonian (finite, variable viscosity η plus **memory** effects)	Cauchy momentum equation with memory constitutive equation	Stress is a function of the **history** of the velocity gradient

Rheological Behavior of Fluids - Newtonian

1. Strain response to imposed shear stress

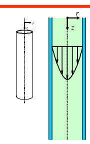
•shear rate is constant





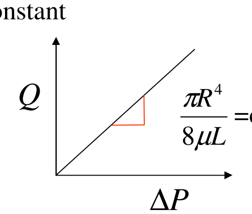
$$\dot{\gamma} = \frac{d\gamma}{dt} = \text{constant}$$

2. Pressure-driven flow in a tube (Poiseuille flow)



•viscosity is constant

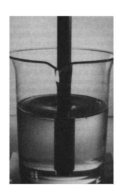
$$Q = \frac{\pi \Delta P R^4}{8\mu L}$$



$$\frac{\pi R^4}{8\mu L} = \text{constant}$$

3. Stress tensor in shear flow

•only two components are nonzero

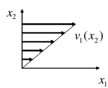


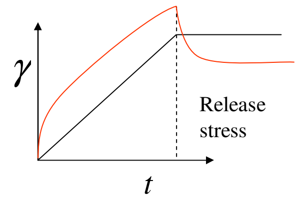
$$\underline{\underline{\tau}} = \begin{pmatrix} 0 & \tau_{12} & 0 \\ \tau_{21} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$

Rheological Behavior of Fluids – non-Newtonian

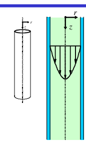
1. Strain response to imposed shear stress

•shear rate is variable

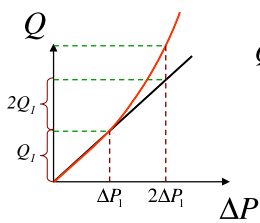




2. Pressure-driven flow in a tube (Poiseuille flow)

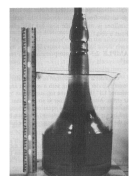


•viscosity is variable



$$Q = f(\Delta P)$$

3. Stress tensor in shear flow



Normal stresses

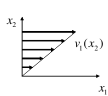
•all 9 components are nonzero

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}_{123}$$

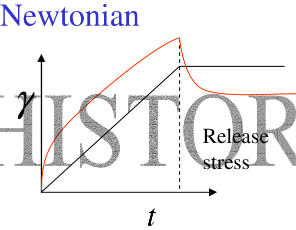
Rheological Behavior of Fluids – non-Newtonian

1. Strain response to imposed shear stress

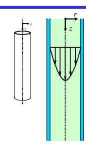
•shear rate is variable



HISTORY

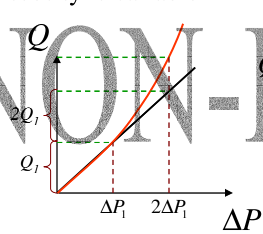


2. Pressure-driven flow in a tube (Poiseuille flow)

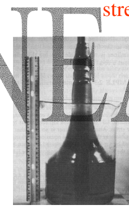


•viscosity is variable

NON-LINEARITY



$$Q = f(\Delta P)$$



Normal stresses

•all 9 components are nonzero

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}_{123}$$

© Faith A. Morrison, Michigan Tech U.

Examples from the film of . . . .

Dependence on the history of the deformation gradient

- Polymer fluid pours, but springs back
- Elastic ball bounces, but flows if given enough time
- Steel ball dropped in polymer solution "bounces"
- Polymer solution in concentric cylinders – has fading memory
- Quantitative measurements in concentric cylinders show memory and need a finite time to come to steady state

Non-linearity of the function $\underline{\underline{\tau}} = f(\underline{\underline{\dot{\gamma}}})$

- Polymer solution draining from a tube is first slower, then faster than a Newtonian fluid
- Double the static head on a draining tube, and the flow rate does not necessarily double (as it does for Newtonian fluids); sometimes more than doubles, sometimes less
- Normal stresses in shear flow
- Die swell

Show NCFM Film on *Rheological Behavior of Fluids*

© Faith A. Morrison, Michigan Tech U.