PH2100 Formula Sheet Knight

Concepts of Motion

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time interval spent traveling}}$$

$$\Delta\vec{r} = \vec{r}_\text{f} - \vec{r}_\text{i}$$

$$\vec{v}_\text{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{a}_\text{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

Kinematics: The Mathematics of Motion

For Motion in a Straight Line:

$$v_s = \frac{ds}{dt} = \text{slope of position-time graph}$$

$$a_s = \frac{dv_s}{dt} = \text{slope of velocity-time graph}$$

$$s_\text{f} = s_\text{i} + \int_{t_\text{i}}^{t_\text{f}} v_s\,dt = s_\text{i} + \begin{cases}\text{area under velocity curve}\\ \text{from } t_\text{i} \text{ to } t_\text{f}\end{cases}$$

$$v_\text{fs} = v_\text{is} + \int_{t_\text{i}}^{t_\text{f}} a_s\,dt = v_\text{is} + \begin{cases}\text{area under acceleration curve}\\ \text{from } t_\text{i} \text{ to } t_\text{f}\end{cases}$$

Uniform Motion:

$$s_\text{f} = s_\text{i} + v_s\Delta t$$

Uniformly Accelerated Motion:

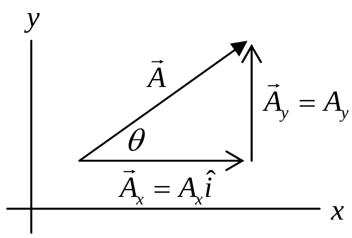
$$v_\text{fs} = v_\text{is} + a_s\Delta t$$

$$s_\text{f} = s_\text{i} + v_\text{is}\Delta t + \tfrac{1}{2}a_s(\Delta t)^2$$

$${v_\text{fs}}^2 = {v_\text{is}}^2 + 2a_s\Delta s$$

Motion on an Inclined Plane: $a_s = \pm g\sin\theta$

Vectors and Coordinate Systems



$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{i} + A_y\hat{j}$$

In the figure above:

$$A_x = A\cos\theta \qquad A_y = A\sin\theta$$

$$A = \sqrt{A_x^2 + A_y^2} \qquad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Force and Motion

$$\vec{a} = \frac{1}{m}\vec{F}_\text{net}$$

where $\vec{F}_\text{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$

Dynamics I: Motion Along a Line

$$\vec{F}_\text{net} = \sum_i \vec{F}_i = m\vec{a}$$

A Model for Friction:

Static: $\vec{f}_\text{s} \le (\mu_\text{s} n,\ \text{direction as necessary to prevent motion})$

Kinetic: $\vec{f}_\text{k} = (\mu_\text{k} n,\ \text{direction opposite the motion})$

Rolling: $\vec{f}_\text{r} = (\mu_\text{r} n,\ \text{direction opposite the motion})$

Drag: $\vec{D} \approx (\tfrac{1}{4}Av^2,\ \text{direction opposite the motion})$

Dynamics II: Motion in a Plane

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

$$(F_\text{net})_x = \sum F_x = ma_x$$

$$(F_\text{net})_y = \sum F_y = ma_y$$

Constant Acceleration:

$$x_\text{f} = x_\text{i} + v_\text{ix}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 \qquad y_\text{f} = y_\text{i} + v_\text{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$v_\text{fx} = v_\text{ix} + a_x\Delta t \qquad v_\text{fy} = v_\text{iy} + a_y\Delta t$$

Projectile Motion:

$$x_\text{f} = x_\text{i} + v_\text{ix}\Delta t \qquad y_\text{f} = y_\text{i} + v_\text{iy}\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

$$v_\text{fx} = v_\text{ix} = \text{constant} \qquad v_\text{fy} = v_\text{iy} - g\Delta t$$

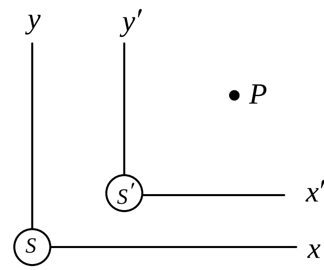
$$v_\text{fy}^2 = v_\text{iy}^2 - 2g(y_\text{f} - y_\text{i})$$

Relative Motion:

$$\vec{r}_{PS} = \vec{r}_{PS'} + \vec{v}_{S'S}t$$

$$\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}$$

$$\vec{a}_{PS} = \vec{a}_{PS'}$$



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Dynamics III: Motion in a Circle

$\theta = \dfrac{s}{r}$

$\text{average angular velocity} = \dfrac{\Delta\theta}{\Delta t}$

$\omega = \dfrac{d\theta}{dt}$

$v_t = \omega r$

Uniform Circular Motion:

$v = \dfrac{2\pi r}{T} \quad \omega = \dfrac{2\pi \text{ rad}}{T}$

$\theta_\text{f} = \theta_\text{i} + \omega\ \Delta t$

$a_r = \dfrac{v^2}{r} = \omega^2 r$

Nonuniform Circular Motion (constant a_t):

$a_t = \dfrac{dv}{dt}$

$\theta_\text{f} = \theta_\text{i} + \omega_\text{i}\Delta t + \dfrac{a_t}{2r}(\Delta t)^2$

$\omega_\text{f} = \omega_\text{i} + \dfrac{a_t}{r}\Delta t$

$(F_\text{net})_r = \sum F_r = ma_r = \dfrac{mv^2}{r} = m\omega^2 r$

$(F_\text{net})_t = \sum F_t = \begin{cases} 0 & \text{uniform motion} \\ ma_t & \text{nonuniform motion} \end{cases}$

$(F_\text{net})_z = \sum F_z = 0$

Newton's Third Law

$\vec{F}_\text{A on B} = -\vec{F}_\text{B on A}$

Impulse and Momentum

$\vec{p} = m\vec{v}$

$J_x = \begin{cases} \int_{t_\text{i}}^{t_\text{f}} F_x(t)dt = \text{area under force curve} \\ F_\text{avg}\Delta t \end{cases}$

$\Delta p_x = J_x$ (impulse-momentum theorem)

$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots$

$\vec{P}_\text{f} = \vec{P}_\text{i}$ (conservation of momentum)

Energy

$K = \tfrac{1}{2}mv^2$

$U_\text{g} = mgy$

$E_\text{mech} = K + U$

$K_\text{f} + U_\text{f} = K_\text{i} + U_\text{i}$ (conservation of mechanical energy)

$(F_\text{sp})_s = -k\Delta s \qquad U_s = \tfrac{1}{2}k(\Delta s)^2$

Perfectly Elastic 1-D Collisions (m_2 initially at rest):

$(v_{\text{f}x})_1 = \dfrac{m_1 - m_2}{m_1 + m_2}(v_{\text{i}x})_1 \qquad (v_{\text{f}x})_2 = \dfrac{2m_1}{m_1 + m_2}(v_{\text{i}x})_1$

Work

$W = \begin{cases} \int_{s_\text{i}}^{s_\text{f}} F_s ds = \text{area under the force-position curve} \\ \vec{F}\cdot\Delta\vec{r} = F\Delta r\cos\theta \quad \text{if } \vec{F} \text{ is a constant force} \end{cases}$

$\Delta K = W_\text{net} = W_\text{c} + W_\text{diss} + W_\text{ext}$ (work-kinetic energy theorem)

$\Delta U = U_\text{f} - U_\text{i} = -W_\text{c}(\text{i} \to \text{f})$

$F_s = -\dfrac{dU}{ds}$

$E_\text{th} = K_\text{micro} + U_\text{micro}$

$\Delta E_\text{th} = -W_\text{diss}$

$E_\text{sys} = K + U + E_\text{th}$

$K_\text{f} + U_\text{f} + \Delta E_\text{th} = K_\text{i} + U_\text{i} + W_\text{ext}$

$P = \dfrac{dE_\text{sys}}{dt} \qquad P = \dfrac{dW}{dt} = \vec{F}\cdot\vec{v} = Fv\cos\theta$

Newton's Theory of Gravity

$F_{M \text{ on } m} = F_{m \text{ on } M} = \dfrac{GMm}{r^2}$

$g_\text{surface} = \dfrac{GM}{R^2}$

Circular Orbit: $v = \sqrt{\dfrac{GM}{r}} \quad T^2 = \left(\dfrac{4\pi^2}{GM}\right)r^3$

Physical Constants

$g = 9.80 \text{ m/s}^2$

$G = 6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$

$M_\text{earth} = 5.98\times10^{24} \text{ kg}$

$R_\text{earth} = 6.37\times10^{6} \text{ m}$

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