PH4210 HW 1 **Due: Monday Oct. 1, 2007**

1. (a) Show that the determinant of matrix M is give by: $\det M = \varepsilon_{ijk} M_{i1} M_{j2} M_{k3}$.

   (b) Show that $\varepsilon_{ijk} M_{im} M_{jn} M_{kr} = \varepsilon_{mnr} \varepsilon_{ijk} M_{i1} M_{j2} M_{k3}$.

   (c) Prove that the determinant of an orthogonal matrix R is 1 (det R = 1).

   (d) Now, show that $\vec{\mathrm{A}} \times \vec{\mathrm{B}}$ is a vector under an orthonormal coordinate transformation $\vec{\mathrm{A}} \to \vec{\mathrm{A}}' = R\vec{\mathrm{A}}, \quad \vec{\mathrm{B}} \to \vec{\mathrm{B}}' = R\vec{\mathrm{B}}$. That is, show that $\vec{\mathrm{A}}' \times \vec{\mathrm{B}}' = R(\vec{\mathrm{A}} \times \vec{\mathrm{B}})$. The book claims this is so on page 14, but equation 2.17 is no proof. Hint: Start with $\vec{\mathrm{A}}' \times \vec{\mathrm{B}}'$ and substitute in the transformations. Insert a judicious choice of the identity matrix in the form $\delta_{pq} = R_{pt} R^T_{tq}$ (you'll have to figure out the subscripts). Then use the identities in (a), (b) and (c) above.

2. Show that $\vec{\nabla}(\nabla \cdot \vec{V})$ and $\nabla^2 \vec{V}$ are fundamentally different by expressing these quantities in suffix notation.

3. Prove that $(\vec{\mathrm{A}} \times \vec{\mathrm{B}}) \cdot (\vec{\mathrm{C}} \times \vec{\mathrm{D}}) = (\vec{\mathrm{A}} \cdot \vec{\mathrm{C}})(\vec{\mathrm{B}} \cdot \vec{\mathrm{D}}) - (\vec{\mathrm{A}} \cdot \vec{\mathrm{D}})(\vec{\mathrm{B}} \cdot \vec{\mathrm{C}})$.

4. Pollack & Stump 2.6

5. Pollack & Stump 2.8

6. Pollack & Stump 2.9

7. Pollack & Stump 2.10

8. Pollack & Stump 2.11

9. Pollack & Stump 2.12

10. Pollack & Stump 2.13