Superposition

Two or more traveling waves in the same medium lead to a resultant wave function that is the algebraic sum of the wave functions of the individual waves.

Linear waves (linear media; wave eqn.)

Small amplitudes

Interference

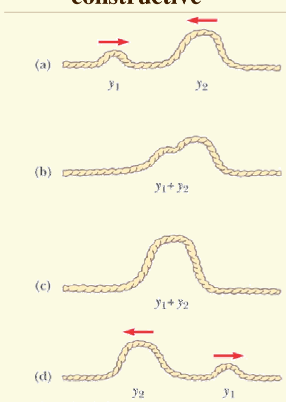
- ✓ A telling property of waves is <u>interference</u>:
- ✓ Two waves at the same place at the same time can add together <u>constructively</u> or <u>destructively</u>.

<u>Constructive</u>: individual displacements in the same direction

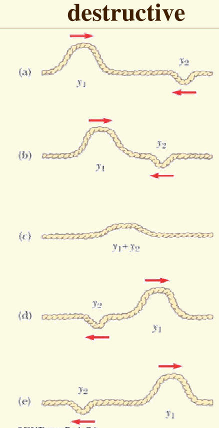
<u>Destructive</u>: individual displacements in opposite directions

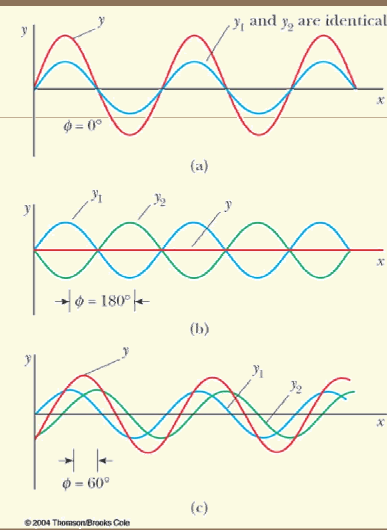
Interference

constructive



destructive





Sum of two waves:

$y_1 = A\sin(kx - \omega t)$

$y_2 = A\sin(kx - \omega t + \phi)$

$$y_1 + y_2 = 2A\cos(\phi/2)\sin(kx - \omega t + \phi/2)$$

Resultant wave is:
(1) Algebraic sum of all displacements
(2) Sinusoidal with the same k and ω
(3) Amplitude is 2Acos(ϕ/2)

Amplitude: 2Acos(ϕ/2)

Constructive interference:

ϕ = 0, 2π, 4π, 6π, etc

Maximum amplitude

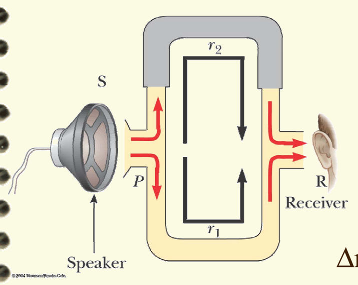
Destructive interference:

ϕ = π, 3π, 5π, etc

Minimum amplitude

Sound Interference

Trombone:



Path lengths: r_1, r_2

Phase difference:

$\phi = k|r_1 - r_2| = k\Delta r$

Path difference controls the phase difference!

$\Delta r = n\lambda$: constructive

$\Delta r = (2n+1)\lambda/2$: destructive

Beats: Interference in Time

Periodic variation in intensity resulting from two waves of slightly different frequency.

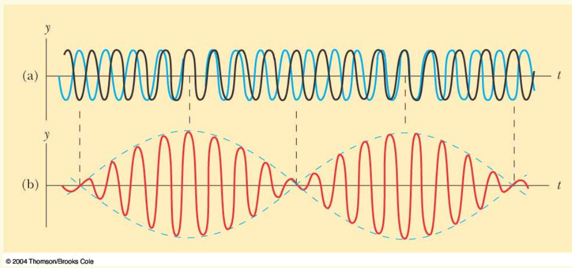
$\Delta P_1 = A\cos 2\pi f_1 t$

$\Delta P_2 = A\cos 2\pi f_2 t$

$$\Delta P = \Delta P_1 + \Delta P_2 = 2A\cos\left[2\pi\left(\overbrace{\frac{f_1 - f_2}{2}}^{\text{small}}\right)t\right]\cos\left[2\pi\left(\underbrace{\frac{f_1 + f_2}{2}}_{\text{Average of } f_1 \text{ and } f_2}\right)t\right]$$

Beats

$$\Delta P = \Delta P_1 + \Delta P_2 = 2A\cos\left[2\pi\left(\frac{f_1 - f_2}{2}\right)t\right]\cos\left[2\pi\left(\frac{f_1 + f_2}{2}\right)t\right]$$



TONE: $(f_1+f_2/2)$ Beat Frequency: f_1-f_2

Standing Waves

Superposition of waves traveling in opposite directions:

$$y_1 = A\sin(kx - \omega t)$$

$$y_2 = A\sin(kx + \omega t)$$

$$y_1 + y_2 = (2A\sin(kx)\cos(\omega t)$$

Q

- Each point (x) in the wave moves sinusoidally in time
- Wave is NOT traveling (no net energy transfer)
- Overall shape y(x) is sinusoidal

Example: Standing Waves on String

Boundary Condition:

Fixed at both ends.

Ends are <u>NODES</u>.

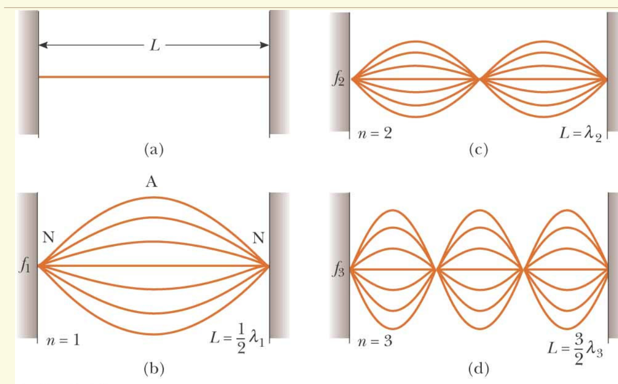
An integral number of half wavelengths must fit in between the ends! **$L=n\lambda/2$**

$\lambda_n = 2L/n$ $n=1, 2, 3, \ldots$

$f_n = v/\lambda_n = nv/2L$

Application: guitar, violin, cello, voice

Standing waves on a string



Standing Waves on Strings:

$f_n = v/\lambda_n = nv/2L$

$f_1 = v/2L$ fundamental (1st harmonic)

$f_2 = v/L$ (2nd harmonic)

$f_3 = 3v/2L$ (3rd harmonic)

$f_n = nf_1$ (nth harmonic)

Application: On string instruments, change L by finger placement!

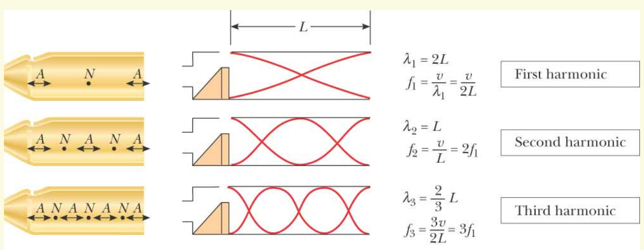
Standing waves in air columns

Open end: Approximate displacement antinode or pressure node

Closed end: Displacement node; pressure antinode

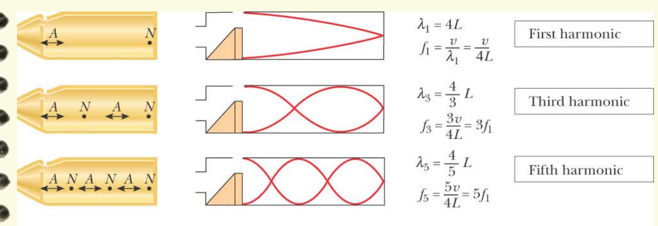
Q2

Open at both ends



(a) Open at both ends

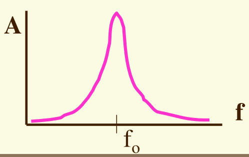
Closed at one end



(b) Closed at one end, open at the other

Resonance

- ✓ Mechanical systems have **natural (resonant) frequencies** they oscillate in.
- ✓ Driving a system at its natural frequency causes it to resonate with the driver.
- ✓ Resonating systems absorb energy from the driver and get larger and larger amplitudes.



Applications: spectroscopy, engineering (Tacoma Narrows bridge), lasers, music….

Tacoma Narrows Bridge