## Problem solving tool chest:

- ✓ Definitions
- ✓ Kinematics
- ✓ Newton's Laws of Motion
- ✓ Energy Conservation

**Energy is ALWAYS conserved in the universe.**

**In a SYSTEM (limited part of the universe) we can track the energy entering, leaving and changing form.**

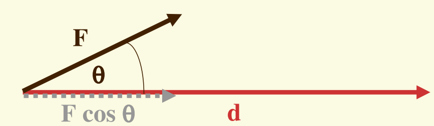
**Accounting for where energy is lets us predict motion!**

## Work and Energy

***WORK***: a type of energy transfer.

1. Definition: For a constant net force **F**, and a displacement **d**,

$$W = F\,d\cos\theta$$




## $W = F\,d\cos\theta$

2. Get used to the definition
3. Work is a scalar
4. Units: [F][d] = N m = J (Joules)
5. If speed is constant, W=0. (WHY?)
6. If **F** ⊥ **d**, then W=0. (WHY?)
7. If displacement is zero, W=0.
8. W can be positive or negative
   - (+) energy transfer to object
   - (−) energy transfer from object

## 1-D example: lift a box

Q1,2,3

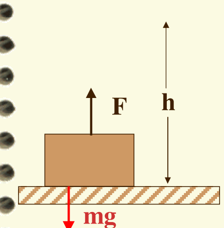
**Careful! Wording is EVERYTHING!**

**Work done BY force F: Fh**

**Work done BY gravity: -mgh**

**Net Work done ON box:**

$$W_{net} = (F-mg)h$$




**If speed = constant, F=mg, and $W_{net} = 0$.**

# Almost 1-D example: slide a box

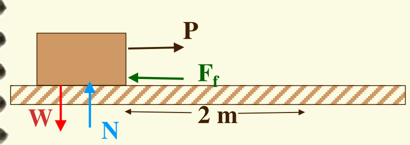
**Work done by P:** **2P Joules**

**Work done by Friction ($F_f$):** **$-2F_f=-2\mu_k N$ Joules**

**Work done by gravity:** **0 Joules** (WHY?)

**Work done by Normal:** **0 Joules** (WHY?)

**Total work ON box: $2(P-F_f)$ Joules OR $W_p+W_f$**

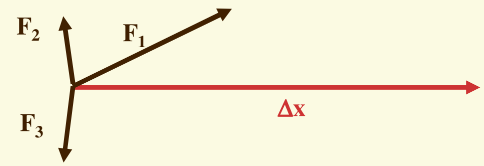



# Don't miss the importance of that last part!

$W_{tot} = \Sigma W_i$

$= F_{1x}\Delta x + F_{2x}\Delta x + F_{3x}\Delta x + F_{4x}\Delta x + \ldots$

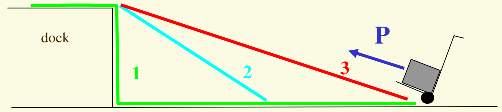
$= (F_{net})_x \Delta x$ for a particle with many forces




# 2D Example:

**Q4,5**

A computer delivery is being made to a loading dock with a frictionless dolly. Which path requires the least work by the worker if the cart dolly moves up with constant speed?



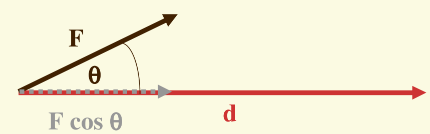

# So what about 2-D?

**Only displacement in the direction of the force (or force in the direction of the displacement) counts!**

$W = F\, d \cos\theta$

**$W = F\,(d\cos\theta)$ OR**

**$W = d\,(F\cos\theta)$**



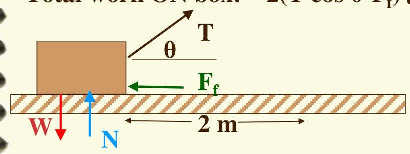
## 2-D example: Pull box with rope

**Work done BY T:** **2(T cos θ)**

**Only the part of T that actually moves the box does work!**

**T sin θ does NO work. **** ****Note: Tsin θ does affect $F_f$ since it changes normal force!**

**Total work ON box: 2(T cos θ-$F_f$) J OR $W_T$-$W_f$**




## Where have I heard that before?

**|F||d| cos θ is like….**

**|A||B| cos θ which is…**

**The definition of a DOT product!**

**-Dot product finds component of one vector in the direction of the other, then multiplies the two.**

**-Dot product "projects" one vector on to the other, then multiplies the projection and the vector.**

## Scalar (Dot) Product

**2 ways to do a dot product:**

**A·B**=ABcosθ

OR, in vector form:

$$\mathbf{A}\cdot\mathbf{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\cdot(B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k})$$
$$= A_xB_x + A_yB_y + A_zB_z$$

Remember:, **i**·**i** = 1, **j**·**j** = 1, **k**·**k**= 1,
**i**·**j** = 0, **i**·**k** = 0, j·**k** = 0, etc.

**KNOW THESE BOTH!**

## Scalar (Dot) Product

To remember about dot products:

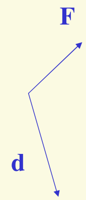
- ✓ **A·B = B·A** (commutative)
- ✓ **A**·(**B+C**) = **A**·**B** + **A**·**C** (distributive)
- ✓ **A**·**A** = $A^2$

**RESULT OF A DOT PRODUCT IS A SCALAR (NUMBER)!!**

# Work:

$W = \mathbf{F} \cdot \mathbf{d} = F\,d\cos\theta$

$= F_x d_x + F_y d_y + F_z d_z$




**Example:**

Constant Force $\mathbf{F} = 3\mathbf{i} + 4\mathbf{j}$ N

displacement $\mathbf{d} = 2\mathbf{i} - 5\mathbf{j}$ m

$W = \mathbf{F} \cdot \mathbf{d} = F_x d_x + F_y d_y + F_z d_z$

$= 3 \cdot 2 + 4 \cdot (-5)$ J $= 6 - 20$ J

$= -14$ J

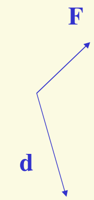
# Why know both definitions?

**Example:**

Constant Force $\mathbf{F} = 3\mathbf{i} + 4\mathbf{j}$ N

displacement $\mathbf{d} = 2\mathbf{i} - 5\mathbf{j}$ m

What is the angle between **F** and **d**?




$W = \mathbf{F} \cdot \mathbf{d} = F\,d\cos\theta$

$$\cos\theta = W/(F\,d) = \frac{-14\text{J}}{(\sqrt{3^2+4^2})(\sqrt{2^2+(-5)^2})}$$

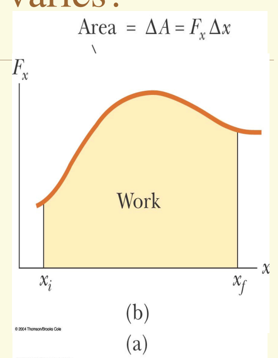
$\theta = 121°$

# OK, but what if force varies?

$$W_{tot} = \sum \lim_{\Delta x \to 0} \Delta W$$

$$= \int_{x_i}^{x_f} F_x\,dx \quad = \quad \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$$

Area under $F_x(x)$ curve!




**If F is constant,**

$$W = \int F_x\,dx = F_x \int dx = F_x (x_f - x_i)$$

# Application: Springs!

For ideal springs or for small displacements from equilibrium,

$$F_s = -kx$$

**Force VARIES.**

**1st cm easier than 2nd easier than 3rd…**

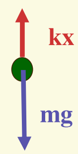
- Hooke's Law
- Restoring Force
- x = 0 is the equilibrium displacement
- k = spring constant (stiffness)
- Force is opposite to displacement

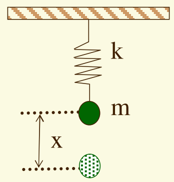
## Finding k—hang a weight

$\Sigma F = kx - mg$

$= 0$

in equilibrium




$k = mg/x$

k has units of N/m

## Work done by springs

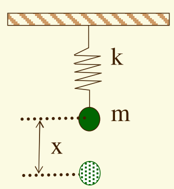
$$W_s = \int_{x_i}^{x_f} (-kx)dx \quad = \frac{-1}{2}kx^2 \Big|_{x_i}^{x_f}$$

$$= \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Caution: not the same as $\frac{1}{2}k(x_i - x_f)^2$

## Example: Springs

A spring stretches 4.6 cm when a 0.75 kg mass is hung on it. How much work must be done on this spring to stretch it 11 cm from equilibrium?




**F=kx=>**
**k=F/m=0.75(9.8)/.046**

**k=159 N/m**

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$W_s = 0 - .5(159)(.11)^2$
$= 0.97$ J **???**