Potential-field continuation: past practice vs. modern methods

Jeffrey D. Phillips, U.S. Geological Survey*

Summary

Several methods for continuing gridded potential-field data from an observation surface to another surface am reviewed. Due to past limitations on computational abilities, two of these methods, the simple Fourier method and the chessboard method, have often been applied in situations that do not meet their basic assumptions. Under certain conditions, this can lead to large errors in the continued field. Equivalent source methods, although computationally slower, am shown to be significantly more accurate under these conditions. By using a combination of equivalent source methods, anomalies at all wavelengths can be accurately and efficiently modeled. Like the chessboard method, equivalent source methods can incorporate variable low-pass filtering during downward continuation to suppress unwanted noise. Unlike the low-pass filter used in the chessboard method, the simple low-pass filtering method suggested here, namely downward continuation less than the prescribed distance, preserves the result as a potential-field.

Introduction

According to theory (Kellog, 1953, Chapter 8), knowledge of a potential field on any continuous surface provides knowledge of the field on any other surface that lies outside the sources. Geophysicists have long used this i&a to evaluate potential-fields on surfaces other than the original measurement surface. The term "potential-field continuation" is used to describe this evaluation process. Continuation may be "upward". away from the sources, or "downward", toward the sources. Upward continuation is a mathematically stable smoothing operation. Downward continuation is a mathematically unstable process, involving the amplification of short wavelengths where the signal-to-noise ratio is typically low.

Due to past limitations on computing ability, methods developed to perform upward or downward continuation have frequently been misapplied to data that do not fit the assumptions of the methods. With today's fast, inexpensive computers, and today's more accurate and efficient algorithms, there is no longer any need to rely on inappropriate methods for potential-field continuation.

The simple Fourier method

The Fourier transform F(u,v,0) of a potential field measured on a horizontal plane at z = 0, can be. converted into the Fourier transform of the same field measured on the plane z = h by a simple multiplication (Bhattacharyya, 1967).

$$F(u,v,h) = F(u,v,0)\ \exp(h(u^2+v^2)^{1/2})$$

If h is negative, the operation is upward continuation; if h is positive, the operation is downward continuation.

The simple Fourier method has been used not only to continue data between horizontal surfaces, but also between parallel, non-horizontal surfaces. To see the danger in this, let us look at a synthetic example. I have used traditional forward methods (Plouff, 1976; Godson, 1983) to compute the magnetic field due to a buried distribution of prismatic magnetic sources (figure la) on a surface with maximum relief of about 1100 m representing actual topography in Nevada (figure lb). The computed field (figure lc) is sampled on a 100 m grid. I have also computed the field of the sources on a surface representing the topography plus 300 m (figure 1d).

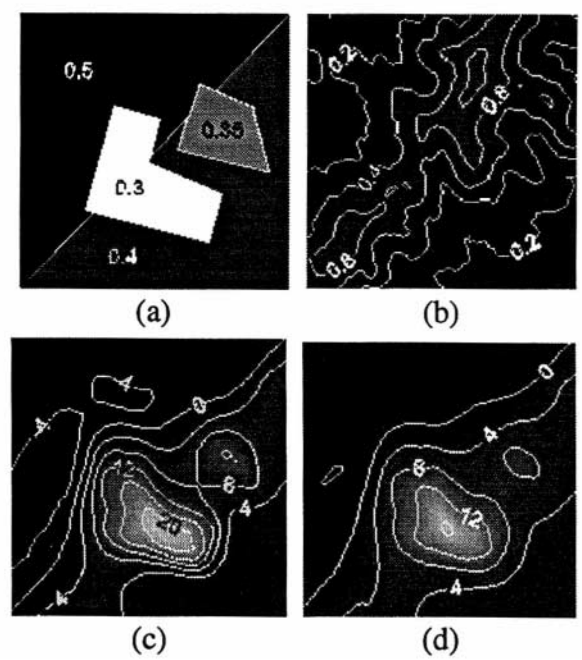
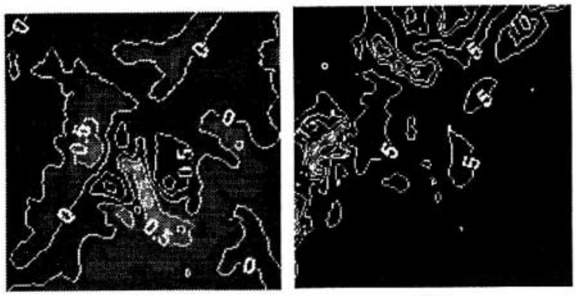


Figure 1 - Synthetic model used for magnetic field continuation studies. The area is 7x7 kilometers, sampled on a 100 m grid. (a) Uniformly magnetized magnetic basement surface (1 A/m); numbers represent depths below sea level in kilometers (after Roest and others, 1992). (b) Initial topographic observation surface contoured at a 0.2 kilometer interval. (c) Total magnetic field computed on the topography assuming vertical magna and vertical regional field, contour interval 4 nT. (d) Total magnetic field computed 300 meters above the topography, contour interval 4 nT.

Using the simple Fourier method to upward continue the first field 300 m, then subtracting the result from the second field, a residual field is generated (figure 2a). The residual field represents the error produced by upward continuing between non-horizontal surfaces using the simple Fourier method. The average error is about 3.5 percent and the maximum error is about 29 percent (figure 2b).



(a) (b)

Figure 2 - Error resulting from a 300 m parallel-surface upward continuation using the simple Fourier method on the model. (a) absolute error, 0.5 nT contour interval; (b) percent error, 5% contour interval.

By repeating this exercise for other values of continuation distance and surface relief, I generated the error surfaces in **figure 3.**

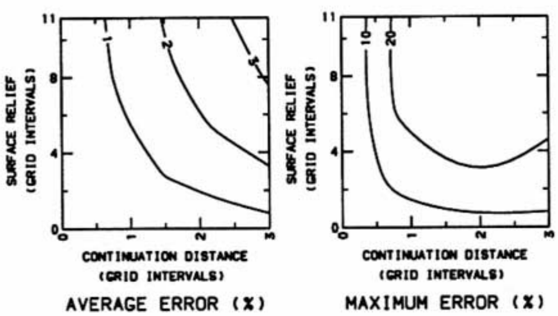


Figure 3 - Average and maximum errors (%) resulting from parallel-surface upward continuation using the simple Fourier method for various combinations of surface relief and continuation distance. Note that the contour values are based on a particular model, and may change for other models.

As might be expected, figure 3 shows that the error in parallel-surface continuation using the simple Fourier method is always small if the continuation distance is small relative to the grid interval, or if the surface relief is small relative to the grid interval. In other situations the error can be large.

The chessboard method

In order to extend the simple Fourier continuation method to the problem of level-to-drape continuation, Cordell (1985) developed the chessboard method. In the chessboard method, a potential field measured on a horizontal surface is continued to several parallel horizontal surfaces spanning the elevation range of a desired draped surface. This three dimensional volume of data resembles the multiple tiers in a three dimensional chessboard. To evaluate the field on a point of the draped surface, vertical interpolation is used between the fields on the nearest bounding horizontal surfaces.

The chessboard method works well for level-to-drape continuation. However, it has also been routinely used for drape-to-level and drape-to-drape continuation.

When the chessboard method is used for parallel-surface continuation, it becomes identical to the simple Fourier method, with the results shown in figure 3.

To test the validity of the chessboard method for drape-to-level surface continuation, we can return to our synthetic example. Sting with the calculated magnetic field on the topography, we pretend that the field is on a level plane, use the simple Fourier method to upward wntinue to a range of parallel levels, then interpolate between the continuation levels to evaluate the field on an actual level surface just above the highest point of the topography. Subtracting this result from the model field calculated on the level surface yields the error, which closely resembles that in shown in figure 2. We can conclude that the restrictions on surface relief, continuation distance, and grid interval that apply to the simple Fourier method also apply to the chessboard method.

Before leaving the chessboard method, I want to point out that it incorporates a unique approach to filtering the downward wntinued portions of the result. Because the method assumes that the initial field is on a horizontal surface, each parallel surface that forms a plane of the three dimensional chessboard is either entirely above or entirely below the starting surface. Continued data on planes that are above the starting surface do not requite low-pass filtering. Continued data on planes that are below the starting surface can be low-pass filtered with increasing severity as the distance from the starting surface increases. In this way, a variable low-pass filter is applied to the field interpolated on the output surface; only the downward continued parts of the output field ate low-pass filtered, and the severity of the filter increases with distance below the input surface. Because the design of the low-pass filter is not based on potential-field theory, it is possible to select a low-pass filter that is too severe, resulting in a downward continued field that is smoother that the original field.

An iterative Fourier equivalent source method

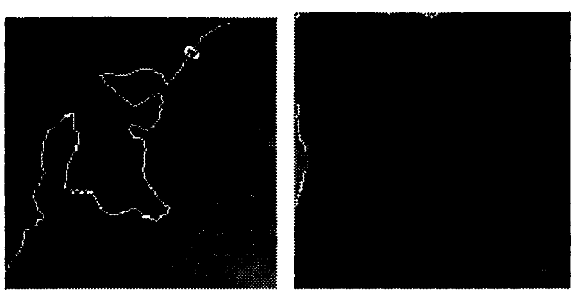
Although it is useful to have level-to-level and level-to-drape continuation methods many aemmagnetic surveys are wllected on surfaces draped over the topography. Draped surfaces are preferred both to maximize resolution of short-wavelength anomalies and to accommodate ancillary geophysical instrumentation such as gamma-ray and electromagnetic sensors. Of course all ground gravity data is wllected on the topography.

Equivalent source techniques provide a way to perform potential-field continuation between general surfaces. They have become more practical with the availability of affordable high-speed computers. The most efficient equivalent source method I have found was developed by Xia and others (1993), and uses an iterative Fourier transform approach. An initial magnetization or density distribution (it can be zero) is assigned to a horizontal plane located just below all the measurements.

The field of this magnetization or density distribution is calculated on the observation surface using a Fourier technique and subtracted from the observed field. This residual field is converted into a residual magnetization or density distribution and added to the initial equivalent magnetization or density distribution. The process is repeated until the residual field becomes sufficiently small or until the process starts to diverge.. Convergence is fastest for magnetic fields if they are first reduced to the magnetic pole, or if we pretend that the field was measured at the pole (by using vertical equivalent dipoles in a vertical ambient field). Once the equivalent source distribution is known, it can be used to calculate the field on any surface that lies above the equivalent layer. It is also easy to compute vertical derivatives or integrals of the field and to change magnetic vector directions.

Determining the equivalent source distribution to reasonable accuracy in rugged terrain can be a slow process, requiring overnight or up to several days on a fast personal computer. Once the source distribution is known, forward calculations can be rapidly carried out.

After applying the Xia method to the synthetic model for 300 m parallel-surface continuation, the largest resulting errors are about 7% (figure 4) and are concentrated at the edges of the data, where missing information has the greatest effect on the upward continuation.



(a) (b)

Figure4 - Error in 300 m parallel-surface upward continuation resulting from using the Xia method on the model. Compare with figure 2. (a) Absolute error, 0.5 nT contour interval. (b) Percent error, 5% contour interval.

Somewhat analogous to the chessboard method, the Xia method can be modified to add a simple type of variable low-pass filtering to areas of the equivalent source field that are downward wntinued below the original observation surface. The approach involves elevating the continuation surface in these areas so that unwanted downward continuation effects are reduced in severity. For example we wuld filter by downward continuing only half as far below the observation surface as the continuation surface requires. This approach has an advantage over that used in the chessboard method in that the low-pass filter maintains the potential-field character of the result; it is not possible to generate a downward continued field that is smoother than the original field.

A hybrid equivalent source technique

TheXia method has alimitation that is clear from the geometry of the equivalent source layer and the observation surface. Namely, the method cannot accurately fit the shortest wavelength anomalies produced at the highest elevations of the terrain. After many iterations, it is these anomalies that slow convergence of the method. This limitation of the method may not be a problem in some situations, where it is desirable to filter out cultural anomalies at high elevations. In other situations, where these anomalies are due to geological sources or where the cultural sources are an important component of the continued field, we may want to preserve them in the equivalent source model using the following hybrid approach..

The Xia method can be used to fit most of the power in the observed magnetic anomaly field by iterating until the convergence becomes unreasonably slow. At this point the resulting equivalent source model can be used to calculate the field on the observation surface. The difference between this calculated field and the observed field represents the residual field that must be fit with an alternate method.

I have chosen to fit this residual field using a simple iterative method, similar to that used by Cordell(1992), in which the equivalent sources are magnetic dipoles (or point masses for gravity data) located on the topographic surface. The first step is to find the maximum amplitude value in the residual field, and fit it exactly with a single dipole source. Subtracting the field of the dipole from the starting residual field generates a new residual field. At each subsequent iteration the method adds a single dipole source or modifies an existing dipole source in order to fit the maximum amplitude of the current residual field. This process continues until up to 10,000 equivalent dipoles have been generated. This is not a sufficient number of dipoles to accurately fit the original anomaly field, but it is a sufficient number to fit the residual field resulting after application of the Xia method.

The process of generating the equivalent dipole sources typically requires a overnight run on a fast personal computer. Note that the dipoles will not necessarily lie on grid nodes unless the magnetic field is at the pole or at the equator. At mid-latitudes, because the residual anomaly peaks are on grid nodes, the equivalent dipoles will be shifted away from the grid nodes along the direction of the magnetic declination.

At the end of this process we have two equivalent source models: a magnetization grid on a horizontal surface that fits most of the observed anomalies, and a set of equivalent dipoles on the topographic surface that fits the rest of the observed anomaly field. To wntinue the observed field to another surface, we merely compute the fields on the new surface due to the two equivalent source distributions and add them together. For areas of downward continuation, the variable low-pass filtering approach outlined for the Xia method can also be applied to the equivalent dipole method. In fact, it is even more critical that it be applied to the short wavelength residual field modeled by the equivalent dipole method.

Software availability

Software to construct the models and perform potential-field continuation using the various methods discussed in this abstract, as well as the Taylor-series method (Grauch, 1984; Cordell and Grauch, 1985), is included in the latest release of the U. S. Geological Survey potential-field software package (Phillips and Grauch, in press).

Conclusions

- The simple Fourier transform method should not be used to continue potential-field data between parallel surfaces, unless the continuation distance is small relative to the sample interval or the relief of the surfaces is small relative to the sample interval..
- The chessboard method should only be used to continue potential-field data from a horizontal surface (or a surface having relief that is small relative to the sample interval) to an arbitrary surface.
- The equivalent source technique presented by Xia and others (1993) is a practical method for continuing potential-field data between arbitrary surfaces.
- A hybrid equivalent source method can be used to accurately fit anomalies at all wavelengths.
- Both equivalent source methods can be modified to permit variable low-pass filtering in areas of downward continuation.

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