Entropy

(2nd law for a Process, as opposed to a Cycle)

Transitional forms of energy:
- Mechanical \( \rightarrow \) work
- Thermal \( \rightarrow \) heat

• Work can be completely and continuously converted to heat.
• Opposite is not true.

Portion of heat that cannot be converted to work is unavailable energy.

That "unavailable energy" has to be rejected as "low-grade" heat.

• Energy is conserved, but "availability" is not.

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Entropy

• Introduced & named by Clausius in 1865
• Entropy is a property
• Units of Btu/lbm \( \rightarrow \) J/kg
  \( \text{Btu/lbm, J/kg for specific enthalpy} \)

Consider Non-Flow Work (gas expansion against a piston)

\[ W_2 = \int PdV \]

- Graphically, the total work is the area under the curve.

It would be convenient to have a similar graph where the area under the curve represents the heat transfer during a process.

For the work above, the pressure is the driving potential.
For heat, temperature is the driving potential.

\[ Q_{2, \text{rev}} = \int TdS \]

\[ Q_2 \geq \int TdS \]

- Graphically, the total heat transfer for the process is the area under the curve.

Caution: the process must be reversible.
Reversible, Adiabatic Process: \( Q = 0 \)

\[ Q_2 = \int_T^2 T \, ds = 0 \]

Since \( T \neq 0 \), \( \int_T^2 ds = s_2 - s_1 = 0 \) \( \rightarrow S \) is constant = isentropic

Isentropic (\( ds = 0 \)) \( \iff \) reversible, adiabatic process

Definitions:

- **Entropy**: the property which remains constant in an adiabatic, reversible process.

- **Temperature**: the property which remains constant in an isothermal process.
The change in entropy, $S$, during an energy conversion process can be a useful measure of the efficiency of the process. While the First Law of Thermodynamics observes that energy is not created or destroyed during conversion, there is nearly always a portion of energy converted that becomes unusable, otherwise known as unavailable. The fraction of available energy to unavailable energy during a conversion process is a measure of conversion efficiency. No energy is lost even though the term lost work is often used to define efficiency. We merely observe that a fraction of energy converted appears impossible to recover. Therefore, determining the entropy change during a conversion process can assist with measuring total efficiency and optimizing energy conversion systems. In order to determine a change in entropy, we must be able to relate $dS$ to other measurable properties.

Consider a closed system on which reversible heat transfer and reversible work are applied to affect an energy change.

Applying the 1st Law to this process:

$$\Delta Q_{\text{rev}} - \Delta W_{\text{rev}} = \Delta E$$

For a closed system, $\Delta E = \Delta U$

$$TdS - PdV = dU$$

On a per mass basis:

$$\boxed{TdS = dU + PdN}$$  \textit{- Gibbs Equation}

Integrating the Gibbs equation will relate changes in entropy, $dS$, to pressure, temperature and volume, provided a property relationship exists for internal energy. The Gibbs equation can also be expressed in terms of enthalpy changes.

$$h = u + Pn \quad \rightarrow \quad dh = d(u + Pn) = \{du + Pdn\} + Ndp$$

$$\boxed{TdS = dh - Ndp}$$
Entropy Change in an Incompressible Substance (liquid or solid)

Incompressibility implies that $dN = 0$, or $dV = 0$. Pressure and temperature changes do not affect specific volume or density. Also, the change in internal energy of an incompressible substance is $du = c dT$. Therefore, the Gibbs equation can be simplified to

$$Td s = d u + P dN = c d T$$

which when integrated becomes:

$$S_2 - S_1 = C_l n \left( \frac{T_2}{T_1} \right) \quad [\text{liquid, solid}]$$

If the temperature of an incompressible substance remains constant during an energy conversion process, then that process is isentropic ($dS = 0$).

Entropy Change in an Ideal Gas

For an ideal gas, the property relationships of interest are $P = RT$ and $du = C v dT$. Substituting these into the Gibbs equation,

$$dS = \frac{du}{T} + \frac{P dV}{T} = C v \frac{dT}{T} + R \frac{dN}{N}$$

Integration, assuming constant specific heats, results in:

$$S_2 - S_1 = C_v n \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{N_2}{N_1} \right) \quad [\text{ideal gas}]$$

When integrating the enthalpy form of the Gibbs equation:

$$S_2 - S_1 = C_p n \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{P_2}{P_1} \right) \quad [\text{ideal gas}]$$

Now changes in entropy of an ideal gas can be related to changes in pressure, temperature and volume.

Entropy Change in a Vapor or Liquid near Saturation (i.e., steam)

There is no simple algebraic expression that relates properties of vapor and liquids residing in energy states at or near saturation. Liquids are not completely incompressible and may evaporate during pressure, temperature or volume change. Vapors do not behave as an ideal gas nor as a non-ideal gas and may condense during an energy conversion process. As such relating changes in entropy to other measurable properties requires the use of the Gibbs equation and tabulated values of $T, P, N, c_v$ and $h$. 
Reference State for Entropy

Although we can only determine a change in entropy during an energy conversion process, the use of entropy involves a single state value. For example, the entropy of saturated steam (vapor) at 340°F and 64 psia is tabulated as 1.6657 Btu/lbm·°R. In actuality, this value of entropy is the difference in entropy at T = 340°F, P = 64 psia and some reference state, S°. Most often the reference state is taken to be at an absolute temperature of 0 K or °R. And the value of entropy at T = 0 K, °R is arbitrarily set to 0 Btu/lbm·°R.

Thus, S₂ - S₁ is really (S₂ - S°) - (S₁ - S°). As long as the reference state is the same for both S₁ and S₂, the difference can be determined. Caution should be exercised if values of entropy are found from different sources or tables since the reference state may not be the same. This caution is also true for changes in internal energy and enthalpy.
Isentropic Process with a liquid (incompressible)

For a liquid, \( \Delta S = c \ln \left( \frac{T_2}{T_1} \right) \). For \( \Delta S = 0 \), \( T_1 = T_2 \). Therefore, the temperature of an incompressible substance remains constant during an isentropic process.

Isentropic Process with an ideal gas

\[
S_2 - S_1 = 0 = c \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{N_2}{N_1} \right)
\]

Rearranging,

\[
\ln \left( \frac{T_2}{T_1} \right) = -\frac{E}{c} \ln \left( \frac{N_2}{N_1} \right) = \ln \left( \frac{N_1}{N_2} \right) \frac{E}{c}
\]

Recall that \( R = \frac{E}{c} \) and \( k = \frac{E}{c} \), substitution of the definitions results in:

\[
\frac{T_2}{T_1} \left| \Delta S = 0 \right. = \left( \frac{N_1}{N_2} \right)^{\frac{k}{c}} \text{ [ideal gas]}
\]

Similarly,

\[
\frac{T_2}{T_1} \left| \Delta S = 0 \right. = \left( \frac{P_2}{P_1} \right)^{\frac{k}{c}} \text{ [ideal gas]}
\]

\[
\frac{P_2}{P_1} \left| \Delta S = 0 \right. = \left( \frac{N_1}{N_2} \right)^{\frac{k}{c}} \text{ [ideal gas]}
\]

These three isentropic process relationships for an ideal gas can be rewritten as:

\[
\begin{align*}
T & \sim k' = \text{constant} \\
T P^{k'} & = \text{constant} \\
P N^k & = \text{constant}
\end{align*}
\]

The last expression, \( P N^k = \text{constant} \), is in form of a polytropic process where the polytropic coefficient is equal to the specific heat ratio \( \gamma \), which represents the most efficient possible polytropic process with an ideal gas because the entropy change is zero.
\[ Q = \int T \, d\theta \]

\[ \delta Q = T \, d\theta \]

\[ d\theta = \frac{\delta Q}{T} \]

Claudius named this entropy and gave it the symbol \( S \).

\[ \text{Entropy} = \text{the property which remains constant during an adiabatic, reversible process} \]

\[ \delta Q \leq T \, dS \]

Reversible and Transfer

\[ \begin{array}{c}
\text{Reversible} \\
\begin{array}{c}
\Delta T \\
\text{Finite Temperature Difference}
\end{array}
\end{array} \]

\[ \frac{n}{V} \]

\[ \frac{q}{s} \]

\[ \frac{q}{s} \]

\[ \frac{q}{s} \]
A polytropic process is described by $P V^n = constant$ and is derived by linear interpolation of a pressure-volume path on a log-log plot. A polytropic process is particularly useful for studying work from expansion or to compression of a gas.

For example, the process that generates the maximum work in the $P V$ diagram shown is obvious. But which process has the least irreversibility? In other words, which process is closest to being isentropic ($\Delta S = 0$)?

If the process is polytropic, which is often an accurate approximation for compression or expansion of a gas, then the work from the process is found by substituting $C/V^n$ for $P$ in the work integral:

$$\int W_2 = \int P dV = \int \frac{C dV}{V^n} = C \left( \frac{V_2}{V_1} \right)^{1-n} \left( \frac{V_1}{V_2} \right)^{1-n}$$

Since $P V^n = P V^n = C$, the work from a polytropic process is:

$$\int W_2 = \frac{P_2 V_2 - P_1 V_1}{1-n}$$

Provided the polytropic coefficient $n \neq 1$.

When $n = 1$, the work is a special case of an isothermal ideal gas.

$$\int W_2 = \int P dV = \int \frac{dV}{V} = C \ln \left( \frac{V_2}{V_1} \right)$$

Since $P V = P V = C$, the work from a polytropic process with $n = 1$ is:

$$\int W_2 = P_2 V_2 \ln \left( \frac{V_2}{V_1} \right) = \frac{P_2 V_2}{V_1} \ln \left( \frac{V_2}{V_1} \right)$$
Isoprocesses for an Ideal Gas

<table>
<thead>
<tr>
<th>Process</th>
<th>$PV^n = CT$</th>
<th>$\Delta U$</th>
<th>$\Delta H$</th>
<th>$\Delta S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant volume; isochoric</td>
<td>$\infty$</td>
<td>$c_v \Delta T$</td>
<td>$c_p \Delta T$</td>
<td>$c_v \ln \left( \frac{T_2}{T_1} \right)$</td>
</tr>
<tr>
<td>constant pressure; isobaric</td>
<td>$0$</td>
<td>$c_v \Delta T$</td>
<td>$c_p \Delta T$</td>
<td>$c_p \ln \left( \frac{P_2}{P_1} \right)$</td>
</tr>
<tr>
<td>constant temperature; isothermal</td>
<td>$1$</td>
<td>$0$</td>
<td>$0$</td>
<td>$R \ln \left( \frac{N_2}{N_1} \right)$</td>
</tr>
<tr>
<td>constant entropy; isentropic (adiabatic, reversible)</td>
<td>$k = \frac{c_p}{c_v}$</td>
<td>$c_v \Delta T$</td>
<td>$c_p \Delta T$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Polytropic

$0 < n < \infty$ : $c_v \Delta T$ : $c_p \Delta T$ : $\frac{c_v}{n} \ln \left( \frac{T_2}{T_1} \right) + \frac{R}{n} \ln \left( \frac{N_2}{N_1} \right)$
Table 1-3  Perfect-gas relationships (constant specific heats)

<table>
<thead>
<tr>
<th>Process</th>
<th>$P, v, T$ relationships</th>
<th>$u_2 - u_1$</th>
<th>$h_2 - h_1$</th>
<th>$s_2 - s_1$</th>
<th>$w$ (nonflow)</th>
<th>$w$ (flow)</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isothermal</td>
<td>$T = \text{constant}$</td>
<td>0</td>
<td>0</td>
<td>($RT\ln(v_2/v_1)$)</td>
<td>($P_1v_1/J\ln(v_2/v_1)$)</td>
<td>($P_1v_1/J\ln(v_2/v_1)$)</td>
<td>($P_1v_1/J\ln(v_2/v_1)$)</td>
</tr>
<tr>
<td>Constant pressure</td>
<td>$P = \text{constant}$</td>
<td>$c_v(T_2 - T_1)$</td>
<td>$c_p(T_2 - T_1)$</td>
<td>$c_p\ln(T_2/T_1)$</td>
<td>$P(v_2 - v_1)J$</td>
<td>0</td>
<td>$c_p(T_2 - T_1)$</td>
</tr>
<tr>
<td>Constant volume</td>
<td>$v = \text{constant}$</td>
<td>$c_v(T_2 - T_1)$</td>
<td>$c_p(T_2 - T_1)$</td>
<td>$c_p\ln(T_2/T_1)$</td>
<td>0</td>
<td>$\nu(P_1 - P_2)J$</td>
<td>$c_v(T_2 - T_1)$</td>
</tr>
<tr>
<td>Isentropic</td>
<td>$s = \text{constant}$</td>
<td>$c_v(T_2 - T_1)$</td>
<td>$c_p(T_2 - T_1)$</td>
<td>0</td>
<td>($P_{v_2} - P_{v_1})/J(1-k)$</td>
<td>$k(P_{v_2} - P_{v_1})/J(1-k)$</td>
<td>0</td>
</tr>
<tr>
<td>(Adiabatic</td>
<td>$P_{v_1}v_1^k = P_{v_2}v_2^l$</td>
<td>0</td>
<td>0</td>
<td>($RT\ln(v_2/v_1)$)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>reversible)</td>
<td>$T_2/T_1 = (v_2/v_1)^{-1}$</td>
<td>0</td>
<td>0</td>
<td>($RT\ln(v_2/v_1)$)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Throttling</td>
<td>$h = \text{constant}$</td>
<td>0</td>
<td>0</td>
<td>($RT\ln(v_2/v_1)$)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$T = \text{constant}$</td>
<td>$P_{v_2}v_2 = P_{v_1}v_1$</td>
<td>0</td>
<td>0</td>
<td>($RT\ln(v_2/v_1)$)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Polytropic</td>
<td>$P_{v_1}v_1^k = P_{v_2}v_2^l$</td>
<td>$c_v(T_2 - T_1)$</td>
<td>$c_p(T_2 - T_1)$</td>
<td>$c_p\ln(P_{v_2}/P_{v_1})$</td>
<td>$P_{v_2} - P_{v_1}J(1-n)$</td>
<td>$n(P_{v_2} - P_{v_1})J(1-n)$</td>
<td>$c_v\left(\frac{k-n}{1-n}\right)(T_2 - T_1)$</td>
</tr>
</tbody>
</table>

$J = \text{Energy Conversion Factor} = \gamma R \Phi / \text{Btu} = 1.0 \text{ Btu/lb}$

$P$ vs. $n$ for polytropic processes

$T$ vs. $n$ for polytropic processes

$S$ vs. $n$ for polytropic processes

$\gamma \leq 1.0$ (1984)
Adiabatic Expansion, \( \dot{Q} = 0 \)

\[
\frac{\dot{Q}}{\dot{W}} = \frac{1}{n - 1} \quad \text{(ideal gas)}
\]

1-2s: Reversible, Adiabatic (\( \Delta S = 0 \)) = Isentropic

\[
\frac{T_2}{T_{is}} = \left( \frac{P_2}{P_{is}} \right)^{\frac{n+1}{n-1}} \quad \text{(ideal gas)}
\]

1-2t: Unrestricted Expansion (throttling) \( \rightarrow \) most irreversible process

* Energy dissipated in fluid friction
* Temperature remains constant

1-2: Irreversible Expansion, but not as bad as throttling

1-2': Slightly greater irreversibilities than 1-2

1st Law for a Turbine:

\[
\dot{W}_f = \dot{m} \left( e_{\text{out}} - e_{\text{in}} \right)
\]

\[
e = \frac{\theta}{\rho} + \frac{1}{2} \rho v^2 + g h
\]

\[
\dot{W}_f = \dot{m} \left[ (\theta + P)_{\text{out}} - (\theta + P)_{\text{in}} \right] = \dot{m} \left( h_{\text{out}} - h_{\text{in}} \right)
\]

\[
\dot{W}_f = h_{\text{out}} - h_{\text{in}} \quad 1-2s: \dot{W}_s = h_1 - h_{2s} \quad \rightarrow \text{reversible}
\]

\[
\dot{W}_f = h_{\text{out}} - h_{\text{in}} \quad 1-2: \dot{W}_2 = h_1 - h_2 \quad \rightarrow \text{irreversible}
\]
The degree of irreversibility is given by the isentropic turbine efficiency, also known as: polytropic turbine efficiency.

$$\eta_t = \frac{\text{energy sought}}{\text{energy cost}} = \frac{\text{actual work}}{(\text{actual work}) + (\text{lost work})} = \frac{\text{actual work}}{\text{ideal work}} = \frac{h_i - h_z}{h_i - h_{zs}}$$

Irreversibilities

- For an ideal gas with constant specific heats, $h_i - h_z = C_p (T_i - T_z)$

$$\eta_t = \frac{h_i - h_z}{h_i - h_{zs}} = \frac{T_i - T_z}{T_i - T_{zs}}$$

$\quad$ only for ideal gas with constant $C_p$

"We can now state that the change in entropy is a measure of the unavailable energy." (E.T. Whitman)

That is, the portion of heat that cannot be converted to work.
Air expands in a gas turbine from 10 atm and 2000°F to 1 atm and 1050°F at the exhaust. Assuming a constant specific heat of 0.240 Btu/lbm°F, find the turbine work (in Btu/lbm) and the adiabatic turbine efficiency.

\[ W_{\text{turbine}} = h_i - h_f \]  
\text{(adiabatic expansion, } T_0 = 0) \]

For an ideal gas, \[ h_i - h_f = C_p(T_i - T_f) \]

\[ W_{\text{turbine}} = (0.240 \frac{\text{Btu}}{\text{lbm} \cdot \text{°F}})(2000 \text{ °F} - 1050 \text{ °F}) = 228 \text{ Btu/lbm} \]

- The adiabatic turbine efficiency is:
  \[ \eta_T = \frac{h_i - h_f}{h_i - h_f} = \frac{T_i - T_f}{T_i - T_{\infty}} \]

\text{Reversible, adiabatic expansion}

- Need to find adiabatic reversible expansion temperature, \( T_{\infty} \)

\text{For an adiabatic process & ideal gas,}

\[ \frac{T_{\infty}}{T_i} = \left( \frac{P_{\infty}}{P_i} \right)^{\frac{k}{k-1}} \text{ where } k = \frac{C_p}{C_v} \]

- For air, \( k = 1.4 \)

\[ T_{\infty} = \left( \frac{2000 \text{ °F} + 460 \text{ °R}}{10 \text{ atm}} \right)^{\frac{1.4}{1.4-1.4}} = 1274 \text{ °R} = 814 \text{ °F} \]

- Therefore,

\[ \eta_T = \frac{(2000 \text{ °F} - 1050 \text{ °F})}{(2000 \text{ °F} - 814 \text{ °F})} = 0.801 = 80.1\% \]

- or look up in table
Case of Vapor Expansion

\[ T_2 = T_2 \] because we are in the two-phase region.

Quality increases with irreversibilities, but temp. does not.

\[ H_2 > H_2^s \], ex\'t enthalpy is greater in the irreversible case.

Degree of irreversibility is

\[ \eta_T = \frac{h_1 - h_2}{h_1 - h_2^s} \]

Revisiting Carnot Cycle

All processes are reversible

Heat which is not available for conversion to work.
- Liquid methane used in cryogenic applications.

- Determine the entropy change in the liquid methane as it passes through a pump:
  (a) from property data.
  (b) assuming totally incompressible liquid, i.e., using this?

\[ \begin{align*}
P_1 &= 1 \text{ MPa} \\
T_1 &= 110 \text{ k} \\
P_2 &= 5 \text{ MPa} \\
T_2 &= 120 \text{ k}
\end{align*} \]

- Critical temperature of methane is 191 K (-82°C) i.e., temperature below which methane remains liquid.

(b) \[ S_2 - S_1 = \frac{0.270 \text{ kJ/kgK}}{3.143 \text{ kJ/kgK}} \]

\[ C_{avg} = 3.486 \text{ kJ/kgK} \]

\[ S_2 - S_1 = (3.486 \text{ kJ/kgK}) \ln \left( \frac{120 \text{ k}}{110 \text{ k}} \right) = 0.303 \frac{\text{kJ}}{\text{kgK}} \]

\[ \text{error} = \frac{\Delta S_{\text{actual}} - \Delta S_{\text{ideal}}}{\Delta S_{\text{actual}}} = 0.122 \ (12.2\%) \]

*Adapted from Cengel & Boles, 4th ed*
Example - Replacing Throttling Valve with Turbine

A cryogenic manufacturing facility handles liquid methane at 115 k and 5 MPa at a rate of 0.280 m³/s. A process requires dropping the pressure of the liquid methane to 1 MPa, which is accomplished by throttling the liquid methane by passing it through a valve (flow restrictor).

A recently hired engineer proposes to replace the throttling valve with a turbine in order to produce power while dropping the pressure to 1 MPa.

Determine the maximum power that the turbine can produce.

Determine how much this turbine could save the facility in electricity costs per year if the turbine operates continuously and the facility pays $0.075/kWh on average for electricity.

\[ \dot{V} = 0.280 \text{ m}^3/\text{s} \]

Assumptions:
- Adiabatic
- Reversible
- Steady flow
- Uniform flow
- Negligible change in kinetic & potential energy

1. \( P_1 = 5 \text{ MPa}, T_1 = 115 \text{ K} \)
2. \( P_2 = 1 \text{ MPa}, S_2 = S_1 \)

\[ h_1 = 232.2 \text{ kJ/kg}, S_1 = 4.9745 \text{ kJ/kgK} \]
\[ \rho_1 = 422.15 \text{ kg/m}^3, \dot{m} = \rho_1 \dot{V} = 118.2 \text{ kg/s} \]

\[ \dot{m} (e_2 - e_1) \]
\[ \text{Lower} \]

\[ h_2 = 222.8 \text{ kJ/kg} \]

\[ \dot{m} \rho_1 \dot{V} = 118.2 \text{ kJ/s} \]

\[ \dot{W}_{\text{turbine}} = (118.2 \text{ kg/s}) [222.8 \text{ kJ/kg} - 232.2 \text{ kJ/kg}] \]

\[ \dot{W}_{\text{turbine}} = 1123 \text{ kW} \]

Annual Power Production = \( \dot{W}_{\text{turbine}} \Delta t = (1123 \text{ kW})(8760 \text{ h/yr}) = 9.837 \cdot 10^6 \text{ kW-hr/yr} \)

At $0.075/kWh, Annual Savings = $737,800/yr

Actual turbine ~ 80% efficient → 900 kW & saves $600,000/yr

* Adapted from Cengel & Boles, 4th ed.
A steam turbine is used to drive a feedwater pump of a large utility boiler. A 17.78 kg/s flow of supercritical steam enters the turbine at 808.3 kPa and 23.76 MPa. The steam exits the turbine at 5.249 kPa with a quality of 0.9566. Determine the power produced by the turbine and the turbine isentropic efficiency.

**State 1 - Supercritical Steam**

\[ \begin{align*}
T_1 & = 808.3 \text{ k} \\
P_1 & = 23.26 \text{ MPa} \\
h_1 & = 3812.1 \text{ kJ/kg} \\
s_1 & = 6.1762 \text{ kJ/kg K} \\
\end{align*} \]

\[ h_2 = (1 - x_2)h_{S2} + \dot{z}_2 h_{g2} \]
\[ = (1 - 0.9566)(14130.9 \text{ kJ/kg}) + (0.9566)(25823.9 \text{ kJ/kg}) \]
\[ = 2457.2 \text{ kJ/kg} \]

\[ s_2 = (1 - x_2)s_{S2} + \dot{z}_2 s_{g2} \]
\[ = (1 - 0.9566)(0.48803 \text{ kJ/kg K}) + (0.9566)(3.2765 \text{ kJ/kg K}) \]
\[ = 8.0341 \text{ kJ/kg K} \]

\[ x_{2s} = \frac{s_{2s} - s_2}{s_{S2} - s_2} = 0.721 \]

\[ h_{2s} = (1 - x_{2s})h_{S2} + x_{2s} h_{g2} = 1887.0 \text{ kJ/kg} \]

**Specific Work**:

\[ w_e = h_1 - h_2 = 3812.1 \text{ kJ/kg} - 2457.2 \text{ kJ/kg} = 854.9 \text{ kJ/kg} \]

**Power**:

\[ W_t = m \cdot w_e = (17.78 \text{ kg/s})(854.9 \text{ kJ/kg}) = 15,200 \text{ kW} \]

**Isentropic Efficiency**:

\[ \eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{3812.1 \text{ kJ/kg} - 2457.2 \text{ kJ/kg}}{3812.1 \text{ kJ/kg} - 1887.0 \text{ kJ/kg}} = 0.60 = 60\% \]
**Process** - a system transitions from one equilibrium state to another.

**Path** - series of states that a system passes through during a process.

**Quasi-equilibrium** - process during which the system remains nearly in equilibrium.

**Iso-processes** - process during which one property is constant

- Isochoric \( \rightarrow \) Volume is constant
- Isobaric \( \rightarrow \) Pressure is constant, also isoplastic
- Isothermal \( \rightarrow \) Temperature is constant
- Isentropic \( \rightarrow \) Entropy is constant
- Adiabatic \( \rightarrow \) no transitional thermal energy conversion

**Cycle** - succession of states in which the system ultimately returns to the initial state.

---

"In order to convert forms of energy, particularly heat, to work on an extended or continuous basis, one needs to operate on a cycle."

"A series of processes that begins and ends at the same state, and thus can repeat indefinitely, or as long as needed."

**Ideal Diesel Cycle (no losses)**

For the Cycle: \( Q_i - W_i = 0 \)  \( \Rightarrow \) \( Q_{net} = Q_{in} - |Q_{out}| = \Delta W_{net} \)

*Alternative form of the first law:
\[
\int_1^5 Q = \int_1^5 W
\]
*Units conversion factor; \( J = 778.16 \frac{ft^3}{lbm} \) = 1.0 \( \frac{N \cdot m}{J} \)
Engine Efficiency, $\eta_{th} = \frac{\text{Work}}{\text{Qin}}$

Gas Cycle Heat Engines:
- Internal Combustion, Spark Ignition (SI)
- Internal Combustion, Compression Ignition (CCI)
- External Combustion, Piston
- External Combustion, Turbine
- Otto Cycle
- Diesel Cycle
- Stirling Cycle
- Ericsson, Brayton

**Otto Cycle**

![Idealized cycle diagram](image-url)
Gas Cycles

S.I. Engine (Otto)
C.I. Engine (Diesel)
Gas Turbine (Brayton & Ericsson)
External Combustion (Stirling)

"Open" Cycle → working fluid does not go through a complete cycle

• Advantageous to devise a "closed" cycle that approximates the "open" cycle.

Air Standard Cycle

• Stott w/Otto cycle
  - description
  - image of engine from notes manual
  - EES example
  - indicator from cabinet
Gas Cycles

Air Standard Cycles

- S.I. Engine
- Diesel Engine
- Gas Turbine

"Open" cycle: working fluid does not go through a complete cycle.

- Advantageous to devise "closed" cycles that approximate the "open" cycle.

Air Standard Cycle

- Fixed mass of air throughout the entire cycle.
- No inlet/exhaust process.
- Air is an ideal gas.
- Compression process is replaced by heat transfer from an external source.
- Cycle is completed by heat transfer to surroundings.
- All processes internally reversible.
- Air has constant specific heat.

A S C: qualitative study of the influence of a number of variables.

- Efficiency, mean effective pressure, etc. don't match actual systems.

\[
\text{Work} = \left(\text{MEP} \cdot A \cdot \text{piston} \right) \text{ stroke per cycle.}
\]

Air Standard Constant Cycle

\[
\frac{V_2}{V_1} = 1 - T_1 \frac{T_H}{T_4} = 1 - T_1 \frac{T_L}{T_4} = 1 - \frac{T_2}{T_4}
\]

Other measures of efficiency:

- Isentropic compression ratio: \( \frac{V_4}{V_1} = \left( \frac{T_L}{T_1} \right)^{\gamma/(\gamma - 1)} \)
- Isentropic expansion ratio: \( \frac{V_2}{V_4} = \left( \frac{T_4}{T_2} \right)^{\gamma/(\gamma - 1)} \)
Second Law

1st Law for a cycle: \[ \Delta Q = \Delta W \]
- Precluded: heat rejection, friction, etc.
- Given a conversion efficiency, can analyze a real system

2nd Law is based on the heat engine; really only applies to a heat engine

Heat Engine:
- Conversion of thermal energy (heat) to mechanical energy (work)
- Using two thermal reservoirs

Thermal Reservoir:
- Constant temperature source/sink
- Heat transfer from thermal reservoir does not change

Steam Turbine (Rankine Cycle)

Thermal Efficiency:
\[ \eta_{\text{thermal}} = \frac{\text{energy output}}{\text{energy cost}} = \frac{W}{Q_H} \]

\[ W = Q_H - Q_L \quad (1\text{st Law}) \]

\[ Q_L \neq 0 \quad (2\text{nd Law}) \]

\[ \eta_{\text{thermal}} = \frac{Q_H - Q_L}{Q_L} = 1 - \frac{Q_H}{Q_L} \]

Refrigeration Cycle

Coefficient of Performance (COP):
\[ \beta = \frac{\text{energy output}}{\text{energy cost}} = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} \]

\[ \beta = \frac{1}{Q_H/Q_L - 1} \]
Second Law Statements

(A) Kelvin-Planck: Impossible to build a 100% efficient heat engine.

\[ W < Q_H \]

(B) Clausius: Impossible to transfer heat from a cold body to a hot body without work input (cyclically)

\[ Q_L \neq Q_H \text{ in a cycle} \]

* cannot prove 'negative' postulates; proof rests on experimental evidence.

Reversible Processes

- deviation from equilibrium is infinitesimal and occurs at an infinitely slow rate
- work expressions from Web lecture

Irreversibilities:

- friction
- uncontrolled expansion
- heat transfer through \( \delta T \) (as opposed to \( dT \))
- refrigeration required to restore system which injects heat to surroundings
- mixing of two substances
- Joule heating
- combustion

External vs. Internal Irreversibilities

Internal \( \Rightarrow \) no irreversibility occurs within boundary during process

External \( \Rightarrow \) no irreversibility occurs outside boundary during process

\[ \begin{align*}
T & \quad V \\
-\Delta T & \quad -\Delta T
\end{align*} \]

(a) Externally reversible

Internally reversible

(b) Internally irreversible

Externally reversible

System passes through exact same state, but (a) is externally reversible and (b) is only internally reversible.
**Canot Cycle** (proposed in 1824 by Sadi Carnot in 1876-1832)

**Canot Heat Engine** - Cycle composed of 4 reversible processes

1. **Reversible Isothermal Expansion**
2. **Reversible Adiabatic Expansion**
3. **Reversible Isothermal Compression**
4. **Reversible Adiabatic Compression**

- **Thermodynamic Process:**
  - **1-2:** Isothermal process where temperature remains constant between 1 and 2.
  - **2-3:** Adiabatic process where heat transfer is zero during expansion.
  - **3-4:** Isothermal process where temperature remains constant between 3 and 4.
  - **4-1:** Adiabatic process where heat transfer is zero during compression.

**Graphical Representation:**

- **Thermodynamic Diagram:**
  - **1-2-3-4-1** cycle

**Propositions:**

1. **Proposition:** It is impossible to construct a heat engine that is more efficient than a reversible engine.

2. **Proposition:** All engines that operate on a Canot cycle between two given constant-temperature reservoirs have the same efficiency.
The dependence of Carnot cycle upon temperatures and not working fluid is basic for the thermodynamic temperature scale.

\[ \text{Thermal} = \frac{\text{Heat}}{\text{In}} = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} \]

\[ \frac{Q_{\text{out}}}{Q_{\text{in}}} = f\left(T_H, T_L\right) \quad \text{for a Carnot cycle} \]

\[ \frac{Q_{\text{in}}}{Q_{\text{out}}} = f\left(T_L, T_H\right) \]

\[ \frac{Q_{\text{in}}}{Q_{\text{out}}} = f\left(T_L, T_H\right) \quad \frac{Q_{\text{out}}}{Q_{\text{in}}} = f\left(T_H, T_L\right) \]

\[ f\left(T_L, T_H\right) f\left(T_H, T_L\right) = f\left(T_L, T_H\right) \]

Therefore, \[ f\left(T_L, T_H\right) = \frac{Q_{\text{out}}}{Q_{\text{in}}} \]

\[ \frac{Q_{\text{out}}}{Q_{\text{in}}} = \frac{\Phi(T_H)}{\Phi(T_L)} \quad (1) \]

Many functional relationships will satisfy eqn (1). Lord Kelvin proposed two different relationships:

1. \[ \frac{Q_{\text{out}}}{Q_{\text{in}}} = \frac{T_H}{T_L} \quad \text{Kelvin Scale} \quad 0 \leq T < \infty \]

2. \[ \frac{Q_{\text{out}}}{Q_{\text{in}}} = \frac{e^{T_H}}{e^{T_L}} \quad \text{relation between (1) & (2):} \]

\[ \log_{10}\left(\frac{Q_{\text{out}}}{Q_{\text{in}}}\right) = T_H - T_L \]

\[ \frac{T}{T_H} = \log_{10} T + L \]

where \( L \) is temp. that corresponds to 0 on the logarithmic scale

\[ -\infty < \frac{T}{T_H} < \infty \]

Carnot Efficiency: \[ \eta_c = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{T_L}{T_H} \]

Minimum efficiency of a thermal power cycle.
Ideal Gas Temperature Scale

\[ P_T = \frac{RT}{V} \]

\[ T = \frac{P_T}{\frac{P}{V_T}} \]

triple point of water

No gas is ideal; however, as \( P \to 0 \), the behavior of all gases approach that of an ideal gas.

No gas is ideal; however, as \( P \to 0 \), the behavior of all gases approach that of an ideal gas.

This is the ideal-gas temperature scale, equivalent to the thermodynamic temperature scale. Can be shown by using an ideal gas as the working fluid in a Carnot cycle.
VALUES OF THE GAS CONSTANT, \( R \)

Coleman J. Major

The numerical value of the gas constant, \( R \), defined by the equation \( PV = nRT \), depends upon the units of \( P \), \( V \), \( n \), and \( T \). A large number of values of the constant may be calculated. The accompanying table gives 84 values of \( R \) in a convenient form using the most common units of pressure and volume. It also incorporates both the pound and gram mole and both Rankine and Kelvin temperature scales. Various combinations of metric and English units may, therefore, be used without the necessity of converting each variable to a common system of units. Conversion factors and constants used for computing the values of \( R \) are listed at the bottom of the table. The following example illustrates the use of the table:

Calculate: The volume in ft\(^3\) occupied by 2 lb. moles of a gas at 15°C at a pressure of 32.2 ft. of water, assuming the ideal gas law.

Solution: \( 15°C + 273.2 = 288.2^°K \)

Enter the top of the table under the column headed “ft H\(_2\)O” and proceed downward to the value of 44.6 for \( R \). (Note that this lines up horizontally with the desired units of ft\(^3\), °K, and lb. moles shown on the left side of the table)

\[
V = \frac{nRT}{P} = \frac{2 \times 44.6 \times 288.2}{32.2} = 798 \text{ ft}^3
\]

Values of Gas Constant, \( R = \frac{PV}{nT} \)

<table>
<thead>
<tr>
<th>Absolute Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>ft H(_2)O</td>
</tr>
<tr>
<td>in H(_2)O</td>
</tr>
<tr>
<td>cm Hg</td>
</tr>
<tr>
<td>mm Hg</td>
</tr>
<tr>
<td>psia</td>
</tr>
<tr>
<td>Atm</td>
</tr>
<tr>
<td>gm</td>
</tr>
<tr>
<td>lb</td>
</tr>
</tbody>
</table>

| gm | 0.00290 | 0.0426 | 2.20 | 0.220 | 0.0867 | 1.18 | 0.0982 |
| lb | 1.31    | 19.31  | 999  | 99.9  | 39.3   | 535  | 44.6   |
| gm | 0.00161 | 0.0266 | 1.22 | 0.122 | 0.0482 | 0.655| 0.0546 |
| lb | 0.730   | 10.73  | 555  | 55.5  | 21.8   | 297  | 24.8   |

| cm | 82.05  | 1206  | 62,400| 6240  | 2450   | 33,400| 2780   |
| lb | 37.200 | 547,000| 2.83 \times 10^3 | 2.83 \times 10^3 | 1.11 \times 10^3 | 1.51 \times 10^3 | 1.26 \times 10^3 |
| gm | 45.6   | 670   | 34,600| 3460  | 1360   | 18,500| 1550   |
| lb | 20.700 | 304,000| 1.57 \times 10^7 | 1.57 \times 10^7 | 619,000 | 8.41 \times 10^7 | 701,000 |

| liters | 0.03205 | 1.206 | 62.4 | 6.24 | 2.45 | 33.4 | 2.78 |
| gm     | 37.2    | 547   | 28,300| 2830 | 1113 | 15,140| 1262 |
| lb     | 20.7    | 304   | 15,700| 1570 | 619  | 8410 | 701   |

Conversion Factors and Constants

\[
1 \text{ lb.} = 453.59 \text{ gm} \quad 359.0 \text{ ft}^3/\text{lb mole}
\]

\[
1 \text{ atm} = 14.696 \text{ psia} \quad 22.414 \text{ cm}^3/\text{gm mole}
\]

\[
1 \text{ atm} = 760 \text{ mm Hg} \quad 1 \text{ inch} = 2.54 \text{ cm}
\]

\[
1 \text{ atm} = 76 \text{ cm Hg} \quad \text{Std. temp.} = 273.16^°\text{K or} 491.69^°\text{R}
\]

\[
1 \text{ atm} = 29.921 \text{ in Hg} \quad 28.31605 \text{ liters} = 1 \text{ ft}^3
\]

\[
1 \text{ atm} = 406.79 \text{ in H}_2\text{O} \quad R = 8.31432 \pm 0.00034 \times 10^7 \text{ erg}^°\text{K}^{-1} \text{ mol}^{-1}
\]

\[
1 \text{ atm} = 33.90 \text{ ft H}_2\text{O}
\]