12/1/2016

Electromagnetic-to-Eletrical Energy Conversion
(Solar-to-Electric; Photovoltaics)

- Photon Energy
- Photon Flux in Solar Spectrum
- Photoelectric Effect
- n-p Junction and charge Carriers
  - maximum electrical power output
  - maximum conversion efficiency

Electromagnetic energy (solar radiation) may be thought of as discrete packets (quanta) of energy; referred to as photons. The energy of a photon is proportional to the frequency of the electromagnetic energy.

$$e_{photon} \sim \nu$$

frequency, Hz [cycles/second]

The proportionality constant between photon energy and electromagnetic frequency is Planck's constant.

$$e_{photon} = \hbar \nu$$

Planck's constant

$$\hbar = 6.626 \times 10^{-34}\ \text{J}\cdot\text{s/quanta}$$

$$h = 4.136 \times 10^{-15}\ \text{eV}\cdot\text{s/quanta}$$

for electromagnetic energy, the quanta is a photon

Photon energy can also be expressed in terms of electromagnetic wavelength,

$$e_{photon} = \hbar \nu = \hbar \frac{c}{\lambda}$$

speed of light in a vacuum, 3×10^8 m/s

- Photon energy in the middle of the visible spectrum, $\lambda = 550$ nm:

$$e_p = \frac{c\hbar}{\lambda} = \frac{(3 \times 10^8\ \text{m/s})(6.626 \times 10^{-34}\ \text{J}\cdot\text{s/photon})}{(550 \times 10^{-9}\ \text{m})} = 3.62 \times 10^{-19}\ \text{J/photon}$$

$$e_p = \left(3.62 \times 10^{-19}\ \text{J/photon}\right)\left(1\ \text{eV}/1.6 \times 10^{-19}\ \text{J}\right) = 2.26\ \text{eV/photon}$$

Photon Flux in Solar Spectrum

photon flux, $\dot{\phi}''_p = \dfrac{\#\text{ of photons}}{\text{time} \cdot \text{area}}$

photon energy flux, $\dot{e}''_i = \phi_{p_i} \nu_i \hbar$

$i \equiv$ frequency band

total photon energy flux

$\dot{e}'' = \sum_i \phi_{p_i} \nu_i \hbar$

using an average frequency, $\dot{e}'' \approx \bar{\Phi}_p \bar{\nu} \hbar = \bar{\Phi}_p \dfrac{c\hbar}{\bar{\lambda}} \quad \left[\dfrac{J}{s m^2} = \dfrac{W}{m^2}\right]$

- extraterrestrial solar energy flux = solar constant, S

$$S = 1359 \frac{W}{m^2}$$

$m_a = 0$ (air mass ratio)

$w = 0$ (water vapor content)

$$\hbar \bar{\nu} = 1.48 \frac{eV}{\text{photon}}$$

average frequency of solar insolation

Thus, the extraterrestrial photon flux is

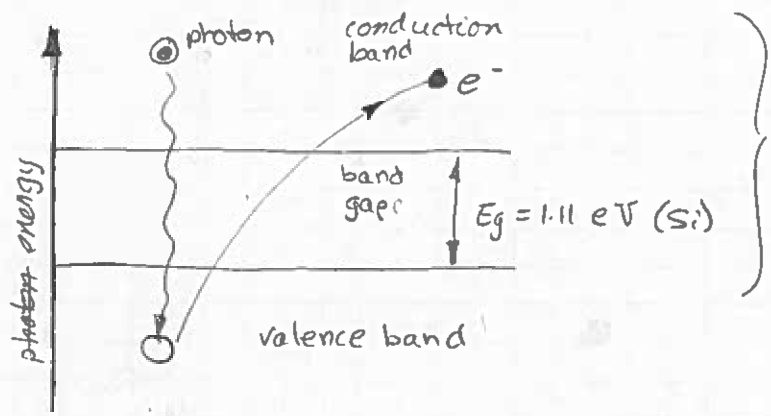
$$\bar{\Phi}_{p,et} = \frac{(0.1359 \ W/cm^2)}{(1.48 \ eV/\text{photon})} \left(1 \ eV / 1.6021 \times 10^{-19} \ J/eV \right) = 5.8 \times 10^{17} \frac{\text{photons}}{cm^2 \cdot s}$$

- For $m_a = 3$ and $w = 5$, $\bar{\Phi}_p \approx \frac{1}{2} \bar{\Phi}_{p,et}$

($\beta_1 \approx 20°$)

metals: many free electrons in conduction band that are free to move in an electric field
↳ conductor ↳ electron gas, electron cloud

silicon (Si): normally, no electrons in conduction band – insulator



Silicon atom in crystal absorbs a photon, which if sufficiently energetic frees an electron into the conduction band forming an electron-hole pair.

Normally, the electron-hole pair disappear spontaneously.

Wavelength of photons that possess sufficient energy to free an electron in a silicon lattice:

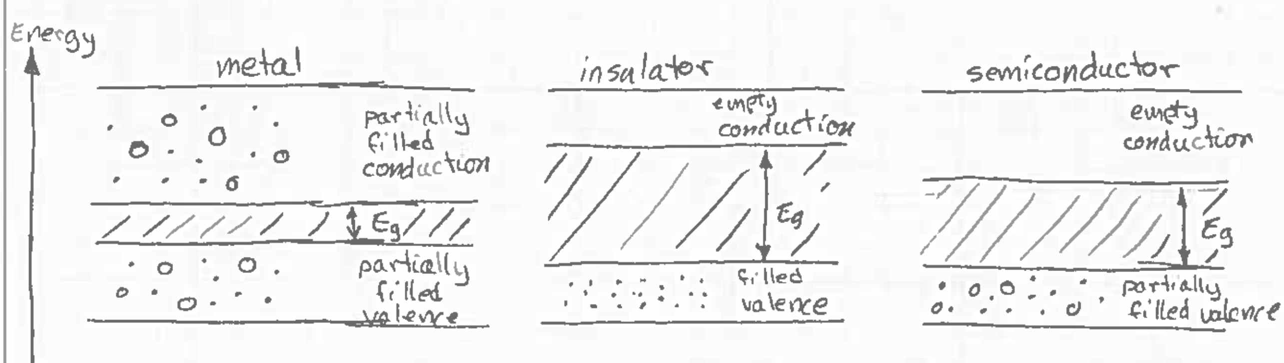
$$E_p = c\frac{h}{\lambda_p} \geq E_g \longrightarrow \lambda \leq 1.12\ \mu m$$

- Photons at wavelengths $> 1.12\ \mu m$ do not have sufficient energy to generate an electron-hole pair; if absorbed these photons are converted to thermal energy through the kinetic energy of valence band electrons.

77% of direct solar energy flux as at a wavelength $\leq 1.12\ \mu m$
↑ non-diffuse

One photon can only generate a single electron-hole pair regardless of how much more energetic than band gap energy.

Band Gaps



n-p Junction

Silicon (Si):
    4 valence electrons (outer shell) all participating in covalent bond
    no free electrons
    insulator

Arsenic (As):
    5 valence electrons
    If As atom replaces Si atom in crystal lattice, then only 4 valence electrons participate in covalent bond leaving one electron free to participate in conduction band.
    Si doped with As is negatively charged with free e^- → n-type semiconductor

Boron (Bo), Aluminum (Al), Indium (In):
    3 valence electrons
    If one of these atoms replaces a Si atom in crystal lattice, then an electron vacancy (hole) is created.
    Si doped with Bo, Al, or In is positively charged → p type semiconductor

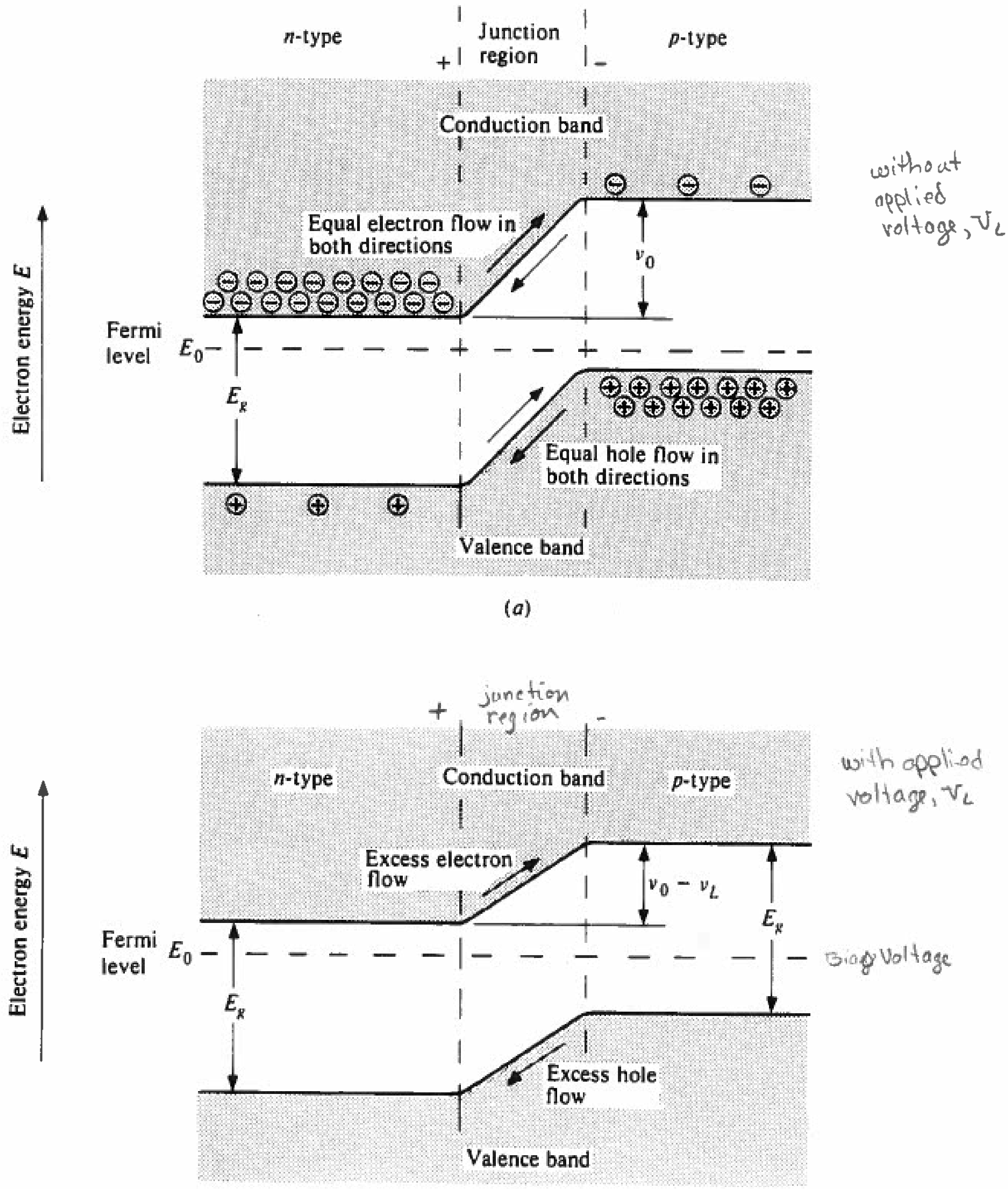


FIGURE 8.12
The charge distribution at an *n-p* junction of semiconductors (*a*) without and (*b*) with an applied voltage v_L. (*From Walsh, 1967.*)

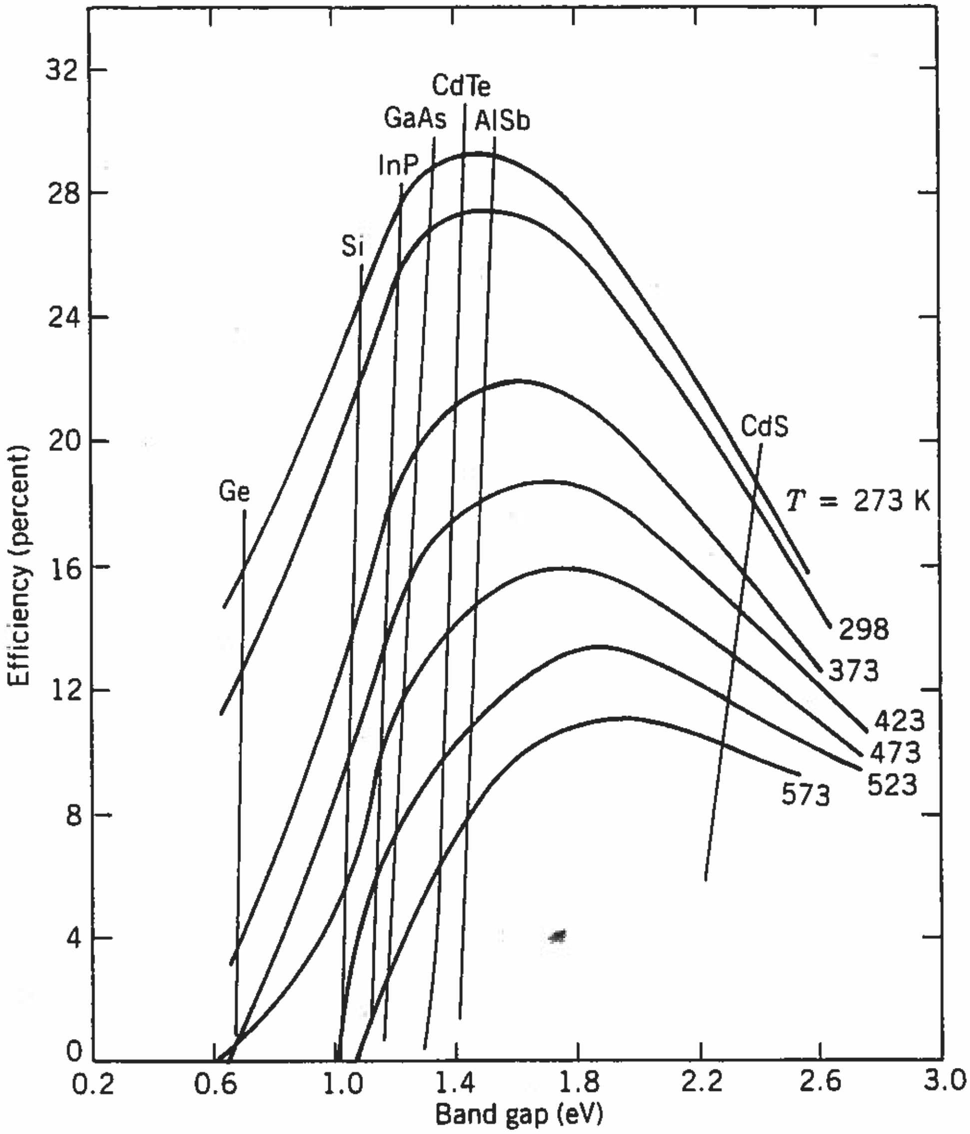


Figure 6.20 Photovoltaic conversion efficiency for several semiconductor materials having different band gaps. For all materials, the efficiencies drop as the temperature increases. Actual devices produce lower efficiencies than the theoretically maximum values.

Example 1

A monochromatic source of photons (red laser pointer) has a power of 1 mW and a wavelength of 638 nm. When pointed at silicon, compute

(a) number of photons per second, and

(b) maximum efficiency of conversion to electrical energy.

Light Intensity, $I_p = \underbrace{N_p}_{\#\text{ of photons}} \overbrace{E_p}^{\text{poton energy}}$

$1\ eV = 1.6021 \cdot 10^{-19}\ J$

$$E_p = \frac{hc}{\lambda} = 3.12 \cdot 10^{-19}\ J = 1.95\ eV \text{ per photon}$$

$$I_p = 1\ mW = 10^{-3}\ \frac{J}{s}$$

$$\text{(a) } N_p = \frac{10^{-3}\ J/s}{3.12 \cdot 10^{-19}\ J/\text{photon}} = 3.21 \cdot 10^{15}\ \frac{\text{photons}}{s}$$

Maximum conversion efficiency is when each photon generates an electron. There are no reflective or scattering losses.

$$\text{Electrical Power} = N_p \cdot E_{bandgap}$$

$$\dot{W}_e = \underbrace{\left(1.11\ \frac{eV}{\text{photon}}\right)}_{E_{bandgap}}\left(1.6021 \cdot 10^{-19}\ \frac{J}{eV}\right)\left(3.21 \cdot 10^{15}\ \frac{\text{photons}}{s}\right) = 5.71 \cdot 10^{-4}\ J/s$$

$$\text{Electromagnetic Power} = \{\text{photon energy}\}\{\#\text{ of photons}\} = 10.03 \cdot 10^{-4}\ J/s$$

$$\eta = \frac{\text{energy sought}}{\text{energy cost}} = \frac{\text{band gap energy}}{\text{photon energy}} = 0.569$$

Example 2

A 7.5-cm-diameter circular photovoltaic solar cell is subjected to a solar energy flux of 2.5×10^{17} photons/s.cm² at an average photon wavelength of 0.838 μm. Calculate

(a) the solar insolation on the cell, in W/m², and
(b) the maximum theoretical power that can be produced by the cell, in W.

solar insolation: $\dot{E}'' = \left\{\begin{matrix}\text{energy per}\\ \text{photon}\end{matrix}\right\}\left\{\begin{matrix}\text{photon}\\ \text{flux}\end{matrix}\right\}$

$$e_p = h\bar{\nu} = \frac{ch}{\lambda} = \frac{(3\times10^{8}\ \mathrm{m/s})(4.13576\times10^{-15}\ \frac{\mathrm{eV\cdot s}}{\mathrm{photon}})}{(0.838\times10^{-6}\ \mathrm{m})} = 1.48\ \frac{\mathrm{eV}}{\mathrm{photon}} = 2.37\times10^{-19}\ \frac{\mathrm{J}}{\mathrm{photon}}$$

$$\bar{\nu} = 3.58\times10^{14}\ 1/\mathrm{s}$$

$$\dot{E}'' = e_p\,\phi = \left(2.37\cdot10^{-19}\ \frac{\mathrm{J}}{\mathrm{photon}}\right)\left(2.5\times10^{17}\ \frac{\mathrm{photons}}{\mathrm{s\cdot cm^2}}\right) = 0.0593\ \frac{\mathrm{J}}{\mathrm{s\cdot cm^2}} = \underline{\underline{592.8\ \frac{\mathrm{W}}{\mathrm{m^2}}}} \quad \underline{\underline{(a)}}$$

• The maximum theoretical power is $\approx \eta_{max}\cdot \dot{E}''\cdot A$

$\eta_{max} \approx 48\%$ (p. 580, El-Wakil)

$A = \frac{\pi}{4}D^2 = 44.18\ \mathrm{cm^2} = 0.0044\ \mathrm{m^2}$

$$\dot{W}_{max,theoretical} = \underline{\underline{1.26\ \mathrm{W_e}}} \quad \underline{\underline{(b)}}$$

Example 3

A 0.08-m-diameter circular photovoltaic cell receives the following solar fluxes, in photons per second per square centimeter:

0.5×10^{17} $(0.3-0.5\,\mu m)$,
0.85×10^{17} $(0.5-0.7\,\mu m)$,
0.50×10^{17} $(0.7-0.9\,\mu m)$,
0.45×10^{17} $(0.9-1.1\,\mu m)$,
0 $(<0.3\,\mu m, >1.1\,\mu m)$.

Estimate:

(a) the number of such cells necessary to produce 10 kW of AC power, and
(b) the array area on which they are mounted.

$\longrightarrow$

Power: $\dot{W}_{em} = \sum \Phi_\lambda h\bar{\nu} = \sum \Phi_\lambda h \frac{c}{\lambda} = ch \sum \frac{\Phi_\lambda}{\lambda}$

$$\dot{W}_{em} = \left(3\cdot10^{8}\,\tfrac{m}{s}\right)\left(6.626\cdot10^{-34}\,\tfrac{J\cdot s}{photon}\right)\left\{\left[\frac{0.5}{0.4}+\frac{0.85}{0.6}+\frac{0.5}{0.8}+\frac{0.45}{1.0}\right]\left(\frac{10^{17}\,photons/cm^2\cdot s}{10^{-6}\,m}\right)\left(\frac{100\,cm}{m}\right)^2\right\}$$

$$\dot{W}_{em} = 743.8\,\frac{W_{em}}{m^2}$$

- with an average conversion efficiency of 8% (Table 13.5),

$$\dot{W}_e = 59.5\,\frac{W_e}{m^2} = 59.5\cdot10^{-3}\,\frac{kW_e}{m^2}$$

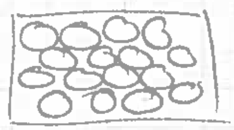
- # of cells required for 10 kW ac power:

$$\text{Area Required} = \frac{(10\,kW_e)}{\left(59.5\cdot10^{-3}\,\frac{kW_e}{m^2}\right)} = 168.1\,m^2$$

$$\text{Area of Cell} = \frac{\pi}{4}(0.08\,m)^2 = 0.005\,m^2$$

(a) $\boxed{\text{\# of Cells Required} = 33{,}435 \text{ cells}}$

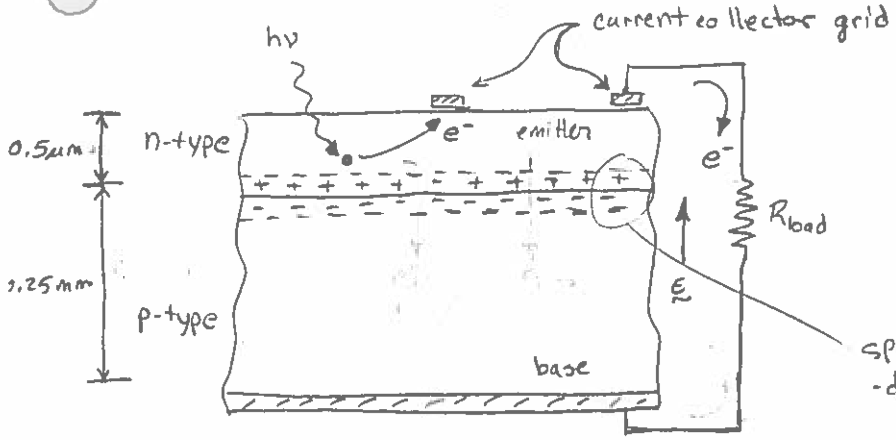
87% packing fraction



(b) $\boxed{\text{area of array} = 193.2\,m^2}$

n-p Junction

The spontaneous recombination of electron-hole pairs, generated from photons, can be suppressed by placing a p-type semiconductor in contact with an n-type semiconductor. A p-n junction forms.



• holes & electrons can diffuse in the Si lattice

At the p-n junction, electrons and holes will diffuse and combine to neutralize one another. The neutralization at the junction effectively charges the interface. Losses of holes in the p-type semiconductor results in a negative charge at the junction. Loss of electrons in the n-type semiconductor results in a positive charge. Thus, an electric field is generated across the p-n junction. The neutralization creates a potential barrier that inhibits the motion of electrons.

A photon-displaced electron moves through the external circuit (R_{load}) because this is easier than crossing the neutralized junction.

The n-type layer is very thin ($0.5\ \mu m = 1/\alpha$) so as to minimize electron-hole recombination.

Intensity of photon flux in the n-type material is a function of thickness:

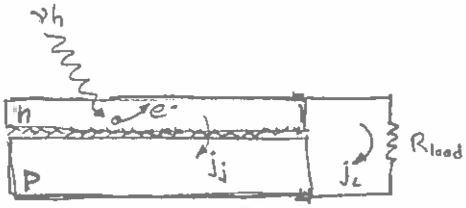
$$I(x) = I_o e^{-\alpha x}$$

$x \equiv$ depth of penetration

$\alpha \equiv$ absorbance coefficient

Photon capture (electron-hole generation) should occur within $1/\alpha$ from the surface to ensure photons absorbed within the diffusion length of the p-n junction.

If an electron-hole pair is generated near the junction, then on average this pair will contribute to the current flow in the external circuit.



net current = source current − junction current

$j_L \qquad j_s \qquad j_j$

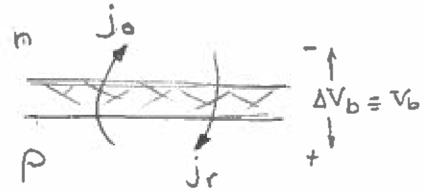
junction current

$$j_j = \left\{ \begin{matrix} \text{light-induced} \\ \text{recombination} \\ \text{current} \end{matrix} \right\} - \left\{ \begin{matrix} \text{reverse} \\ \text{saturation} \\ \text{current} \end{matrix} \right\}$$

$j_r \qquad j_o \rightarrow f(T)$

a.k.a. dark current

current density [A/cm²]



j_r: Electrons in the conduction band of the n-type semiconductor do not have sufficient energy to cross the n-p junction.

These electrons may cross with sufficiently large bias voltage ΔV_b.

A bias voltage is induced by photons.

j_r depends on the bias voltage (induced by photons) and the reverse saturation current j_o

The junction current is proportional to the potential for electron flow $[\Delta V_B]$ and the reverse saturation current j_o.

$$\frac{dj_r}{dV_b} \sim j_o \longrightarrow \frac{dj}{j} = \lambda\, dV_b \longrightarrow j_r = j_o e^{\lambda V_b}$$

proportionality constant

where $\lambda = \frac{e_o}{k_B T}$

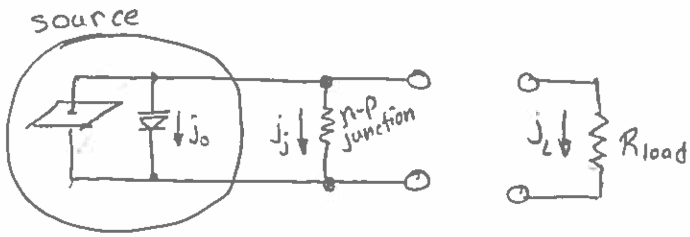
$e_o \equiv$ charge of electron, $1.602 \cdot 10^{-19} \frac{J}{V}$ (Coulomb)

$k_B \equiv$ Boltzmann's constant, $1.381 \cdot 10^{-23} \frac{J}{K}$

Thus, the junction current is:

$$j_j = j_o \left[e^{\lambda V_b} - 1 \right]$$

# of charge carriers within n-p junction



$I = jA$
↑ area of cell

$j_L = j_s - j_j$
↑ junction current
↑ source current; generated by photons

$$j_L = j_s - j_o\left[e^{\frac{e_o V_L}{k_B T}} - 1\right]$$

$$\dot{W}_e = V_L (j_L A)$$

$$\dot{W}_e = V_L A j_s - V_L A j_o \left[e^{\frac{e_o V_L}{k_B T}} - 1\right]$$

differentiating with respect to V_L and setting equal to zero,

$$e^{\frac{e_o V_{L,mp}}{k_B T}} = \frac{1 + j_s/j_o}{1 + \frac{e_o V_{L,mp}}{k_B T}}$$

solve iteratively to find voltage corresponding to maximum power

- If the source current, j_s, and the reverse saturation current, j_o, are known, then $V_{L,max}$ can be determined. Likewise, the maximum possible power can be determined.

$$\dot{W}_{e,max} = \frac{A V_{L,mp} (j_o + j_s)}{1 + \frac{e_o V_{L,mp}}{k_B T}}$$

$$\eta_{max} = \frac{\dot{W}_{e,max}}{\dot{W}_{e,em}}$$

For a short-circuit, $j_j = 0$.
For an open circuit, $j_L = 0$.

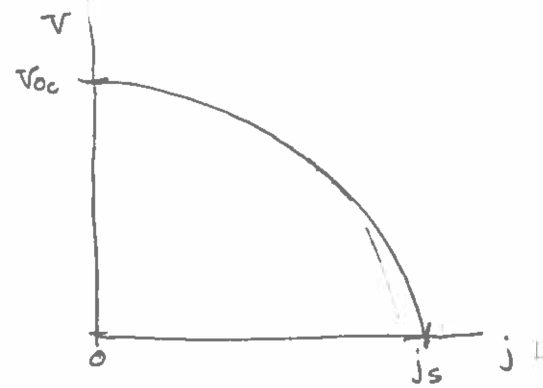
open circuit ; $j_L = 0$; $V_L = V_{oc}$

$$j_L = 0 = j_s - j_o \left[e^{\frac{e_o V_{oc}}{k_B T}} - 1 \right]$$

$$V_{oc} = \frac{k_B T}{e_o} \ln\left(\frac{j_s}{j_o} - 1 \right)$$

short circuit ; $j_L = j_s$; $V_L = 0$

$$\dot{W}_e = V_L A j_L = V_L I_L = I_L^2 R_L$$



example (STC)

short circuit current density: $j_s = 180 \frac{A}{m^2}$

reverse-saturation current density: $j_o = 8.10^{-9} \frac{A}{m^2}$

At 27°C and maximum power, determine the area required to generate 1000 $\dot{W}$.

$$\left. e^{\frac{e_o V_{L,mp}}{k_B T}} = \frac{1 + j_s/j_o}{1 + \frac{e_o V_{L,mp}}{k_B T}} \right\} \longrightarrow V_{L,max} = 0.5368\ V$$

$$\frac{\dot{W}_{e,max}}{A} = \frac{V_{L,mp}(j_o + j_s)}{1 + \frac{e_o V_{L,mp}}{k_B T}} = 92.1277 \frac{W_e}{m^2}$$

at 1000 $\dot{W}$, $A = 10.85\ m^2$

If the solar insolation is 930 $\frac{W_{em}}{m^2}$, then the maximum conversion efficiency is

$$\eta_{max} = 9.7\%.$$

Table 13-4 Ideal spectral solar energy utilized by silicon cells

Wavelength range, μm	Solar energy, %	Fraction converted, by cell	Solar energy converted, %
<0.3	0	0	0
0.3–0.5	17	0.36	5
0.5–0.7	28	0.55	15
0.7–0.9	20	0.73	15
0.9–1.1	13	0.91	12
>1.1	22	0	0
		Total	48

Table 13-5 Typical energy balance of a nonconcentrating silicon photovoltaic conversion array, arbitrary units

<table>
<tr><th>Input on array</th><th colspan="6">Energy distribution</th></tr>
<tr><td rowspan="7">100</td><td rowspan="2">In nonphotovoltaic material</td><td>12</td><td colspan="4">Reflection by and absorption in cover glass</td></tr>
<tr><td>13</td><td colspan="4">Absorption by frames, structures, earth</td></tr>
<tr><td rowspan="5">In photovoltaic material</td><td rowspan="5">75</td><td>Nonelectric</td><td>64</td><td colspan="2">Dissipation as heat in silicon</td></tr>
<tr><td rowspan="4">Electric</td><td rowspan="4">11</td><td>1.5</td><td>Losses due to cell temperature above 28°C</td></tr>
<tr><td>0.5</td><td>Losses due to cell and module mismatch</td></tr>
<tr><td>1.0</td><td>Losses in wiring and dc-to-ac conversion</td></tr>
<tr><td>8.0</td><td>Delivered as ac power</td></tr>
</table>

maximum theoretical efficiency ~ 48%

40-50 cells together generate 20-25V

Photovoltaics (PV)

- Electromagnetic energy conversion into electrical energy
- PV performance rated according to maximum DC power output under standard test conditions (STC):

temperature = 25°C
solar irradiance = 1000 W/m²
air mass = 1.5

- actual performance is typically 80-90% of STC.

- Balance of Plant:
  - system & battery control
  - overcurrent protection
  - surge protection
  - battery bank
  - power conditioner → DC-DC or DC-AC inverter

- DOE Goals: (2004)

	1991	1995	2000	2010-2030
$ per kWe	0.40-0.75	0.25-0.50	0.12-0.20	<0.06
efficiency (flat plate & concentrator)	5-14%	7-17%	10-20%	15-25%
system cost, $ per We	0.10-0.20	0.07-0.15	0.03-0.07	0.01-0.015
system life, years	5-10	10-20	>20	>30