Electromagnetic-to-Electrical Energy Conversion (Solar-to-Electric; Photovoltaics)

- Photon Energy
- Photon Flux in Solar Spectrum
- Photoelectric Effect
- n-p Junction and Charge Carriers
  - maximum electrical power output
  - maximum conversion efficiency
Electromagnetic energy (solar radiation) may be thought of as discrete packets (quanta) of energy, referred to as photons. The energy of a photon is proportional to the frequency of the electromagnetic energy,

\[ E_{\text{photon}} \sim \nu \]

\[ \text{frequency, } \nu \text{ [cycles/second]} \]

The proportionality constant between photon energy and electromagnetic frequency is Planck's constant.

\[ E_{\text{photon}} = \hbar \nu \]

Planck's constant:
- \( \hbar = 6.626 \times 10^{-34} \text{ J s/quantum} \) for electromagnetic energy, the quantum is a photon
- \( \hbar = 4.136 \times 10^{-15} \text{ eV s/quantum} \)

Photon energy can also be expressed in terms of electromagnetic wavelength,

\[ E_{\text{photon}} = \hbar \nu = \hbar \frac{c}{\lambda} \] \[ \text{speed of light in a vacuum, } c = 3 \times 10^8 \text{ m/s} \]

- Photon energy in the middle of the visible spectrum, \( \lambda = 550 \text{ nm} \):

\[ E_\lambda = \frac{\hbar c}{\lambda} \]

\[ = \frac{(3 \times 10^8 \text{ m/s})(6.626 \times 10^{-34} \text{ J s/quantum})}{(550 \times 10^{-9} \text{ m})} = 3.62 \times 10^{-19} \text{ J/quantum} \]

\[ E_\lambda = (3.62 \times 10^{-19} \text{ J/quantum})(1 \text{ eV/16 \times 10^{-19} J}) = 2.26 \text{ eV/quantum} \]
Photon flux in Solar Spectrum

photon flux, $\Phi_{\nu}^p = \frac{\text{# of photons}}{\text{time \cdot area}}$

photon energy flux, $\dot{E}_i^p = \Phi_{\nu_i} \nu_i h$

using average frequency, $\dot{E}'' = \bar{\Phi_{\nu}^p} \overline{\nu h} = \bar{\Phi_{\nu}^p} \frac{c^2 h}{2}$ \[\frac{\text{J/m}^2}{\text{s}} \equiv \frac{\text{W}}{\text{m}^2}\]

extraterrestrial solar energy flux = solar constant, S

$S = 1359 \frac{\text{W}}{\text{m}^2}$

$\eta_k = 0$ \text{ (air mass ratio)}

$W = 0$ \text{ (water vapor content)}

Thus, the extraterrestrial photon flux is

$\bar{\Phi}_{p,et} = \frac{(0.1359 \ \text{W/cm}^2)}{(1.48 \text{ eV/photon})} \left( \frac{1 \text{ eV}}{1.6021 \times 10^{-19} \text{ J/eV}} \right) = 5.8 \times 10^{17} \frac{\text{photons}}{\text{cm}^2 \cdot \text{s}}$

For $\eta_k = 3$ and $W = 5$, $\bar{\Phi}_{p} \approx \frac{1}{2}\bar{\Phi}_{p,et}$

($h = 20^\circ$)
Photovoltaic Effect

metals: many free electrons in conduction band that are free to move in an electric field

Silicon atom in crystal absorbs a photon, which if sufficiently energetic frees an electron into the conduction band forming an electron-hole pair.

Normally, the electron-hole pair disappear spontaneously.

Wavelength of photons that possess sufficient energy to free an electron in a silicon lattice:

\[ E_p = \frac{C}{\lambda_p} \geq E_g \rightarrow \lambda \leq 1.12 \text{\,um} \]

Photons at wavelengths > 1.12 \, \text{um} do not have sufficient energy to generate an electron-hole pair; if absorbed these photons are converted to thermal energy through the kinetic energy of valence band electrons.

75% of direct solar energy flux occurs at a wavelength \leq 1.12 \, \text{um}

The photon can only generate a single electron-hole pair regardless of how much more energetic than band gap energy.

Band Gaps

<table>
<thead>
<tr>
<th>Energy</th>
<th>Metal</th>
<th>Insulator</th>
<th>Semiconductor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>partially filled conduction</td>
<td>empty conduction</td>
<td>empty conduction</td>
</tr>
<tr>
<td></td>
<td>partially filled valence</td>
<td>partially filled valence</td>
<td>partially filled valence</td>
</tr>
<tr>
<td></td>
<td>[ E_g ]</td>
<td></td>
<td>[ E_g ]</td>
</tr>
</tbody>
</table>
Silicon (Si):
4 valence electrons (outer shell) all participating in covalent bond
no free electrons
insulator

Arsenic (As):
5 valence electrons
If As atom replaces Si atom in crystal lattice, then only 4 valence
electrons participate in covalent bond leaving one electron free
to participate in conduction band.
Si doped with As is negatively charged with free e\textsuperscript{-} \rightarrow n-type semiconductor

Boron (B), Aluminum (Al), Indium (In):
3 valence electrons
If one of these atoms replaces a Si atom in crystal lattice, then an
electron vacancy (hole) is created.
Si doped with B, Al, or In is positively charged \rightarrow p-type semiconductor
FIGURE 8.12
The charge distribution at an $n$-$p$ junction of semiconductors (a) without and (b) with an applied voltage $V_L$. (From Walsh, 1967.)
Figure 6.20 Photovoltaic conversion efficiency for several semiconductor materials having different band gaps. For all materials, the efficiencies drop as the temperature increases. Actual devices produce lower efficiencies than the theoretically maximum values.
Example 1

A monochromatic source of photons (red laser pointer) has a power of 1 mW and a wavelength of 638 nm. When pointed at silicon, compute:

(a) number of photons per second, and
(b) maximum efficiency of conversion to electrical energy.

\[ E_p = \frac{h \cdot c}{\lambda} = 3.12 \times 10^{-19} \text{ J} = 1.95 \text{ eV per photon} \]

\[ I_p = 1 \text{ mW} = 10^{-3} \text{ W} = \frac{1}{3} \text{ J/s} \]

(a) \[ N_p = \frac{10^{-3} \text{ J/s}}{3.12 \times 10^{-19} \text{ J/photon}} = 3.21 \times 10^{15} \text{ photons/s} \]

Maximum conversion efficiency is when each photon generates an electron. There are no reflective or scattering losses.

Electrical Power = \[ N_p \cdot E_{\text{bandgap}} \]

\[ W_e = \left( \frac{1.11 \text{ eV/photon}}{1.6021 \times 10^{-19} \text{ J/eV}} \right) \left( 3.21 \times 10^{15} \frac{\text{photons}}{s} \right) = 5.71 \times 10^{-9} \text{ J/s} \]

Electromagnetic Power = \[ \left( \text{photon energy} \right) \left( \frac{\text{# of photons}}{\text{#}} \right) = 10.03 \times 10^{-4} \text{ J/s} \]

\[ \eta = \frac{\text{energy sought}}{\text{energy cost}} = \frac{\text{band gap energy}}{\text{photon energy}} = 0.569 \]
A 7.5 cm-diameter circular photovoltaic solar cell is subjected to a solar energy flux of $2.5 \times 10^{17}$ photons/s/cm² at an average photon wavelength of 0.838 μm. Calculate

(a) the solar insolation on the cell, in W/m², and
(b) the maximum theoretical power that can be produced by the cell, in W.

Solar insolation: $E = \frac{\text{energy}}{\text{flux}}$

$$E_p = h \cdot \frac{\nu}{\lambda} = \frac{\text{constant}}{(3 \times 10^8 \text{ m/s})(4.13576 \times 10^{-15} \text{ eV/photons})}{(0.838 \times 10^{-6} \text{ m})} = 1.48 \frac{\text{eV}}{\text{photons}} = 2.37 \times 10^{-19} \frac{\text{J}}{\text{photons}}$$

$$\nu = 3.58 \times 10^{14} \text{ Hz}$$

$$E' = E_p \phi = (2.37 \cdot 10^{-19} \frac{\text{J}}{\text{photons}})(2.5 \times 10^{17} \frac{\text{photons}}{\text{s} \cdot \text{cm}^2}) = 0.0593 \frac{\text{J}}{\text{s} \cdot \text{cm}^2} = 592.8 \frac{\text{W}}{\text{m}^2} \quad (a)$$

The maximum theoretical power is $P_{max} = E' \cdot A$

$$P_{max} = 48 \% \quad (p. 580, E1. Wekal)$$

$$A = \frac{\pi D^2}{4} = 94.18 \text{ cm}^2 = 0.0044 \text{ m}^2$$

$$P_{max, \text{theoretical}} = 1.26 \text{ W} \quad (b)$$
Example 3

A 0.08 m diameter circular photovoltaic cell receives the following solar fluxes, in photons per second per square centimeter:

- \(0.5 \times 10^{17} \text{ photons} \) (0.3 - 0.5 um)
- \(0.85 \times 10^{17} \text{ photons} \) (0.5 - 0.7 um)
- \(0.50 \times 10^{17} \text{ photons} \) (0.7 - 0.9 um)
- \(0.45 \times 10^{17} \text{ photons} \) (0.9 - 1.1 um)
- \(0 \text{ photons} \) (> 1.1 um)

Estimate:

(a) the number of such cells necessary to produce 10 kW of AC power, and
(b) the array area on which they are mounted.

\[
\text{Power: } \dot{W}_{\text{em}} = \sum \Phi_i \eta \bar{V} = \sum \Phi_i h \frac{c}{\Delta} = c h \sum \frac{\Phi_i}{\Delta}
\]

\[
\dot{W}_{\text{em}} = \left(3 \times 10^{8} \text{ W} \right) \left(6.626 \times 10^{-34} \text{ J s} \right) \left[0.5 \times \frac{1}{0.4} + 0.85 \times \frac{1}{0.6} + 0.5 \times \frac{1}{0.8} + 0.45 \times \frac{1}{1.0} \right] \left(10^{17} \text{ photons/cm}^2 \right) \left(100 \text{ cm}^2 \right) = 743.8 \text{ W/m}^2
\]

With an average conversion efficiency of 6% (Table 13.5),

\[
\dot{W}_e = 59.5 \frac{\dot{W}_{\text{em}}}{\text{m}^2} = 59.5 \times 10^{-3} \text{ kW/m}^2
\]

(a) \# of cells required for 10 kW ac power:

\[
\text{Area Required} = \frac{(10 \text{ kW})}{(59.5 \times 10^{-3} \text{ kW/m}^2)} = 168.1 \text{ m}^2
\]

\[
\text{Area of Cell} = \frac{\pi}{4} (0.08 \text{ m})^2 = 0.005 \text{ m}^2
\]

\[
\# \text{ of Cells Required} = \frac{168.1}{0.005} = 33,620 \text{ cells}
\]

87% packing fraction

(b) Area of array = 193.2 m^2
**n-p Junction**

The spontaneous recombination of electron-hole pairs, generated from photons, can be suppressed by placing a p-type semiconductor in contact with an n-type semiconductor. A p-n junction forms.

At the p-n junction, electrons and holes will diffuse and combine to neutralize one another. The neutralization at the junction effectively charges the interface. Losses of holes in the p-type semiconductor results in a negative charge of the junction. Loss of electrons in the n-type semiconductor results in a positive charge. Thus, an electric field is generated across the p-n junction. The neutralization creates a potential barrier that inhibits the motion of electrons.

A photon-displaced electron moves through the external circuit (R_{load}) because this is easier than crossing the neutralized junction.

The n-type layer is very thin (0.5 μm = \(\frac{1}{2}\)d) so as to minimize electron-hole recombination.

**Intensity of photon flux is a function of thickness:**

\[
I(x) = I_0 e^{-\alpha x}
\]

- \(x\) = depth of penetration
- \(\alpha\) = absorbance coefficient

Photon capture (electron-hole generation) should occur within \(\frac{1}{2}d\) from the surface to ensure photons absorbed within the diffusion length of the p-n junction.

If an electron-hole pair is generated near the junction, then on average this pair will contribute to the current flow in the external circuit.
The junction current is proportional to the potential difference \( \Delta V_b \) and the reverse saturation current \( j_0 \).

\[
\frac{dJ_j}{\Delta V_b} \sim j_0 \quad \Rightarrow \quad \frac{dJ_j}{J_j} = \frac{e^\lambda V_b}{j_0} \quad \text{proportionally}
\]

where \( \lambda = \frac{e_0}{k_B T} \)

\( e_0 \equiv \text{charge of electron, } 1.602 \times 10^{-19} \text{ C} \)

\( k_B = \text{Boltzmann's constant, } 1.38 \times 10^{-23} \text{ J/K} \)

Thus, the junction current is:

\[
J_j = j_0 \left[ e^{\lambda V_b} - 1 \right]
\]

- \# of charge carriers within n-p junction
\[ I = jA \]
\[ j_L = j_s - j_j \]
\[ j_L = \text{junction current} \]
\[ j_s = \text{source current generated by photons} \]

\[ \begin{align*}
  j_L &= j_s - j_j \\
  W_e &= V_L (j_L A) \\
  W_e &= V_L A j_s - V_L A j_0 \left( e^{\frac{eV_e}{k_B T}} - 1 \right) \\
  \text{(differentiating with respect to } V_e \text{ and setting equal to zero)} \\
  V_e &= \frac{eV_{lim}}{k_B T} \left( 1 + \frac{j_s}{j_0} \right) \\
  \text{solve iteratively to find voltage corresponding to maximum power} \\
\end{align*} \]

If the source current, \( j_s \), and the reverse saturation current, \( j_0 \), are known, then \( V_{e, \text{max}} \) can be determined. Likewise, the maximum possible power can be determined:

\[ W_{e, \text{max}} = \frac{A V_{lim} (j_0 + j_s)}{1 + \frac{eV_{e, \text{max}}}{k_B T}} \]

\[ P_{\text{max}} = \frac{W_{e, \text{max}}}{W_{e, \text{em}}} \]

For a short circuit, \( j_j = 0 \).
For an open circuit, \( j_L = 0 \).
open circuit: \( j_L = 0 \); \( V_L = V_{oc} \)

\[
\begin{align*}
\bar{j}_L &= 0 = j_S - j_0 \left( e^{\frac{e\bar{V}_{oc}}{kT}} - 1 \right) \\
V_{oc} &= \frac{kT}{e} \ln \left( \frac{j_S}{j_0} - 1 \right)
\end{align*}
\]

short circuit: \( j_L = j_S \); \( V_L = 0 \)

\[
\begin{align*}
\bar{W}_e &= V_L A \bar{j}_L = V_L I_L = I_L^2 R_L
\end{align*}
\]

**Example (S7C)**

short circuit current density: \( j_S = 180 \text{ A/m}^2 \)

reverse-saturation current density: \( j_0 = 8 \times 10^{-9} \text{ A/m}^2 \)

At 27°C and maximum power, determine the area required to generate 1000 W.

\[
\begin{align*}
\frac{e^{\frac{eV_{oc}}{kT}}}{1 + \frac{j_S}{j_0} \left( e^{\frac{eV_{oc}}{kT}} - 1 \right)} \rightarrow V_{oc, \text{max}} = 0.5368 \text{ V}
\end{align*}
\]

\[
\frac{\bar{W}_e, \text{ max}}{A} = \frac{V_{oc, \text{max}} (j_0 + j_S)}{1 + \frac{eV_{oc, \text{max}}}{kT}} = 92.1277 \text{ W/m}^2
\]

At 1000 W, \( A = 10.85 \text{ m}^2 \)

If the solar insolation is 930 \( \text{W/m}^2 \), then the maximum conversion efficiency is

\[\eta_{\text{max}} = 9.7\%\]
Table 13-4 Ideal spectral solar energy utilized by silicon cells

<table>
<thead>
<tr>
<th>Wavelength range, μm</th>
<th>Solar energy, %</th>
<th>Fraction converted, by cell</th>
<th>Solar energy converted, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.3-0.5</td>
<td>17</td>
<td>0.36</td>
<td>5</td>
</tr>
<tr>
<td>0.5-0.7</td>
<td>28</td>
<td>0.55</td>
<td>15</td>
</tr>
<tr>
<td>0.7-0.9</td>
<td>20</td>
<td>0.73</td>
<td>15</td>
</tr>
<tr>
<td>0.9-1.1</td>
<td>13</td>
<td>0.91</td>
<td>12</td>
</tr>
<tr>
<td>&gt;1.1</td>
<td>22</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>48</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 13-5 Typical energy balance of a nonconcentrating silicon photovoltaic conversion array, arbitrary units

<table>
<thead>
<tr>
<th>Input on array</th>
<th>Energy distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>In nonphotovoltaic material</td>
<td>12 Reflection by and absorption in cover glass</td>
</tr>
<tr>
<td></td>
<td>13 Absorption by frames, structures, earth</td>
</tr>
<tr>
<td>100</td>
<td>Electric 64 Dissipation as heat in silicon</td>
</tr>
<tr>
<td>In photovoltaic material</td>
<td>1.5 Losses due to cell temperature above 28°C</td>
</tr>
<tr>
<td></td>
<td>0.5 Losses due to cell and module mismatch</td>
</tr>
<tr>
<td></td>
<td>1.0 Losses in wiring and dc-to-ac conversion</td>
</tr>
<tr>
<td></td>
<td>8.0 Delivered as ac power</td>
</tr>
</tbody>
</table>

Maximum theoretical efficiency ~ 40% 40-50 cells together generate 20-25V
Photovoltaics (PV)

- Electromagnetic energy conversion into electrical energy
  - PV performance rated according to maximum DC power output under standard test conditions (STC):
    - temperature = 25°C
    - solar irradiance = 1000 W/m²
    - air mass = 1.5

  - Actual performance is typically 80-90% of STC.

- Balance of Plant:
  - system & battery control
  - overcurrent protection
  - surge protection
  - battery bank
  - power conditioner → DC-DC or DERAC inverter

- DOE Goals:
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td># per kW_e</td>
<td>$0.40-0.75</td>
<td>$0.25-0.50</td>
<td>$0.12-0.20</td>
<td>&lt;$0.06</td>
</tr>
<tr>
<td>efficiency</td>
<td>5-14%</td>
<td>7-17%</td>
<td>10-20%</td>
<td>15-25%</td>
</tr>
<tr>
<td>(flat plate%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(concentrated)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>system cost,</td>
<td>$0.10-0.20</td>
<td>$0.07-0.15</td>
<td>$0.03-0.07</td>
<td>$0.01-0.015</td>
</tr>
<tr>
<td># per kW_e</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>system life,</td>
<td>5-10</td>
<td>10-20</td>
<td>20-20</td>
<td>&gt;30</td>
</tr>
<tr>
<td>(years)</td>
<td></td>
<td></td>
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