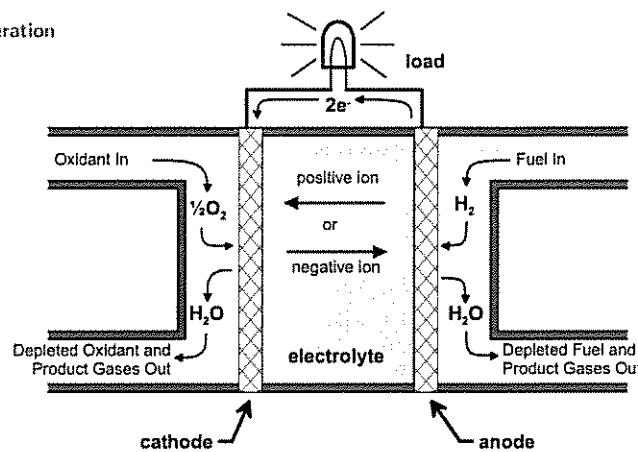


Fuel Cell Operation



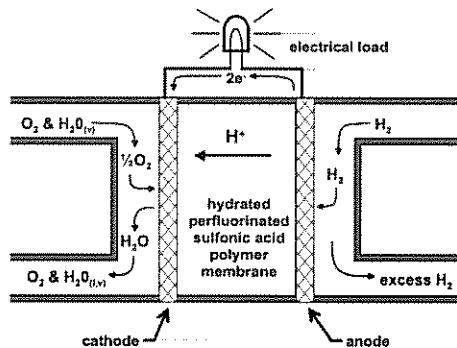
- electrochemical energy conversion device
- nonconsumable anode and cathode
- oxidation-reduction reaction converted directly into electrical energy without an intermediary conversion into heat – more efficient than the Carnot cycle – as high as 90% efficiency theoretically possible
- reactants (typically gaseous) external to energy conversion device
- electrolyte layer in contact with a porous anode and cathode, provides reactant and ionic transport as well as physical barrier to separate fuel and oxidant
- ionic transport can be positive or negative

Fuel Cell Classification

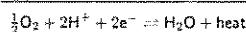
	PEMFC*	AFC	PAFC	MCFC	SOFC
electrolyte	ion exchange membrane	KOH in asbestos matrix	phosphoric acid in silicon carbide	alkali carbonates in $LiAlO_2$ matrix	ceramics yttria-zirconia
charge carrier	H^+	OH^-	H^+	CO_3^{2-}	O^{2-}
operating temperature	$\sim 80^\circ C$	$\sim 100^\circ C$	$\sim 200^\circ C$ $\sim 220^\circ C$	$\sim 650^\circ C$	$\sim 1000^\circ C$
catalyst	platinum	various	platinum	nickel	perovskites
product water management	evaporative (hopefully)	evaporative	evaporative	gaseous product	gaseous product
product heat management	process gas & cooling system	cooling system	process gas & cooling system	internal reforming & process gas & generation	internal reforming & process gas & generation
fuel	pure H_2 , tolerates CO_2	pure H_2 -no CO_2 -	pure H_2 tolerates CO_2 ~ 1% CO	H_2, CO, CH_4 , other hydrocarbons	H_2, CO, CH_4 , other hydrocarbons

* Also known as solid polymer fuel cell (SPFC)

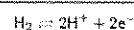
PEM Fuel Cell Kinetics



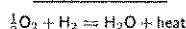
O₂ reduction half reaction - cathode



H₂ oxidation half reaction - anode

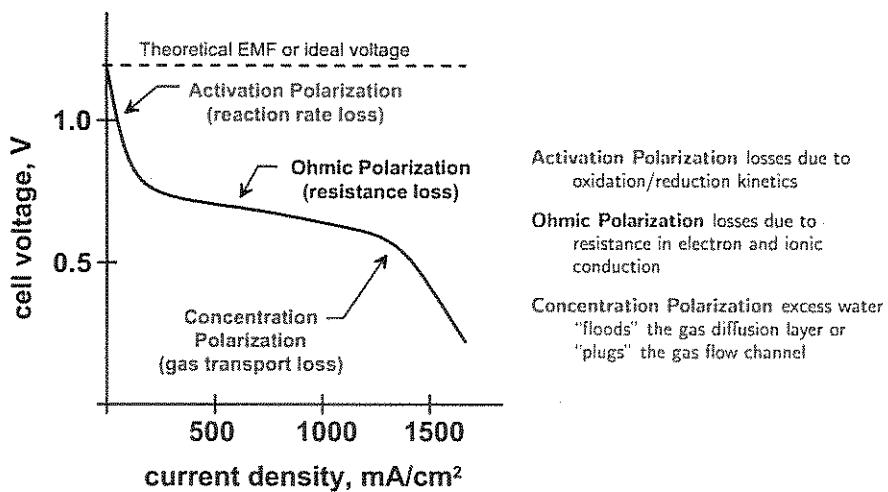


overall cell reaction



performance limited by rate of O₂ reduction which
is more than 100 times slower than the H₂ oxidation

Polarization Losses



water management is critical to
the performance of a PEM fuel cell

Electrochemical Reactions

lect

Acid Electrolyte

common fuels

H₂

CH₄

C₃H₈

CO

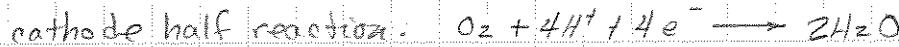
C

CH₃OH

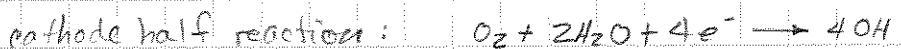
CH₂O₂

NH₃

N₂H₄



Alkaline Electrolyte



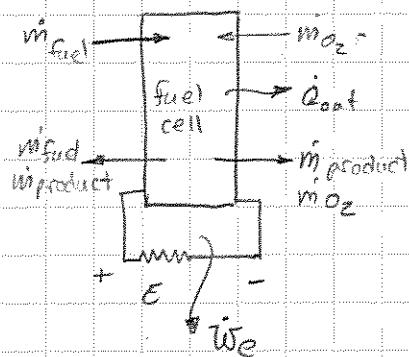
Energy Available for Conversion

The total energy change of the reactants is the difference in enthalpy of formation.

$$\Delta H_f = \sum_{\text{products}} \Delta H_f - \sum_{\text{reactants}} \Delta H_f$$

$\Delta H_f > 0$ exothermic

$\Delta H_f < 0$ endothermic



Applying Cons. of Energy to the Control Volume

$$\dot{Q}_{\text{out}} - \dot{W}_{\text{e}} = \dot{\Delta H} = \dot{\Delta U} + \dot{m}_{\text{product}} g \frac{\Delta h}{\Delta m} \quad \begin{cases} \text{no changes in kinetic or} \\ \text{gravitational potential energies} \end{cases}$$

total change
in enthalpy

The maximum work (power) will occur if the thermal power portion of the enthalpy change is reversible.

$$\dot{S}_{\text{Rev}} = T \dot{dS}$$

Fuel cells typically operate isothermally. Therefore,

$$\dot{Q}_{\text{out}} = T \dot{\Delta S}$$

$$-\dot{W}_{\text{e}} \leq \underbrace{\dot{\Delta H} - T \dot{\Delta S}}$$

known as change in Gibbs Free Energy, $G = H - TS$

$$-\dot{W}_{\text{e}} \leq \sum_{\text{products}} \dot{G} - \sum_{\text{reactants}} \dot{G}$$

Criteria for a spontaneous reaction:
 $dG_{r,p} = \delta Q - TdS \leq 0$

Gaseous Reactants

• Treating the reactants as an ideal gas, $PV = n\bar{R}T$
 $T \neq \text{of mols}$

• for an ideal gas $TdS = dh - \Delta dP$ (Gibbs equation)

$$\hookrightarrow dG = dH - TdS = \Delta dP$$

$$\hookrightarrow dG = n\bar{R}T \frac{dP}{P}$$

• for an isothermal process

$$G_2 - G_1 = n\bar{R}T \ln\left(\frac{P_2}{P_1}\right)$$

• Tabulated values of Gibbs Free Energy (and enthalpy of formation) are given relative to a reference state.

$$G = G^\circ + nRT \ln\left(\frac{P}{P_0}\right)$$

Reference state; typically at $P_0 = 1 \text{ atm}$ (but NOT ALWAYS!)

$\frac{P}{P_0}$ is also known as activity

Electrical Work

$$We = neFE$$

\uparrow voltage difference across electrodes
 \downarrow Faraday's constant = $96,487 \text{ g mol e}^{-1}$
 mol of electron per mol of fuel

The change in Gibbs Free Energy at reference state is $\Delta G^\circ = -neFE^\circ$

$$\text{Thus, } E^\circ = \frac{-\Delta G^\circ}{neF}$$

open circuit potential
 (no current flow)

Maximum Conversion Efficiencies

$$\text{Heat Engine: } \eta_{th} = 1 - \frac{T_{cool}}{T_{high}}$$

$$\text{Fuel Cell: } \eta_{th} = \frac{We}{HV} = \frac{\Delta G}{\Delta H} = \frac{neFE}{\Delta H}$$

$$\eta_{th,\max} = \frac{neFE^\circ}{\Delta H} ; \text{ For } \text{H}_2 + \text{O}_2 \text{ reaction at } 25^\circ\text{C and } 1 \text{ atm, } E^\circ = 1.23 \text{ V.}$$

3.6.3. Thermodynamic Efficiency

For the case of an electrochemical energy converter working ideally, it has been shown that the free-energy change of the reaction may be totally converted to electrical energy. Thus, an electrochemical energy converter has a thermodynamic efficiency given by:

$$\eta_{th} = \frac{\Delta G}{\Delta H} = 1 - \frac{T\Delta S}{\Delta H} \quad (3.65)$$

Table 3-1. Thermodynamic Data for some Candidate Fuel-Cell Reactions Under Standard Conditions at 25 °C.

Fuel	Reaction	n	-ΔH° [kJ/mol]	-ΔG° [kJ/mol]	E° rev. [V]	%
Hydrogen	H ₂ + 0.5 O ₂ → H ₂ O _(l)	2	286.0	237.3	1.229	83.0
	H ₂ + Cl ₂ → 2 HCl _(aq)	2	335.5	262.5	1.359	78.3
	H ₂ + Br ₂ → 2 HBr	2	242.0	205.7	1.066	85.0
Methane	CH ₄ + 2 O ₂ → CO ₂ + 2 H ₂ O _(l)	8	890.8	818.4	1.060	91.9
Propane	C ₃ H ₈ + 5 O ₂ → 3 CO ₂ + 4 H ₂ O _(l)	20	2221.1	2109.9	1.093	95.0
Decane	C ₁₀ H ₂₂ + 15.5 O ₂ → 10 CO ₂ + 11 H ₂ O _(l)	66	6832.9	6590.5	1.102	96.5
Carbon monoxide	CO + 1.5 O ₂ → CO ₂	2	283.1	257.2	1.066	90.9
Carbon	C + 0.5 O ₂ → CO	2	110.6	137.3	0.712	124.2
	C + O ₂ → CO ₂	4	393.7	394.6	1.020	100.2
Methanol	CH ₃ OH + 1.5 O ₂ → CO ₂ + 2 H ₂ O _(l)	6	726.6	702.5	1.214	96.7
Formaldehyde	CH ₂ O _(g) + O ₂ → CO ₂ + 2 H ₂ O _(l)	4	561.3	522.0	1.350	93.0
Formic-acid	HCOOH + 0.5 O ₂ → CO ₂ + H ₂ O _(l)	2	270.3	285.5	1.480	105.6
Ammonia	NH ₃ + 0.75 O ₂ → 0.5 N ₂ + 1.5 H ₂ O	3	382.8	338.2	1.170	88.4
Hydrazine	N ₂ H ₄ + O ₂ → N ₂ + 2 H ₂ O _(l)	4	622.4	602.4	1.560	96.8

Finding ΔG

1003

		heat of formation \bar{h}_f° [kJ/kmol]	Gibbs free energy \bar{g}° [kJ/kmol]
hydrogen gas	H ₂	0	0
Oxygen gas	O ₂	0	0
water vapor	H ₂ O(v)	-241820	-228590
liquid water	H ₂ O(l)	-225830	-237180
carbon monoxide	CO	-110530	-137150
carbon dioxide	CO ₂	-393320	-393360

from Sengel & Boles, Table A-22

On a molar basis, $\bar{g} = \bar{g}^\circ - RT \ln(P/P_0)$; P₀ is the reference state pressure, which is typically 1 atm

- For a generic chemical reaction



where A is the fuel and a, b, c, and d are the stoichiometric conditions.

$$\Delta \bar{g} = \underbrace{(m\bar{g}_m - n\bar{g}_N)}_{\text{products}} - \underbrace{(a\bar{g}_A - b\bar{g}_B)}_{\text{reactants}}$$

- On a per mol of fuel basis,

$$\begin{aligned} \Delta \bar{g} &= \left(\frac{m}{a} \right) \bar{g}_m + \left(\frac{n}{a} \right) \bar{g}_N - \bar{g}_A - \left(\frac{b}{a} \right) \bar{g}_B \quad \left[\frac{\text{kJ}}{\text{kmol fuel(A)}} \right] \\ &= \left\{ \left(\frac{m}{a} \right) \bar{g}_m^\circ + \left(\frac{n}{a} \right) \bar{g}_N^\circ - \bar{g}_A^\circ - \left(\frac{b}{a} \right) \bar{g}_B^\circ \right\} - \left\{ \left(\frac{m}{a} \right) RT \ln \left(\frac{P_m}{P_0} \right) + \left(\frac{n}{a} \right) RT \ln \left(\frac{P_N}{P_0} \right) \dots \right\} \end{aligned}$$

- Collecting Terms,

$$\Delta \bar{g} = \Delta \bar{g}^\circ - RT \ln \underbrace{\left\{ \frac{\left(\frac{P_m}{P_0} \right)^{\frac{m}{a}} \left(\frac{P_N}{P_0} \right)^{\frac{n}{a}}}{\left(\frac{P_A}{P_0} \right)^b \left(\frac{P_B}{P_0} \right)^{\frac{b}{a}}} \right\}}_{\text{known as the reaction coefficient, } K}$$

$$\Delta \bar{g} = \Delta \bar{g}^\circ - RT \ln K$$

- When the reference pressure is 1 atm, and reactant/product pressures are expressed in atm, and when the stoichiometric coefficient for the fuel is 1 mol (a=1), then the change in Gibbs free energy is commonly written as:

$$\Delta g = \Delta \bar{g}^\circ + RT \ln \left(\frac{P_A P_B^b}{P_m P_N^n} \right) \quad \text{Note the sign change.}$$

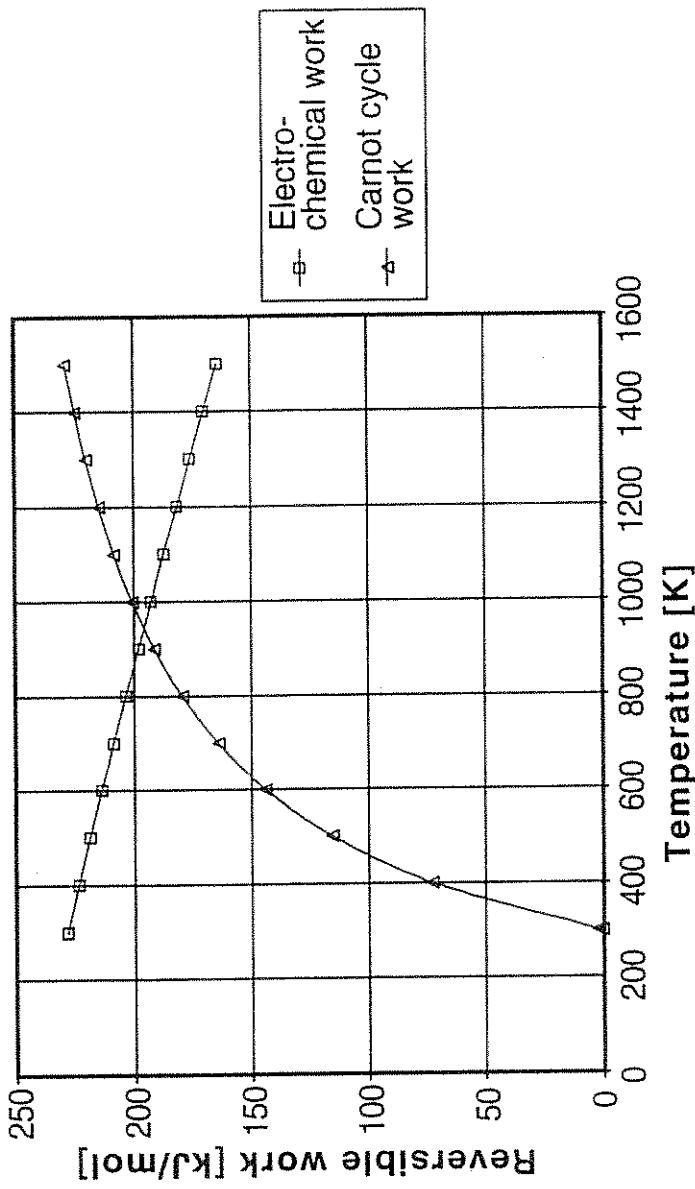
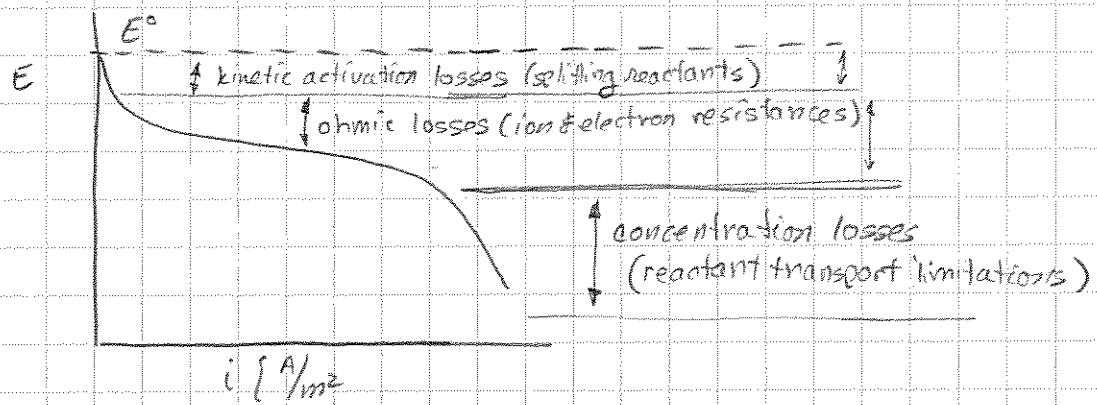


FIGURE 3.10 The reversible work produced by a H_2/O_2 fuel cell is greater than that of a Carnot engine at temperatures below 950 K. At higher temperatures, the Carnot engine is able to convert more of the HHV of H_2 (285.840 kJ/mol) into work. The data for the standard Gibbs energy of formation for water vapor was taken from Lide (1995, pp. 5–64).

Polarization Curve



At non-standard states,

$$E = E^\circ + \frac{RT}{nF} \ln \left[\frac{\sum (\text{Products})^m}{(\text{Reactants})^n} \right] \quad \text{Nernst Equation}$$

H₂ heating value at 300°C

Determine the heating value of H₂ at 300°C. Reactants and oxidants are at 1 atm.



at 300°C & 1 atm, this will be in vapor form

$$\bar{\Delta h}^\circ = \sum_{\text{products}} n_p \bar{h}_f^\circ - \sum_{\text{reactants}} n_r \bar{h}_f^\circ$$

$$\bar{\Delta h}^\circ = \left[\left(1 \frac{\text{kmol H}_2\text{O}}{\text{kmol H}_2} \right) \left(-285830 \frac{\text{kJ}}{\text{kmol H}_2\text{O}} \right) \right] - \left[\left(1 \frac{\text{kmol H}_2}{\text{kmol H}_2} \right) (0) + \left(\frac{1}{2} \frac{\text{kmol O}_2}{\text{kmol H}_2} \right) (0) \right]$$

$$\bar{\Delta h}^\circ = -285830 \frac{\text{kJ}}{\text{kmol H}_2\text{O}} = -\overline{\text{HHV}}_{\text{H}_2} = -(\overline{\text{LHV}} + \bar{h}_{fg})$$

{liquid water}
product {vapor}
product {enthalpy of
vaporization}

$$\bar{\Delta h}_{(v)}^\circ = \bar{\Delta h}_{(o)}^\circ - \bar{h}_{fg}(300^\circ\text{C}) = -241820 \frac{\text{kJ}}{\text{kmol H}_2}$$

Hz Fuel Cell

Determine the maximum conversion efficiency and open circuit voltage for a hydrogen-oxygen (in air) fuel cell operating at (a) 75°C and (b) 300°C. The products exhaust to 1 atm, the hydrogen is supplied at 2 atm and the oxygen is supplied (in air) at 1.5 atm.

(a) At 75°C → product water is in liquid form

$$\bar{\Delta h_f^\circ} = -285830 \frac{\text{kJ}}{\text{kmol Hz}}$$

$$\bar{\Delta g^\circ} = \left[\left(\frac{1 \text{ kmol H}_2}{\text{kmol H}_2} \right) \left(-237180 \frac{\text{kJ}}{\text{kmol H}_2 \text{O}_{(l)}} \right) \right] - \left[\left(\frac{1}{2} \right) (0) + \left(\frac{1}{2} \right) (0) \right] = -237180 \frac{\text{kJ}}{\text{kmol Hz}}$$

$\underbrace{\bar{g}^\circ}_{\text{H}_2\text{O}(l)}$ $\underbrace{\bar{g}^\circ}_{\text{H}_2}$ $\underbrace{\bar{g}^\circ}_{\text{O}_2}$ $\underbrace{\text{Liquid water product}}$

$$\begin{aligned} \bar{\Delta G}_{75^\circ\text{C}} &= \bar{\Delta g^\circ} + RT \ln \left[\frac{\left(\frac{P_{\text{O}_2}}{P_0} \right)^{\frac{1}{2}} \left(\frac{P_{\text{H}_2\text{O}}}{P_0} \right)^{\frac{1}{2}}}{\left(\frac{P_{\text{H}_2\text{O}(l)}}{P_0} \right)^{\frac{1}{2}}} \right] \\ &= -237180 \frac{\text{kJ}}{\text{kmol Hz}} + (8.314 \frac{\text{kJ}}{\text{kmol K}})(348\text{K}) \ln \left[\frac{(2)^{\frac{1}{2}} (1.5 \times 0.2)^{\frac{1}{2}}}{(1)^{\frac{1}{2}}} \right] \\ &= \left(-237180 \frac{\text{kJ}}{\text{kmol Hz}} \right) + \left(+334 \frac{\text{kJ}}{\text{kmol Hz}} \right) \quad \underbrace{-0.1156}_{=} \end{aligned}$$

$$\bar{\Delta G}_{75^\circ\text{C}} = -236846 \frac{\text{kJ}}{\text{kmol Hz}}$$

(b) at 300°C → product water is in vapor form

$$\bar{\Delta h_f^\circ} = -241820 \frac{\text{kJ}}{\text{kmol Hz}}$$

$$\bar{\Delta g^\circ}_{\text{H}_2} = -228590 \frac{\text{kJ}}{\text{kmol Hz}}$$

$\underbrace{\text{vapor water product}}$

$$\bar{\Delta G}_{300^\circ\text{C}} = \bar{\Delta g^\circ}_{\text{H}_2} + RT \ln \left(2 \cdot (1.5 \cdot 0.2)^{0.5} \right) = -228040 \frac{\text{kJ}}{\text{kmol Hz}}$$

Maximum Conversion Efficiency

$$(a) \eta_{75^\circ\text{C}} = \frac{\bar{\Delta G}}{\bar{\Delta h_f^\circ}} = \frac{-236846 \frac{\text{kJ}}{\text{kmol Hz}}}{-285830 \frac{\text{kJ}}{\text{kmol Hz}}} = 0.829$$

$$(b) \eta_{300^\circ\text{C}} = \frac{-228040 \frac{\text{kJ}}{\text{kmol Hz}}}{-285830 \frac{\text{kJ}}{\text{kmol Hz}}} = 0.799$$

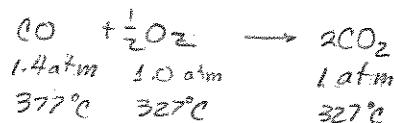
For fuelcells, the higher heating value is used even when the product water is in vapor form.

Open Circuit Voltage : $E = E^\circ - RT \ln(K)$; $E^\circ = \frac{\bar{\Delta G}^\circ}{nF}$; $nF = \frac{2 \text{ kmol e}^-}{\text{kmol Hz}}$

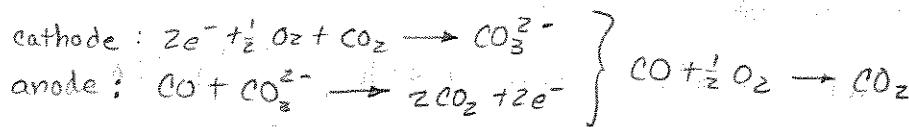
$$(a) E_{75^\circ\text{C}} = 1.227 \text{ V} ; P_{\text{H}_2} = 2 \text{ atm}, P_{\text{O}_2} = 1.5 \text{ atm in air}, P_{\text{H}_2\text{O}} = 1 \text{ atm}$$

$$(b) E_{300^\circ\text{C}} = 1.18 \text{ V} ; \quad " \quad " \quad "$$

CO Fuel Cell @ 327°C



Determine the cell voltage and efficiency for CO reacting in air.



$$E = E^\circ - \frac{RT}{neF} \ln \left(\frac{P_{\text{CO}_2}}{P_{\text{CO}} P_{\text{O}_2}^{1/2}} \right) \quad ; \quad P_0 = 1 \text{ atm}$$

$$E^\circ = \frac{\Delta \bar{G}^\circ}{neF}$$

$$\Delta \bar{G}^\circ = 1 \bar{g}_{\text{CO}_2}^\circ - \bar{g}_{\text{CO}}^\circ - 0.5 \bar{g}_{\text{O}_2}^\circ = (-394360 \frac{\text{kJ}}{\text{kmol}}) - (-137150 \frac{\text{kJ}}{\text{kmol}}) - \frac{1}{2}(0)$$

$$\frac{1 \text{ kmol CO}_2}{1 \text{ kmol O}_2}$$

$$E^\circ = - \frac{(-257210 \frac{\text{kJ/kmol CO}}{\text{kmol e}^-})}{(2 \frac{\text{kmol e}^-}{\text{kmol CO}})(96489 \frac{\text{C}}{\text{kmol e}^-})} = 1.33 \text{ V} \quad (\text{open circuit potential at reference state})$$

$$E = E^\circ - \frac{RT}{neF} \ln \left(\frac{P_{\text{CO}_2}}{P_{\text{CO}} P_{\text{O}_2}^{1/2}} \right) = 1.33 \text{ V} - \frac{(8.134 \frac{\text{kJ}}{\text{kmol K}})(600 \text{ K})}{(2 \frac{\text{kmol e}^-}{\text{kmol CO}})(96489 \frac{\text{C}}{\text{kmol e}^-})} \ln \left(\frac{1}{1.4(1.0 \times 0.2)^{0.5}} \right)$$

↑ fraction of O₂
in air.

$$= 1.33 \text{ V} - (0.0253 \text{ V})(0.44555)$$

$$E = 1.319 \text{ V} \quad (\text{open circuit potential at } 327^\circ\text{C \& } P_{\text{CO}} = 1.4 \text{ atm})$$

$$\left. \begin{array}{l} \Delta \bar{G} = -neFE = -254494 \frac{\text{kJ}}{\text{kmol CO}} \\ \Delta \bar{H}^\circ = -282990 \frac{\text{kJ}}{\text{kmol CO}} \end{array} \right\} \eta_{\text{max}} = \frac{\Delta \bar{G}}{\Delta \bar{H}^\circ} = 0.8993$$