Energy Conversion Lecture Notes:
Wind Energy

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1 Introduction to Wind Energy

Where does wind come from?

- local and global pressure differences

What forms of energy are involved with wind energy harvesting?

- kinetic energy converting to...
- **shaft work**
- electric potential
- gravitational potential
- chemical potential

2 Types of Turbines

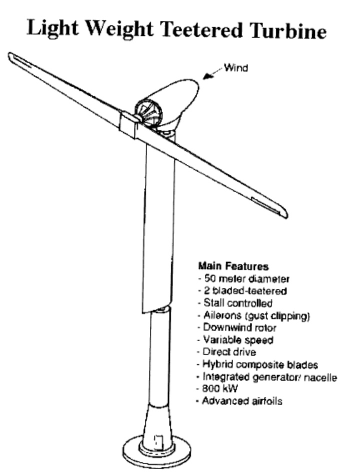


Figure 4-1 Future - NREL/DOE

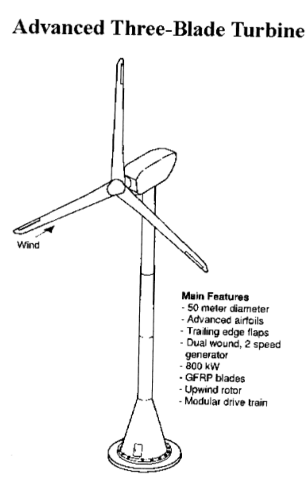


Figure 4-2 Future - NREL/DOE

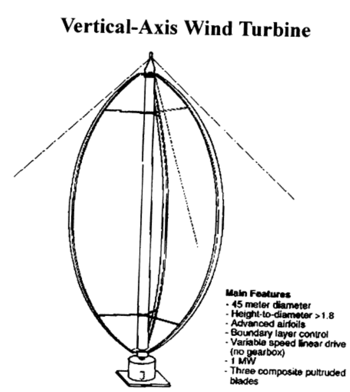
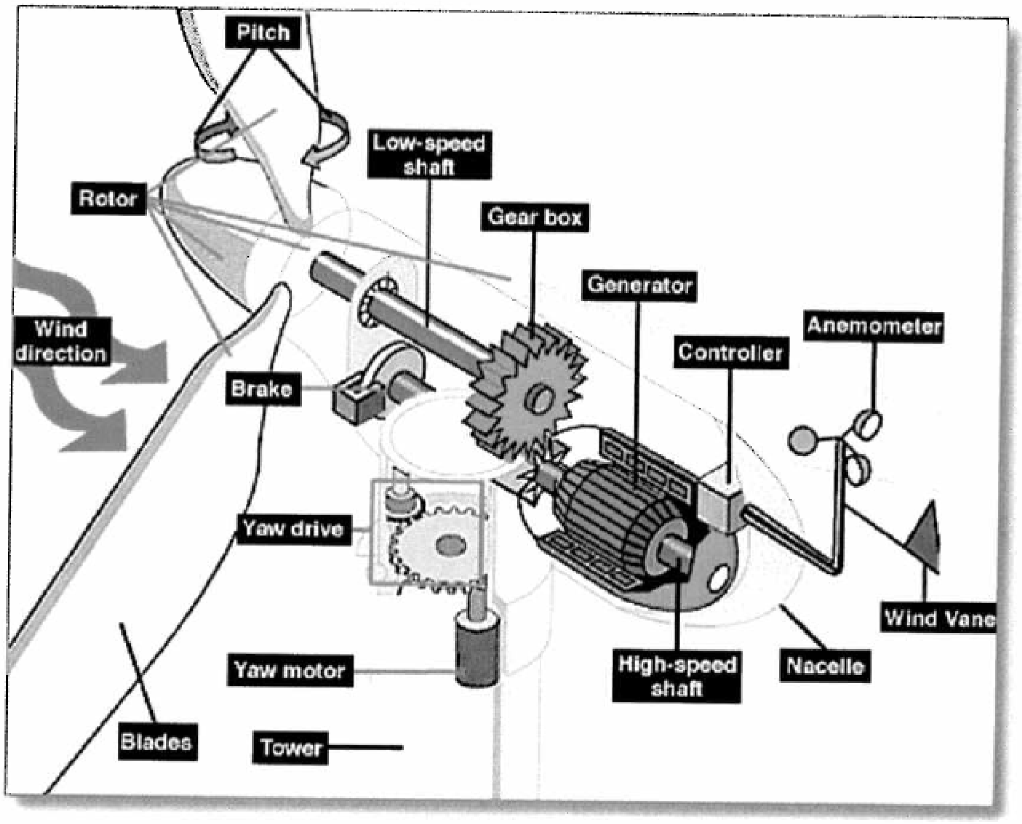
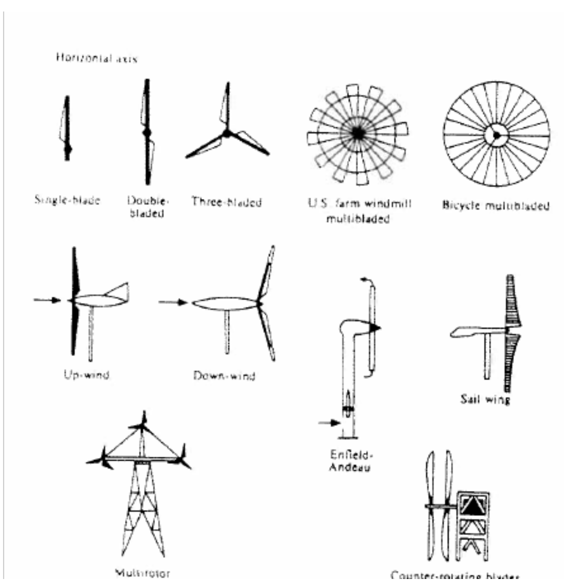
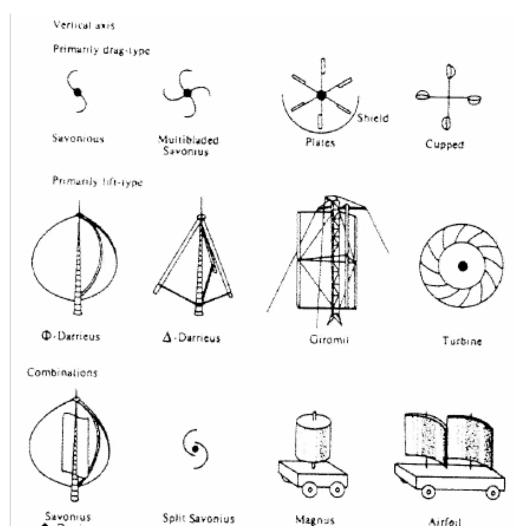


Figure 4-3 Future - NREL/DOE



2.1 HAWTs vs VAWTs





2.1.1 Horizontal Axis Wind Turbines

Advantages

- variable pitch
- tall tower - higher wind speeds
- high efficiency
- steady angle of attack

Disadvantages

- tall tower/large blades - difficult to transport, challenging to install
- main components installed at top of tower
- high visibility
- yaw control is necessary

2.1.2 Vertical Axis Wind Turbines

Advantages

- smaller support structure compared with HAWTs
- yaw control not needed
- generator components typically located on the ground
- typically less noise compared to HAWTs
- can take advantage of higher windspeeds produced by local stuctures and geography

Disadvantages

- cyclic loading makes fatigue failure more likely
- lower wind speeds due to shorter structure

2.2 Lift Driven vs Drag Driven Turbines

- Lifting turbines always more efficient
- Savonius Turbine - simple drag driven VAWT

3 Betz Limit

- model to determine the power extraction of an *ideal* turbine
- model is attributed to Betz (1926) and is a simple linear momentum model based on ship propeller performance
- model considers a control volume analysis of a stream tube

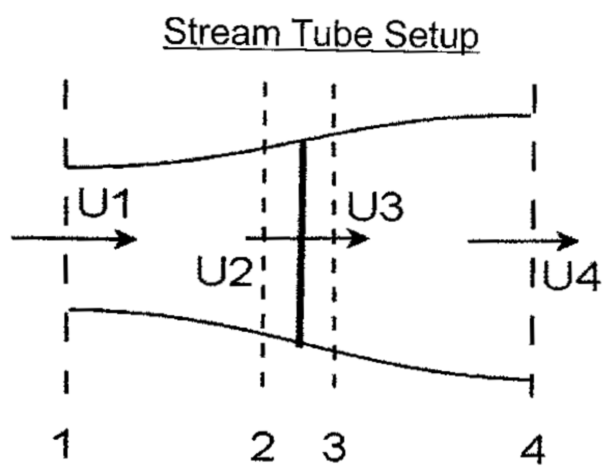
3.1 Available Power in the Wind

$$P = \frac{1}{2}\rho U^3 A \tag{1}$$

3.2 Betz Limit Model Assumptions

- homogeneous, imcompressable, steady flow
- no frictional drag
- infinite number of blades (power extracted over entire cross section)
- uniform thrust over entire cross section
- non-rotating wake
- static pressure far up and down stream equal to ambient static pressure

3.3 Stream Tube Control Volume



Refer to this figure for the following analysis

3.4 Thrust Over the Control Volume

The thrust over the control volume is calculated by:

$$T = U_1 \left(\rho A U\right)_1 - U_4 \left(\rho A U\right)_4 \tag{2}$$

Recognizing that the mass flow rate must remain constant through the control volume:

$$\left(\rho A U\right)_1 = \left(\rho A U\right)_4 = \dot{m} \tag{3}$$

This simplifies the thrust equation to:

$$T = \dot{m}\left(U_1 - U_4\right) \tag{4}$$

3.5 Bernoulli's Equation Before and After the Rotor

Before the rotor (state 1 to state 2):

$$p_1 + \frac{1}{2}\rho {U_1}^2 = p_2 + \frac{1}{2}\rho {U_2}^2 \tag{5}$$

After the rotor (state 3 to state 4):

$$p_3 + \frac{1}{2}\rho {U_3}^2 = p_4 + \frac{1}{2}\rho {U_4}^2 \tag{6}$$

Also note that from the assumptions: $p_1 = p_4$, $U_2 = U_3$ & $A_2 = A_3$

3.6 Thrust at the Rotor

The definition of pressure is used to determine the thrust at the rotor:

$$\begin{aligned} F &= pA \\ T &= A_2 (p_2 - p_3) \\ T &= A_2 \left[\frac{1}{2}\rho \left(U_1^2 - U_4^2\right)\right] \end{aligned} \tag{7}$$

3.7 Combining Deffinitions of Thrust

Thus far the thrust has been defined two ways, over the control volume (Eqn 4) and at the rotor (Eqn 7). Equating (Eqn 4) and (Eqn 7) and substituting in the mass flow rate at state 2 gives:

$$\rho A_2 U_2 (U_1 - U_4) = \frac{1}{2}\rho A_2 \left(U_1^2 - U_4^2\right) \tag{8}$$

Simplifying Eqn 8 above gives:

$$U_2 = \frac{U_1 - U_4}{2} \tag{9}$$

which physically means the wind speed at the rotor is simply an average of the upstream and downstream velocities.

3.8 Axial Induction Factor

To examine the physical limitations of the system, the *Axial Induction Factor* (a) is defined as the fractional decrease in wind speed from state 1 to state 2:

$$a = \frac{U_1 - U_2}{U_1} \tag{10}$$

Rearranging for U_2, the wind speed at the rotor becomes:

$$U_2 = U_1 (1 - a) \tag{11}$$

Since U_2 is the average of U_1 and U_4, the same fractional decrease in wind speed must occur from state 3 to state 4 giving:

$$U_4 = U_1 (1 - 2a) \tag{12}$$

The wind velocity downstream of the rotor is now in terms of the upstream velocity and axial induction factor only. Physically, the a term is related to the energy extracted from the wind. The limits on a are explored mathematically below.

3.9 Limits on the Axial Induction Factor

Considering (Eqn 12) from above and examining the limits on a:

$$U_4 = U_1 \left(1 - 2a\right)$$

- If $a = 0$ then the downstream velocity is the same as the upstream velocity as if no rotor is present.
- If $a = \frac{1}{2}$ the downstream velocity must equal zero. This sets the upper limit of a.

3.10 Power Extracted at the Rotor and Power Coefficient

The power extracted at the rotor is determined by:

$$\begin{aligned}
P &= \frac{1}{2}\dot{m}\Delta U^2 \\
P &= \frac{1}{2}\rho A_2 U_2 \left(U_1^2 - U_4^2\right) \\
P &= \frac{1}{2}\rho A_2 U_2 \left(U_1 - U_4\right)\left(U_1 - U_4\right) \\
P &= \frac{1}{2}\rho A_2 \left(U_1 \left(1-a\right)\right)\left(U_1 - \left(U_1 \left(1-2a\right)\right)\right)\left(U_1 - \left(U_1 \left(1-2a\right)\right)\right) \\
P &= \frac{1}{2}\rho A_2 U_1^3 4a \left(1-a\right)^2 \qquad (13)
\end{aligned}$$

Using this expression, the power coefficient can be calculated as the power extracted by the rotor compared to the power available in the wind:

$$C_P = \frac{\text{power extracted}}{\text{power available}} = \frac{\frac{1}{2}\rho A_2 U_1^3 4a \left(1-a\right)^2}{\frac{1}{2}\rho U_1^3 A} \qquad (14)$$

This simplifies to:

$$\boxed{C_P = 4a \left(1-a\right)^2} \qquad (15)$$

3.11 Max Power Coefficient (Betz Limit)

Differentiating the expression for the power coefficient derived above and finding where it equals zero (by adjusting a) will reveal potential maximums and minimums of the power coefficient.

$$C_P' = 4a \left(1-a\right)^2 - 8a \left(1-a\right) \qquad (16)$$

When $C_P' = 0$, $a = (1, \frac{1}{3})$. Earlier it was shown that a can range only from 0 to $\frac{1}{2}$ therefore the maximum possible power coefficient (Betz Limit) is shown to be:

$$C_{P,max} = 4a \left(1-a\right)^2 = 4\left(\frac{1}{3}\right)\left(1 - \left(\frac{1}{3}\right)\right)^2 \approx \boxed{0.5926} \qquad (17)$$

This is the best possible power coefficient achievable by a wind turbine. If a continues to grow from $\frac{1}{3}$ the power coefficient decreases. It should be noted that the theory only holds to $a = \frac{1}{2}$. At this value of axial induction factor, the downstream velocity has been reduced to zero, according to the assumptions.

The thrust coefficient (C_T) is defined as:

$$C_T = \frac{\text{Thrust Force}}{\text{Dynamic Force}} = \frac{\frac{1}{2}\rho A U_1^2 \left[4a \left(1-a\right)\right]}{\frac{1}{2}\rho A U_1^2} \qquad (18)$$

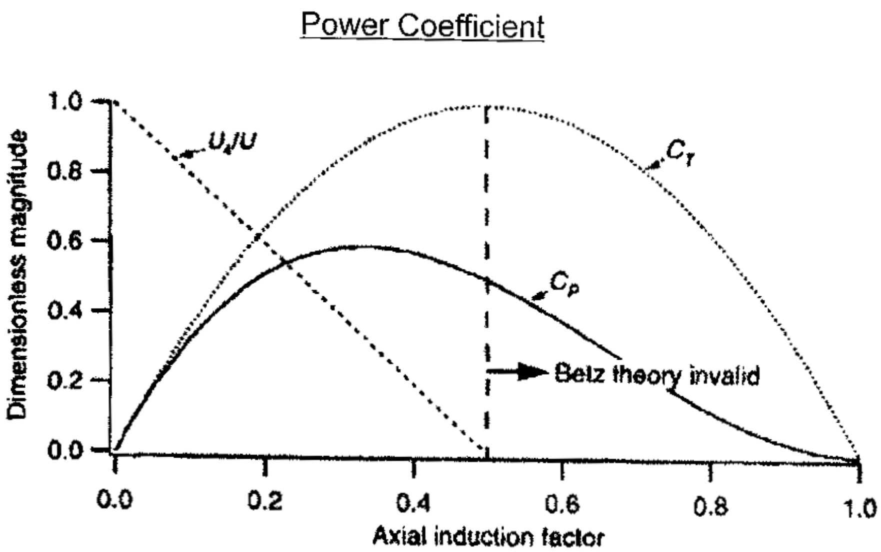
which has a maximum at $a = \frac{1}{2}$

The Betz limit represents the highest theoretical efficiency that can be achieved for this ideal case. For the case of real turbines, three main aerodynamic effects reduce the efficiency (C_P):

1. Rotating wake
2. Finite number of blades and tip losses
3. Aerodynamic drag

Mechanical inefficiencies in the rest of the system will also reduce the amount of power that can be extracted from the wind. The most power that can be extracted by the wind turbine can be calculated by:

$$\eta_{overall} = \frac{P_{out}}{\frac{1}{2}\rho A U^3} = \eta_{mech} C_P \Rightarrow P_{out} = \frac{1}{2}\rho A U^3 \left(\eta_{mech} C_P\right) \tag{19}$$



The remainder of the notes and figures in this lecture has has been borrowed from Wind Energy Systems by Dr. Gary L. Johnson.

4 C_P and Tip Speed Ratio

Power Extracted

$$P_m = C_P \left(\frac{1}{2}\rho A u^3\right) = C_P P_w \tag{20}$$

Tip Speed Ratio - the ratio of linear blade speed to incoming wind speed

$$\lambda = \frac{r_m \omega_m}{u} \tag{21}$$
$$r_m = \text{max swept radius}$$
$$\omega_m = \text{rotor angular velocity}$$
$$u = \text{wind speed}$$

- for a given turbine, there exists a tip speed ratio that optimizes the C_P

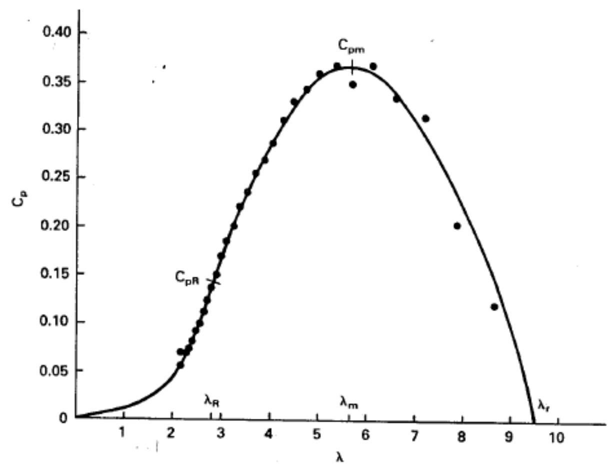


Figure 9: Coefficient of performance C_p versus tip-speed ratio λ for Sandia 17-m Darrieus turbine. Two blades; 42 r/min.

5 Effects of Efficiency

Electrical Power Output

$$P_e = C_P \eta_m \eta_g P_W \qquad (W) \tag{22}$$
$$C_P = \text{Power Coefficient}$$
$$\eta_m = \text{Transmission Efficiency}$$
$$\eta_g = \text{Generator Efficiency (or pump, compressor, ect...)}$$

Rated Power Output:

$$P_{eR} = C_{PR} \eta_{mR} \eta_{gR} \frac{\rho}{2} A u_R^3 \qquad (W) \tag{23}$$

where the R subscripts denote values at the rated power. The rated overall efficiency is simply:

$$\eta_o = C_{PR} \eta_{mR} \eta_{gR} \tag{24}$$

6 Energy Production and Capacity Factor

The average power of a turbine can be calculated by:

$$P_{e,ave} = \int_0^\infty P_e f(u) du \qquad (W) \tag{25}$$

$f(u)$ is the probability density function of wind speeds, the Weibull distribution in this case:

$$f(u) = \frac{k}{c}\left(\frac{u}{c}\right)^{k-1} \exp\left[-\left(\frac{u}{c}\right)^k\right] \tag{26}$$

The electrical power out over all possible ranges of wind speed:

$$\begin{aligned} P_e &= 0 && (u < u_c) \\ P_e &= a + bu^k && (u_c < u < u_R) \\ P_e &= P_{eR} && (u_R < u < u_F) \\ P_e &= 0 && (u > u_F) \end{aligned} \tag{27}$$

where k is the Weibull shape parameter, c is a scale parameter (with units of velocity) related to the wind speed regime and

$$a = \frac{P_{eR} u_c^k}{u_c^k - u_R^k} \qquad \text{and} \qquad b = \frac{P_{eR}}{u_R^k - u_c^k} \tag{28}$$

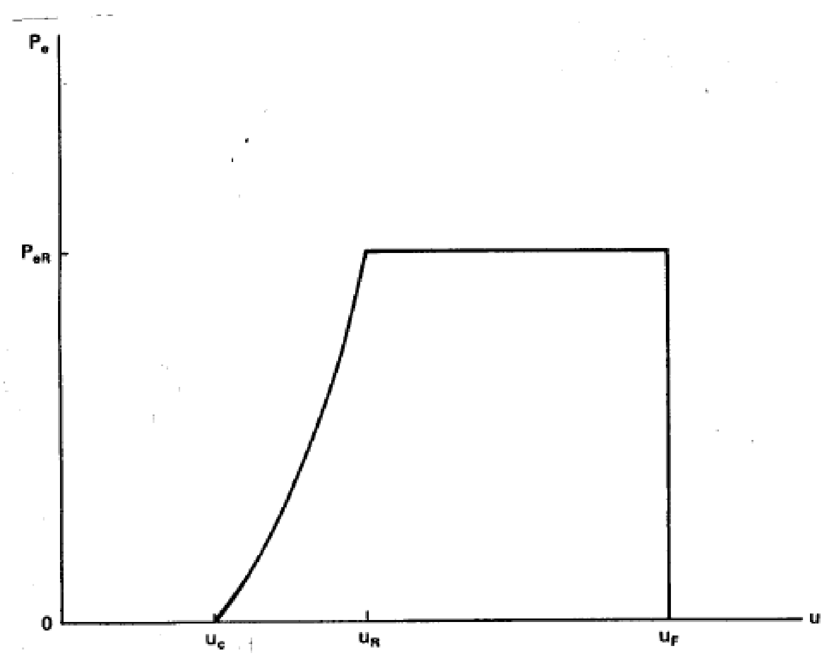


Figure 16: Model wind turbine output versus wind speed.

Considering all windspeeds:

$$P_{e,ave} = \int_{u_c}^{u_R} \left(a + bu^k\right) f(u) du + P_{eR} \int_{u_R}^{u_F} f(u) du \qquad (W) \tag{29}$$

Integrating the above equation gives:

$$P_{e,ave} = P_{eR} \left\{ \frac{\exp\left[-\left(\frac{u_c}{c}\right)^k\right] - \exp\left[-\left(\frac{u_R}{c}\right)^k\right]}{\left(\frac{u_R}{c}\right)^k - \left(\frac{u_c}{c}\right)^k} - \exp\left[-\left(\frac{u_F}{c}\right)^k\right] \right\} \qquad (W) \tag{30}$$

Grouping all of the terms inside the braces into a coefficient called the capacity factor (CF) gives:

$$P_{e,ave} = P_{eR}(CF) = \eta_o \frac{\rho}{2} A u_R^3 (CF) \qquad (W) \tag{31}$$

The normalized average power is:

$$P_N = \frac{P_{e,ave}}{\eta \left(\frac{\rho}{2}\right) A c^3} = (CF) \left(\frac{u_R}{c}\right)^3 \tag{32}$$

The yearly energy production for a turbine is given by:

$$W = P_{e,ave}(\text{time}) = (CF) P_{eR} (8760) \qquad (kWh) \tag{33}$$

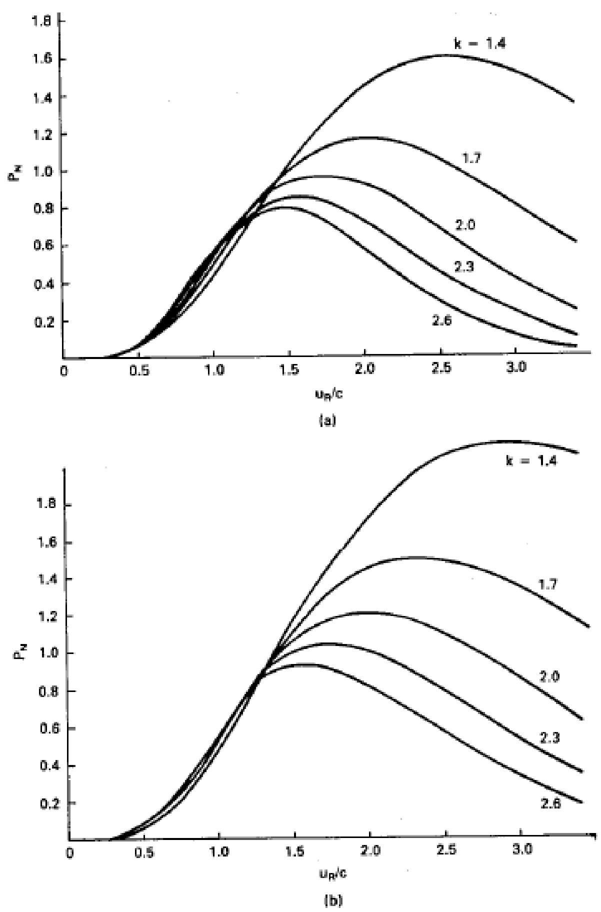


Figure 17: Normalized power versus normalized rated speed: (a) $u_c = 0.5u_R$, $u_F = 2u_R$; (b) $u_c = 0.4u_R$, $u_F = 2u_R$.

7 Basic Airfoil Aerodynamics

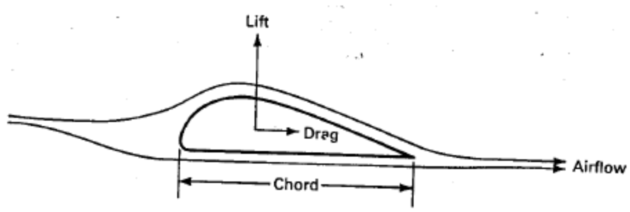


Figure 3: Lift and drag on a stationary airfoil.

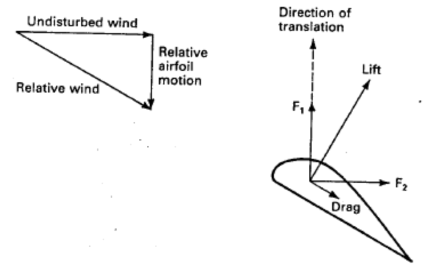


Figure 4: Lift and drag on a translating airfoil.

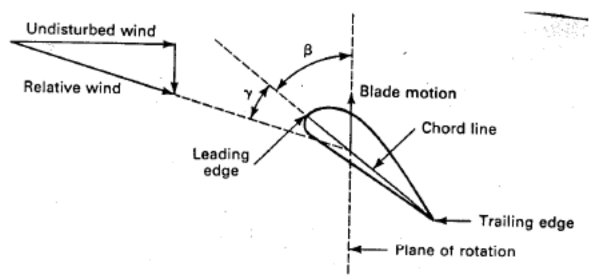
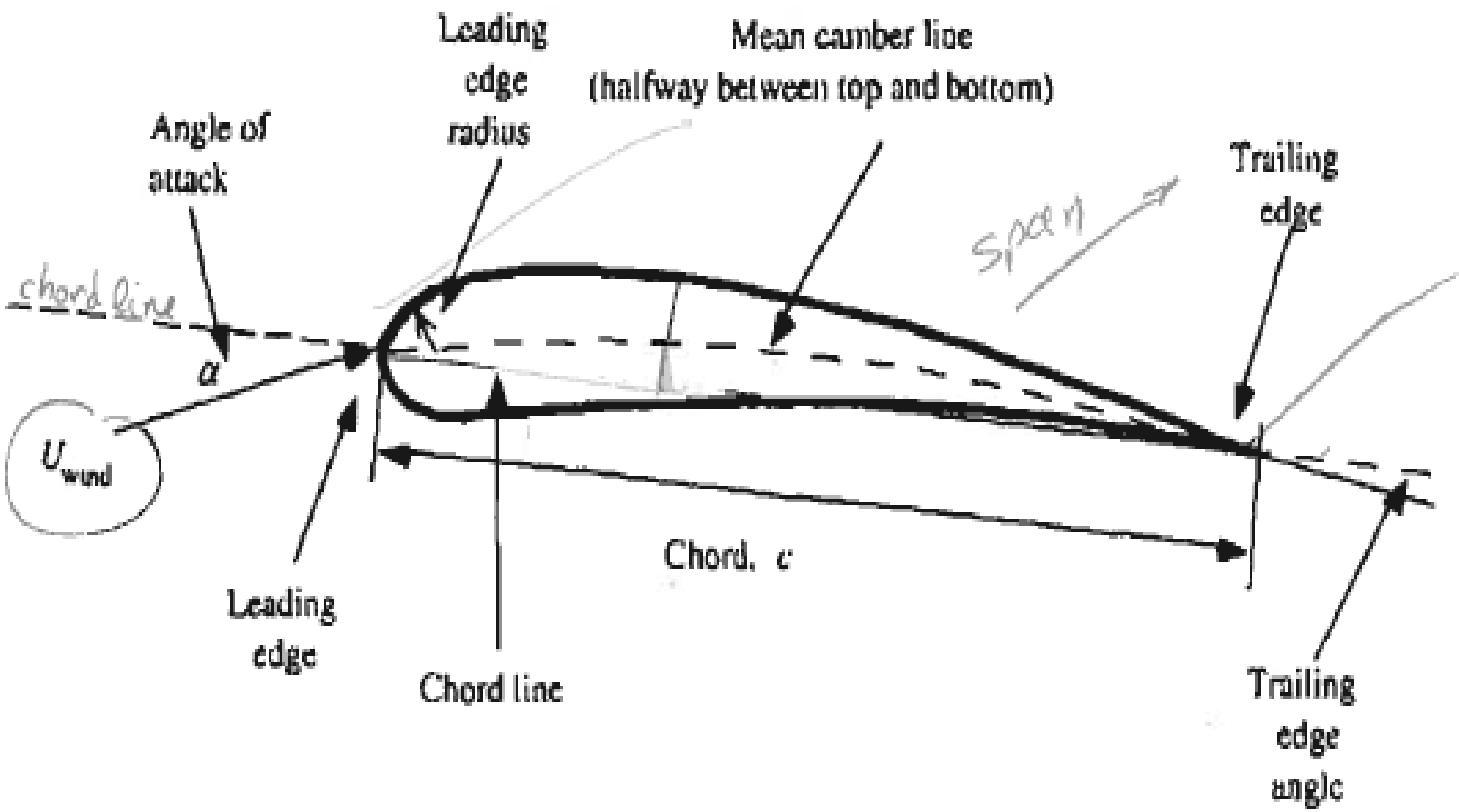
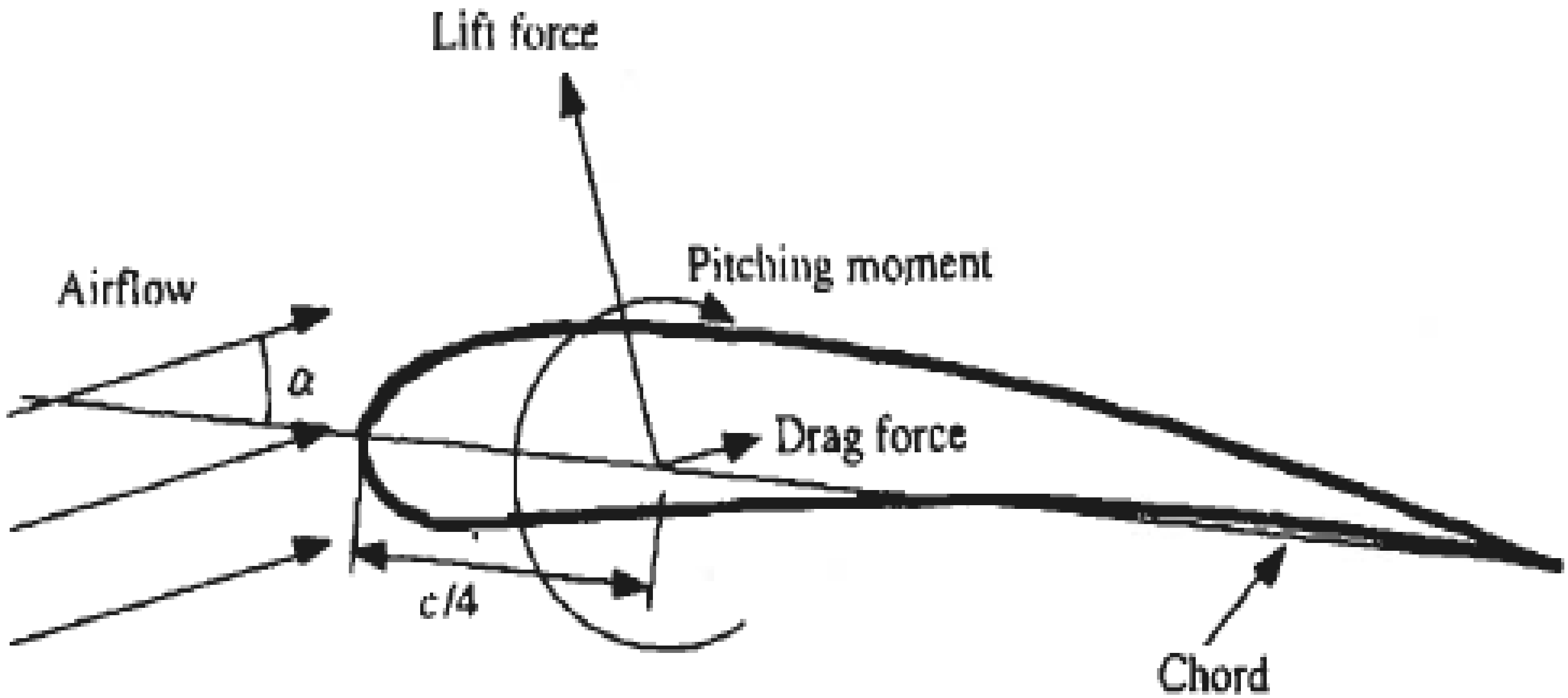


Figure 6: Definition of pitch angle β and angle of attack γ.

Airfoil Terminology



Airfoil Forces



Airfoil Forces

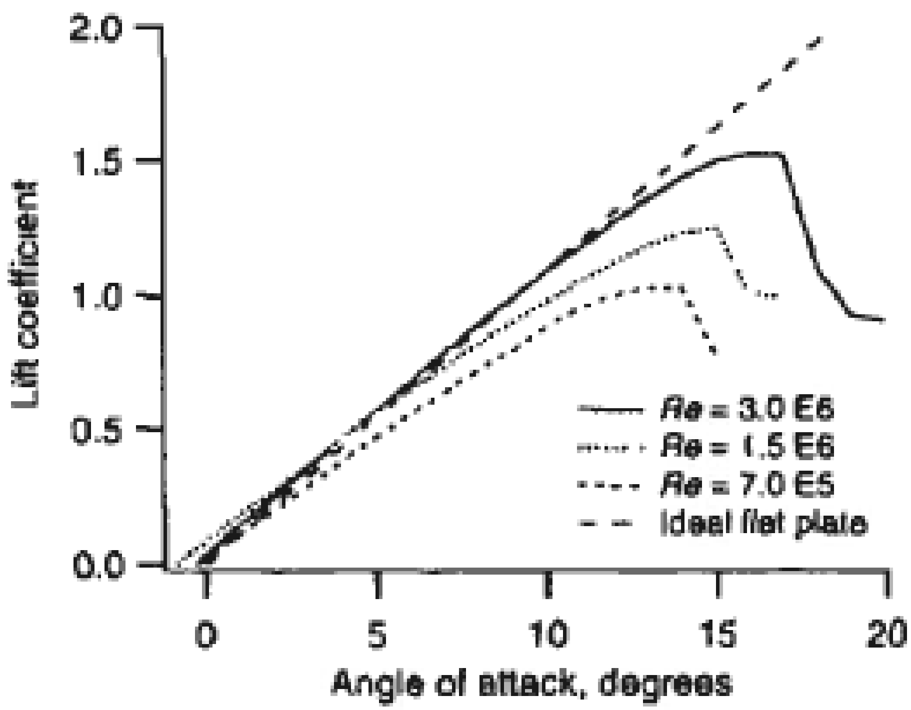
$$C_l = \frac{L/l}{\frac{1}{2}\rho U^2 c} = \frac{Lift\ Force}{Dynamic\ Force}$$

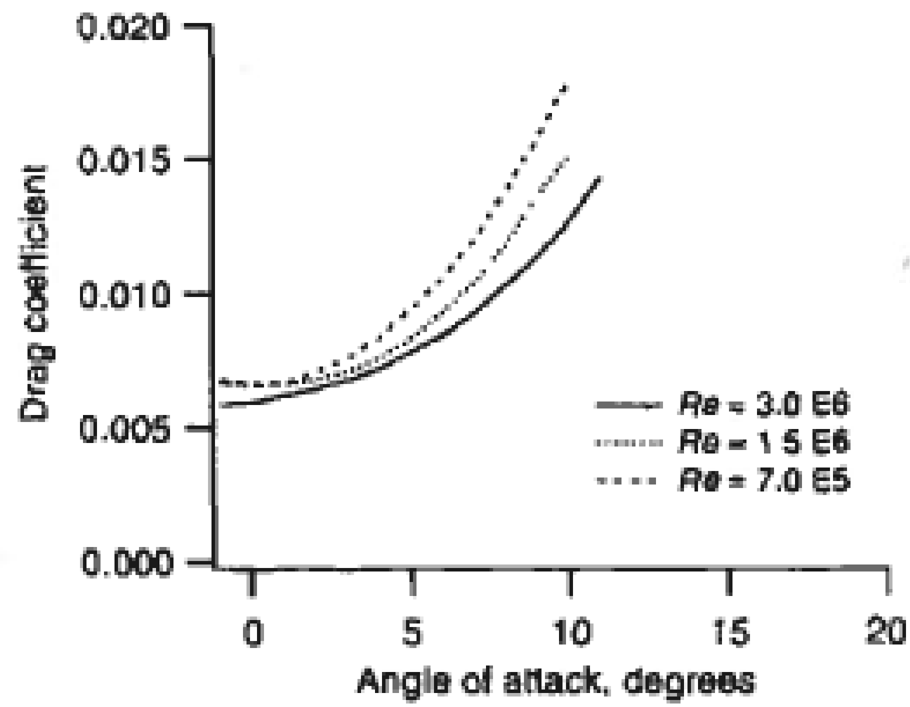
$$C_d = \frac{D/l}{\frac{1}{2}\rho U^2 c} = \frac{Drag\ Force}{Dynamic\ Force}$$

$$Re = \frac{U L}{\nu} = \frac{\rho U L}{\mu} = \frac{Inertial\ Force}{Viscous\ Force}$$

$$C_m = \frac{M}{\frac{1}{2}\rho U^2 l c^2} = \frac{Pitching\ Moment}{Dynamic\ Moment}$$

Airfoil Forces





NACA 0012 symmetric airfoil (Miley, 1982)