Dynamics of Evaporating Films: A Numerical Model and an Experimental Approach

by

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Abstract

The phenomenon of evaporation is observed through regular processes such as cooling, drying, hardening of plastic, evaporation from open reservoirs. Evaporation is of great importance in many engineering problems. It is a complicated fluid phenomenon even in the absence of vapor shear, involving combined effects of several physical mechanisms including microconvective stirring, thermocapillary flows, and body forces due to density differences. The combined effect of all these mechanisms leads to a complicated, three-dimensional fluid motion. In addition to these physical mechanism there can be a destabilizing effect due to recoil of the ejected vapor.

The scientific investigation is on the competing fluid physical mechanisms operative during evaporation processes in reduced gravity and impact of these mechanisms on the stability and structure of fluid film. The experimental approach is supported by numerical modeling which initiates and diagnoses instabilities and motion of fluid films undergoing evaporation unique to the micro-gravity environment. The research is focused on instability mechanisms; namely, the Rayleigh-Taylor instability, surface tension effects, flow structure and implications of film instability for heat transfer. The numerical approach is based on finite difference techniques where in the film thickness variation is analyzed based on nonlinear evolution equation. The numerical results are obtained for distinct cases of fluid film; such as isothermal film, quasi-equilibrium, non-evaporating film and comprehensive evolution equation. The findings from numerical analysis will be used in future to design the experimental test set up and the control parameters for the experiments to be conducted on NASA's Reduced Gravity Aircraft.

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Chapter 1 Introduction

1.1 Evaporation

The process wherein the atoms or molecules in a liquid state gain sufficient energy to enter the gaseous state is termed as evaporation. It is a gradual change of state from liquid to gaseous phase at the liquid's surface. At higher vapor pressure, evaporation advances promptly at higher temperature and higher flow rate from liquid to gaseous state. Evaporation phenomenon is of great importance in many engineering problems. It is a prime factor in spacecraft thermal control applications.Devices namely capillary pumped loops and loop heat pipes depend critically on evaporation.

The ability to maintain a stable evaporating film is critical to a number of life support and thermal control technolgies necessary for human exploration is space. A variety of physical mechanisms acting at a liquid-vapor interface may tend to stabilize or destabilize the surface when subjected to small perturbations. Capillary effects tend to amplify long-wave length instabilities while gravitational body forces may stabilize or destablize an interface.Cooling at the interface via evaporation tends to give rise to microconvective stirring of the fluid due to thermocapillary flows as well as buoyancy-induced flows in the presence of natural or artificial gravity. Rapid evaporation can perturb the liquid surface due to the inertia of the vapor; a condition referred to as vapor recoil. The combined effect of all the these mechanisms leads to a complex three dimensional fluid motion within the liquid film. In terrestrial applications, an evaporating film can be rendered stable by proper orientation with respect to the gravitational acceleration. The absence of gravity or in reduced gravity environments such as found on the Moon or Mars, it is unclear at present how to stably maintain an evaporating liquid layer.

1.2 Research overview

The objective of the research presented here is to establish the design criteria for lowgravity experiments intended to investigate the stability of evaporating liquid films under variable gravitational accelerations. The results of numerical models of a perturbed viscous liquid layer bounded at the top by its vapor, placed over a uniformly heated rigid plate are presented. The numerical model is based on the nonlinear evolution equation [?, Burelbach]ormulated by Davis, et al. [?, Burelbach] The inverted orientation of the liquid surface induces instabilities due to the gravitational body force giving rise to pendant drops on the underside of a plate when the liquid film is thick. Should the liquid film being sufficiently thin, film rupture is possible due to disjoining pressure effects; also referred to as van der Waals forces. The surface tension tend to amplify long wave length instabilities and suppress short wavelength instabilities. In the conditions where surface tension is only acting mechanism on the vapor liquid interface, its lone presence tend to stabilize the perturbed liquid film, the long wave length instabilities generated by the surface tension stabilizes the liquid layer depending on the magnitude of initial disturbances and its orientation. The combination of evaporative cooling and surface tension forces in the absence of disjoining pressure force results into film rupture depending on magnitude of evaporative cooling and surface tension. On addition of the vapor recoil effect in the case of rapid evaporation acts on the liquid surface accelerates the process of film rupture. In the absence of surface tension, the process of film rupture accelerates but there is absence of pendant drop formation with gravity forces stabilizing the liquid vapor interface. In the presence of vapor recoil forces in addition to evaporative cooling and body forces, destabilizes the liquid vapor interface and initiates dry spot or film rupture. These combinations of different stabilizing- destabilizing mechanisms is discussed elaborately along with film profile variations and magnitudes of fast growing wavelengths of instabilities. Also, combination of all these mechanisms is included in a numerical model which is the result of single evolution equation and its numerical results are discussed in the subsequent chapter based on thin liquid films (100 - 1000A)and thick liquid films $(1 - 100 \mu m)$. The purpose of these simplified numerical models is to make use of the resulting findings in the experimental approach with investigation of the effects of above mentioned instability mechanisms on evaporating liquid film dynamics and behavior of the vapor-liquid interface.

The experimental test set up is devised to investigate the time scales of evaporating liquid film placed over uniformly heated copper block along with study of film behavior inside the leak free vacuum environment. Liquid media under investigation is ethanol. The dissimilar response of system components to evaporating/condensing liquid film is observed. The Glimpses of bouyancy effect is noticed. The experimental approach is initiated with test for prime purpose validity of components, calibration of power supplies, pressure sensors and gauges. Installation procedure for each components is explained in conjunction to illustrative representation.

Chapter 2 One-sided model of evaporation

2.1 The Basic concept

A one-sided model of thin viscous layer bounded by its vapor and below by a uniformly heated rigid plane is formulated based on the works of [1]. The liquid film is thin enough such that gravity effects in the formulation may be neglected whereas the van der Waals forces of attractions are considered important. The film is an incompressible, non-draining and laterally unbounded Newtonian fluid with constant material properties. There is mass loss, momentum transfer and energy consumption at the vapor-liquid interface. Figure (2.1) shows the physical configuration describing an evaporating liquid film on a uniformly heated horizontal plate [1]. The vaporliquid interface is located at z = h(x, t), where z is the vertical co-ordinate, and the film thickness h is a function of the lateral co-ordinate x and time t. J(x, t) is the mass flux due to evaporation. $T^{(I)}(x, t)$ is the temperature of the interface.

The van der Waals forces are accounted using potential function ϕ which depends



Figure 2.1: Evaporating thin liquid film on a heated horizontal plate. The gravitational acceleration is in the \pm z-directions

on layer thickness: $\phi = A/h^3$ The Navier-Stokes equation for the liquid layer with an extra body force, $\nabla \phi$ which models the van der Waals attractions:

$$\nabla(V_t + V \cdot \nabla V) = -\nabla p - \nabla \phi + \mu \nabla^2 V \qquad (2.1)$$

Based on *Davisetal* the continuity and energy equations in the liquid are,

$$\nabla \cdot V = 0 \tag{2.2}$$

$$T_t + V \cdot \nabla T = \kappa \nabla^2 T \tag{2.3}$$

The jump mass balance,

$$J = \rho(V - V_i) \cdot n = \rho_v(V_v - V_i) \cdot n \tag{2.4}$$

The jump energy balance is given by

$$J h_{fg} + \frac{1}{2} \left[\frac{J}{\rho_v} \right]^2 = -\kappa \nabla \cdot n$$
(2.5)

A linearized constitutive equation for evaporative flux is derived from kinetic theory

$$J = \left(\frac{\alpha \rho_v h_{fg}}{T_s^{3/2}}\right) \left(\frac{MW}{2\pi R_g}\right)^{\frac{1}{2}} (T_i - T_s)$$
(2.6)

A single evolution equation for dimensionless film shape, h(x, t), may be obtained using one-sided model and the above scaling. The reader is referred to the work of Davisetal[1, 2] for details. The evolution equation for an evaporating liquid film is

$$h_t + E\left(h + K^{-1}\right) + S\left(h^3 h_{xxx}\right)_x + \left[Ah^{-1} + E^2 D^{-1}(h + K^{-1})h^3 + KMP^{-1}(h + K)^{-2}h^2\right]h_x = 0$$
(2.7)

The ratio of viscous to evaporative time scales is the Evaporation number:

$$E = \frac{k\Delta T}{\rho\nu h_{fg}} \tag{2.8}$$

The non-dimensional measure of van der Waals forces is the Disjoining Pressure number:

$$A = \frac{\dot{A}}{6\pi\rho\nu^2\dot{h_0}} \tag{2.9}$$

The non-dimensional measure of surface tension is the Surface tension number:

$$S = \frac{\sigma h_0}{3\rho\nu^2} \tag{2.10}$$

which is the ratio of surface tension and viscous forces. The Prandtl number is

$$P = \frac{\nu}{\kappa} \tag{2.11}$$

which charactizes the convection based on the ratio of fluid viscosity to thermal conductivity of the fluid. The parameter K measures the degree of non-equilibrium at the evaporating interface:

$$K = \left(\frac{kT_s^{3/2}}{\alpha \dot{h_0} \rho_v h_{fg}^2}\right) \left(\frac{2\pi R_g}{MW}\right)^{1/2}$$
(2.12)

The Marangoni number is

$$M = \frac{\gamma \Delta T \dot{h_0}}{2\rho\nu\kappa} \tag{2.13}$$

where ΔT is the temperature difference across the interface. The ratio of the vapor to liquid densities is defined as

$$D = \frac{3\rho_v}{2\rho} \tag{2.14}$$

Finally, ratio of gravitational to viscous effects is the Gravitational number:

$$G = \frac{g\dot{h_0}^3}{3\nu^2} \tag{2.15}$$

Although all of the parameters identified above are relevant to an evaporating film, the current analysis consider the stability characteristics of only isothermal film.

Chapter 3 Numerical Model

3.1 Formulation of Numerical Model

The evolution equation (2.7) is highly nonlinear partial differential equation. A stable numerical scheme is selected by validating and reproducing the results for the case of isothermal thin film from Burelbach[1]. This numerical scheme could be employed for all different cases of simplified evolution equation. The evolution equation is solved as an initial value problem for spatially periodic solutions on fixed interval $-\pi/k_m < X < \pi/k_m$, where k_m is the dimensionless maximizing wavenumber derived from linear stability theory.

The theoretical period of the fastest growing interfacial disturbance is $2\pi/k_m$. Centered difference is applied in space while midpoint Crank-Nicholson rule is used in time with the following initial condition:

$$h(X,0) = 1 + 0.1\sin(k_m X) \tag{3.1}$$

Considering van der Waals and surface tension effects of leading order. Assuming neglegible stirring effect due to evaporative cooling and vapor recoil effect due to rapid evaporation. The evolution equation(2.7)reduces to

$$h_t + A \left(h^{-1} h_x \right)_x + S \left(h^3 h_{xxx} \right)_x = 0$$
(3.2)

Equation 3.2 can be further simplified using the following variable transformations

$$X = \acute{x}\sqrt{\frac{A}{S}} \tag{3.3}$$

and

$$T = \acute{t}\left(\frac{A^2}{S}\right) \tag{3.4}$$

The result is a simplified evolution equation in canonical form:

$$h_T + (h^{-1}h_X)_X + (h^3h_{XXX})_X = 0$$
(3.5)



Figure 3.1: Film profiles at N = 30 for thin film computations $(100 - 1000 \text{\AA})$



Figure 3.2: Film profiles at N = 20 for thin film computations $(100 - 1000 \text{\AA})$

This approach is identical (other than the choice of programming language) to that of Burelbach [1]. The results are reproduced by using MATLAB as the programming language.

For 30 elements and at $\Delta T = 0.001$, the dimensionless rupture time is calculated to be T = 5.988 at a minimum film thickness of h = 0.03253. For N = 20 elements and $\Delta T = 0.001$, a rupture time of T = 5.988 and h = 0.02343 are calculated.

These results are consistent with Burelbach[1] thus validating the basic computational approach. This proves the numerical codes written and the build in logic is correct and gives confidence for its future usage.

The evolution equation (2.7) is strongly nonlinear. It is comprehensive equation which includes all the stabilizing/destabilizing mechanisms acting on liquid layer. This evolution equation is simplified for varied cases from isothermal thin film to the whole evolution equation retaining each parameter that represents different mechanisms acting on liquid layer.

3.2 Isothermal thin film

The results for isothermal thin film from Davis et.al [1] were used for validation of the numerical technique. This is the case of evaporating liquid layer being sufficiently thin where film rupture is possible due to disjoining pressure forces. There is considerable surface tension at the interface which tends to amplify long wavelength instabilities and suppress short wavelength instabilities due to the mass loss due to slow evaporation. At the initiation of perturbation the surface tension acts at the interface to stabilize the liquid surface but the film being adequately thin the disjoining pressure effects or popularly termed as van der Waals attraction overcomes the stabilizing forces and makes the film to dry out and rupture.

For isothermal thin film the evolution equation (2.7) simplifies, assuming there is slow mass loss due to evaporation and microconvective stirring due evaporative cooling at the interface is negligible with E = K = 0 and no temperature gradients along the liquid surface (M = 0). In addition, the gravitational body force acting on the film are small enough to neglect as compared to disjoining pressure forces which implies $G \ll A$. The resulting isothermal thin film evolution becomes:

$$h_t + A \left(h^{-1} h_x \right)_x + S \left(h^3 h_{xxx} \right)_x = 0$$
(3.6)

Equation 3.6 can be further simplified using the following variable transformations

$$X = \acute{x}\sqrt{\frac{A}{S}} \tag{3.7}$$

and

$$T = t \left(\frac{A^2}{S}\right) \tag{3.8}$$

The result is a simplified evolution equation in canonical form:

$$h_T + \left(h^{-1}h_X\right)_X + \left(h^3h_{XXX}\right)_X = 0 \tag{3.9}$$

where X and T are non-dimensional length and time variables related to their respective dimensional counterparts by scales $\dot{h_0}\sqrt{A/S}$ and $(\nu/\dot{h_0}^2)(A^2/S)$.

As solved in the validation of the numerical techique in the preceding discussion the film profile variation is plotted over fastest growing wavelength. As shown in the figure (3.3)



Figure 3.3: Film profile variation for van der Waals attraction and surface tension effects. $(100 - 1000 \text{\AA})$

Time taken for the film to rupture is T = 5.988 at a minimum film thickness h = 0.03253. It is time taken for film to see the point of first dry out.

From the perturbation analysis based on work of Davis[2], the maximum growth rate is found to be $k_m = 1/\sqrt{2}$. Recalling that the theoretical period of the fastest growing interfacial disturbance is $2 \pi/k_m$, we find that $\lambda_m = X$. From the definition of X 3.7, the physical wavelength most likely to appear on a perturbed isothermal thin film is:

$$\dot{\lambda_m} = 2\pi \dot{h_0} \sqrt{\frac{2S}{A}} \tag{3.10}$$

Substituting for A and S, the physical wavelength of the fastest growing disturbance on a thin isothermal liquid film is:

$$\lambda_{thin} = 4\pi^{\frac{3}{2}} \dot{h_0} \left(\frac{\sigma \dot{h_0}^2}{\dot{A}}\right)^{\frac{1}{2}}$$
(3.11)

For a thin isothermal liquid film, the fastest growing wavelength is proportional to the square of the mean film thickness, $\dot{h_0}$.

The term $\sigma \dot{h_0}^2/\dot{A}$ is the ratio of capillary pressure to disjoining pressure. For the case of room temperature ethanol at mean film thickness of $10^{-9}m$ ($\sigma = 0.02N/m$, $\dot{h_0} = 10^{-6}m$, $\dot{A} = 10^{-20}J$, $h_{fg} = 8.8 \cdot 10^5 J/Kg$, $k = 0.0017 J/ms^{\circ}K$, $\nu = 5 \cdot 10^{-7}m^2/s$, $\Delta T = 10^{\circ}K$) the maximum wavelength is calculated as $\lambda_{thin} = 3.15\mu m$

The analysis shows that for sufficiently thin film with surface tension and disjoining pressure forces controlling the dynamics the maximum wavelength of long wave instabilities would be $\lambda'_{thin} = 3.15 \mu m$. The disturbances generated by stabilities with smaller wavelengths then $3.15 \mu m$ would be suppressed and instabilities with higher wavelengths would make the film to rupture.

3.3 Isothermal thick film

The film being adequately thick enhances the gravitational body force acting on the liquid layer, surface tension acts at the evaporating interface but film is thick enough to neglect the magnitude of disjoining pressure effects or van der Waals attraction. The mass loss due to slow evaporation is considered to be negligible. The vapor recoil effects are neglected as they are at lower order then mass loss due to evaporation.

The approach for the isothermal thin film analysis is adapted for the case of isothermal thick $(1 - 100\mu m)$ film. Disjoining pressure is considered negligible compared to gravitational effects $(A \ll G)$ and surface tension forces are considered constant. The evolution equation 2.7 becomes:

$$h_t + (Sh^3h_{xxx})_x + (Gh^3h_x)_x = 0 ag{3.12}$$

Equation 3.12 can be written in canonical form using the variable transformations

$$X = \acute{x}\sqrt{\frac{G}{S}} \tag{3.13}$$

and

$$T = t \left(\frac{G^2}{S}\right) \tag{3.14}$$

The resulting evolution equation is

$$h_T + (h^3 h_{XXX})_X + (h^3 h_X)_X = 0 ag{3.15}$$

The equation (3.15) is solved using validated numerical technique. The film profile variation is plotted over the wavelength using similar initial perturbation as before.

Figure (3.4) shows a continuous thinning of the film, but no dry out is observed without the disjoining pressure term. Note that the gravitational acceleration is vertically upward in (3.4) resulting in a very thin liquid film at approximately $-\pi/2k_m$ and a pendant drop centered at approximately $\pi/2k_m$. The final non-dimensional time shown is beyond T = 9 with time steps of $\Delta T = 0.0001$.



Figure 3.4: Film profiles variations for thick film with surface tension and acting body forces $(1 - 100\mu m)$ Lobe drops formations observed.

Similar to the isothermal thin film, the fastest growing physical wavelength on a thick isothermal liquid film is:

$$\lambda_{thick} = 2\pi \dot{h_0} \left(\frac{2S}{G}\right)^{\frac{1}{2}} \tag{3.16}$$

substituting for G and S, the fastest growing wavelength is:

$$\dot{\lambda_{thick}} = 2\pi \left(\frac{2\sigma}{\rho g}\right)^{\frac{1}{2}} = 2\pi\sqrt{2}L_c \tag{3.17}$$

where $L_c = \sqrt{\frac{\sigma}{\rho g}}$ is the capillary length. For an isothermal thick film, the fastest growing wavelength is approximately nine times the capillary length. From equation (3.17) we find that the wavelength is dependent on fluid properties but not on the mean film thickness, $\dot{h_0}$, opposed to the isothermal thin film case (3.11) where the wavelength is linearly proportional to the mean film thickness.

Som[3], as part of a study on the stability of condensing liquid films in variable gravitational environments, conducted a series of experiments to investigate the effect of liquid film growth on the disturbance wavelength. The experiments consisted of pumping various viscosity silicon oils through a stack inverted perforate disks to simulate condensation on the under side of a plate. The liquid films were isothermal and sufficiently thick so as to neglect disjoining pressure effects. They found that the mean distance between pendant drops was independent of pumping rate. In other words, the disturbance wavelength was constant for all film thickness which is consistent with the result expressed in equation (3.17). For 50cSt Silicon oil ($\sigma = 0.0207N/m, \rho = 963kg/m^3$), the average wavelength of gravitationally-induced pendant drops was $\lambda = 13mm$. Using these physical properties, the capillary length is $L_c = 1.48mm$ and fastest growing wavelength for an isothermal thick film (3.17) is 13.15mm; showing remarkable agreement with the results of Som [3].

The fastest growing wavelength being so large due to surface tension and stabilizing body force the film never ruptures because the wavelengths of perturbations/ disturbances are suppress by the long wavelengths generated by the surface tension and gravitational body forces. Due to positive orientation of gravitational body force lobe drop or pendant drop formation is seen in profile variations.

3.4 Transition Film Thickness

The first two cases of isothermal thin and thick films is showed the possibilities of film rupture or dry out depending on the magnitude of wavelengths of instabilities. It is seen that when there is dependance of wavelength on mean film thickness film rupture is observed as in the case of isothermal thin film dry out occurs after non-dimensional time of 5.988 whereas continuous thinning is seen in the case of isothermal thick film without any dry out. This transition of occurence of dry out and continuous thinning is seen from when film grows from thin to thick film or vice versa.

Up to this point, the distinction between thin and thick liquid films has been arbitrarily set at $1\mu m$. Using the two models, however, the transition between thin and thick liquid films may be better defined. Figure(3.5) shows the wavelength variations over the mean film thickness for isothermal ethanol liquid film at room temperature. The two plots (thin and thick films) intersect at mean film thickness $h_{0_{TR}} \approx 0.73\mu m$ which is termed as the transition film thickness. An exact expression for the transition film thickness may be found by equating (3.11)and (3.17) and solving for the mean film thickness, $\dot{h_0}$,

$$\dot{h_{0_{TR}}} = \left(\frac{\dot{A}}{2\pi\rho g}\right)^{\frac{1}{4}} \tag{3.18}$$

which for room temperature ethanol results in $h_{0_{TR}} = 0.673 \mu m$.

The value $0.673\mu m$ is the transition film thickness for ethanol where disjoining pressure forces dominate the dynamics of the system in isothermal conditions of evaporation. The film could show dry out, thinning or even pendant drop formation based on magnitude of varied stabilizing/destabilizing mechanisms.



Figure 3.5: Variation of fastest growing wavelength, $\dot{\lambda_m}$, versus the mean film thickness, $\dot{h_0}$, for the isothermal thin and thick film analysis for n-pentane at room temperature

3.5 Mass loss and Surface Tension

For the adequately thick film with mass loss and surface tension acting on the vaporliquid interface the assuming negligible disjoining forces and interfacial temperature being constant in case of quasi-equilibrium state. The terms E,S,D are of higher order and interfacial temperature being constant K = 0 at saturation state. The evolution equation (2.7) is simplified considering vapor recoil forces acting on the evaporating interface to be considerably less then mass loss due to evaporation at the interface. The van der Waals attraction of lower order such that it won't affect the dynamics of liquid layer. The equation (2.7) becomes,

$$h_t + Eh^{-1} + S(h^3 h_{xxx})_x + [E^2 D^{-1} h_x]_x = 0$$
(3.19)

Equation (3.19) can be further simplified using the following variable transformations

$$X = \acute{x} \left(\frac{E}{S}\right)^{\frac{1}{4}} \tag{3.20}$$

and

$$T = \acute{t}E \tag{3.21}$$

The resulting evolution equation is

$$h_T + h^{-1} + \left(h^3 h_{XXX}\right)_X + \left(\frac{E^{\frac{3}{2}}}{DS^{\frac{1}{2}}}\right)(h_X)_X = 0$$
(3.22)

Similar to previous analysis, the expression of physical wavelength on quasi-equilibrium



Figure 3.6: Film profile variation for mass loss due to evaporation and surface tension effects. Rupture time $T_r=0.404$

state thick film is

$$\dot{\lambda_{thick}} = 2\pi\sqrt{2}\dot{h_0} \left(\frac{S}{E}\right)^{\frac{1}{4}}$$
(3.23)

Substituting the expression for S and E, expression for fastest growing wavelength is

$$\lambda_{thick} = 2\pi \sqrt{2} \dot{h_0}^{\frac{5}{4}} \left(\frac{\sigma h_{fg}}{3\nu k\Delta T}\right)^{\frac{1}{4}}$$
(3.24)

For the case of room temperature ethanol of $1\mu m$ mean film thickness ($\sigma = 0.02N/m$, $\dot{h_0} = 10^{-6}m$, $\dot{A} = 10^{-20}J$, $h_{fg} = 8.8 \cdot 10^5 J/Kg$, $k = 0.0017 J/ms^{\circ}K$, $\nu = 5 \cdot 10^{-7} m^2/s$, $\Delta T = 10^{\circ}K$) the maximum wavelength is calculated as $\lambda_{thick} = 256.1\mu m$. This is fastest growing wavelength for ethanol at mean film thickness of $1\mu m$ for higher order evaporative mass loss and surface tension effects over liquid surface.

3.6 Surface tension

Existence of surface tension effects with very slow evaporation along the liquid surface is sometimes a typical case observed on thermal control systems in space explorations. The liquid layer is considered to be of sufficient thickness such that van der Waals attractions could be assumed to be of lower order. Due to slow evaporation across the interface the forces acting due to rapid evaporation or vapor recoil effects are very small. Body forces acting on the liquid interface could be considered to be negligible. The interfacial temperature is assumed constant which gives us the case of K = 0 and the evolution equation (2.7 reduces to,

$$h_t + S\left(h^3 h_{xxx}\right)_x = 0 \tag{3.25}$$

Simplifying (3.25) further using variable transformation

$$X = \acute{x} \tag{3.26}$$

and

$$T = tS \tag{3.27}$$

The resulting evolution equation is

$$h_T + \left(h^3 h_{XXX}\right)_X = 0 (3.28)$$

The physical wavelength for quasi-equilibrium surface tension dominant film is

$$\dot{\lambda_{thick}} = 2\pi\sqrt{2}\dot{h_0} \tag{3.29}$$

For the case of room temperature ethanol for thick film over $1\mu m$ ($\sigma = 0.02N/m$, $\dot{h_0} = 10^{-6}m$, $\dot{A} = 10^{-20}J$, $h_{fg} = 8.8 \cdot 10^5 J/Kg$, $k = 0.0017J/ms^{\circ}K$, $\nu = 5 \cdot 10^{-7}m^2/s$, $\Delta T = 10^{\circ}K$, $\rho = 790kg/m^3$) the maximum wavelength is calculated as $\lambda_{thick} = 8.86\mu m$. The fastest growing wavelength $\lambda_{thick} = 8.86\mu m$ is calculated for the case of mean film thickness $1\mu m$ and from the numerical technique the film shows no dry out or rupture. This fastest growing maximum wavelength of evaporating film at $1\mu m$ thickness with surface tension being major stabilizing mechanism over the liquid surface.

3.7 Mass loss, van der Waals attraction, and surface tension

Conditions where the evaporating liquid layer is at a transition film thickness for the case of ethanol, film thickness $0.673\mu m$ using equation (3.18) assuming that the liquid layer has disjoining pressure forces acting as one of instabilities alongwith surface tension with mass loss at the interface with small amount of vapor recoil effects due evaporation at the vapor-liquid interface. But at the same time we neglect the effects of body force acting on the liquid layer as a typical case in space exploration with



Figure 3.7: Film profile variation for surface tension effects as the sole acting mechanism over the liquid surface. No film rupture observed.

thin liquid layer formations. Similar to previous analysis the interfacial temperature is constant at K = 0. This simplifies the evolution equation (2.7) to,

 $h_T + \epsilon h^{-1} +$

$$h_t + Eh^{-1} + \left[\left(Ah^{-1} + E^2 D^{-1} \right) h_x \right]_x + S \left(h^3 h_{xxx} \right)_x = 0$$
(3.30)

Simplifying (3.31) further using variable transformation

$$X = \acute{x}\sqrt{\frac{A}{S}} \tag{3.31}$$

and

$$T = \acute{t}\left(\frac{A^2}{S}\right) \tag{3.32}$$

The resulting evolution equation is

 $[(h^{-1} + D)h_X]_X + (h^3 h_{XXX})_X = 0 (3.33)$

where, $\epsilon = ES/A^2$, $D = E^2 D^{-1}/A$.



Figure 3.8: Film profile variation for mass loss due to evaporation, disjoining pressure forces and surface tension. $T_r=0.989$

The fastest growing physical wavelength for quasi-equilibrium film is

$$\dot{\lambda_{tr}} = 2\pi \dot{h_0} \sqrt{\frac{2S}{A}} \tag{3.34}$$

Similar to the isothermal thin films, substituting for S and A gives

$$\dot{\lambda_{tr}} = 4\pi^{\frac{3}{2}} \dot{h_0} \left(\frac{\sigma \dot{h_0}^2}{\dot{A}}\right)^{\frac{1}{2}}$$
(3.35)

For the case of room temperature ethanol for transition film thickness $0.673\mu m$ $(\sigma = 0.02N/m, h_0 = 10^{-6}m, A = 10^{-20}J, h_{fg} = 8.8 \cdot 10^5 J/Kg, k = 0.0017 J/ms^{\circ}K, \nu = 5 \cdot 10^{-7}m^2/s, \Delta T = 10^{\circ}K, \rho = 790 kg/m^3)$ the fastest growing wavelength is calculated as $\lambda_{tr} = 14.3mm$. Here 14.3mm is the fastest growing wavelength of ethanol film at a transition film thickness of $0.673\mu m$. The $\lambda_{tr} = 14.3mm$ is the maximum fastest growing wavelength generated by ethanol film at transition film thickness of $0.673\mu m$. The $\lambda_{tr} = 14.3mm$ is the maximum fastest growing wavelength generated by ethanol film at transition film thickness of $0.673\mu m$. In the above described situation the film shows first dry out at $T_r = 0.989$ with maximum wavelengths of instabilities at 14.3mm. This conditions could observed for very thin evaporating film formations where disjoining pressure and evaporative mass loss are major destabilizing forces and surface tension being a stabilizing mechanism.

3.8 Mass loss and body forces

Sometimes there are situations when there is mass loss due to evaporation largely but vapor recoil effects due to exiting liquid molecules are considerably low. Surface tension effects across the evaporating interface are negligible and gravitational forces acting on the liquid layer which thick enough to respond to acting body forces. The liquid interface is considered to be at constant temperature at K = 0. The terms Eand G are of higher order and terms D, M, P, K are of lower order.

Resulting evolution equation is,

$$h_t + Eh^{-1} + \left[\left(-Gh^{-1} \right) h_x \right]_x = O$$
(3.36)

Simplifying further using variable transformation,

$$X = \acute{x} \tag{3.37}$$

and

$$T = \acute{t}G \tag{3.38}$$

The resulting evolution equation is

$$h_T + \frac{E}{G}h^{-1} - \left[h^{-1}h_X\right]_X = 0 \tag{3.39}$$

Fastest growing wavelength for evaporating film with evaporation and gravitation forces dominant is

$$\lambda_{thick} = 2\pi\sqrt{2}\dot{h_0} \tag{3.40}$$

For the case of room temperature ethanol for mean film thickness $1\mu m$ ($\sigma = 0.02N/m$, $\dot{h_0} = 10^{-6}m$, $\dot{A} = 10^{-20}J$, $h_{fg} = 8.8 \cdot 10^5 J/Kg$, $k = 0.0017J/ms^{\circ}K$, $\nu = 5 \cdot 10^{-7}m^2/s$, $\Delta T = 10^{\circ}K$, $\rho = 790kg/m^3$) the fastest growing wavelength is calculated as $\lambda'_{thick} = 8.86\mu m$. This fastest growing wavelength is similar to the case for liquid film with surface tension being lone stabilizing mechanism. The fastest growing wavelengths are dependent on surface tension, density, gravity forces and disjoining pressure forces. In the absence surface tension and van der Waals attraction the fastest growing wavelengths are lengths are proportional to mean film thickness. In the present case of ethanol at mean film thickness of $1\mu m$ will give fastest growing wavelength of $8.86\mu m$ at with initial perturbation but will show first dry outs after $T_r = 0.496$. Due to the lack of surface tension over the liquid surface the evaporating liquid layer won't show pendant drop formations seen in the cases where surface tension and body forces are acting mechanisms for inducing or supressing instabilities.

3.9 Higher mass loss, body force and vapor recoil effect

Similar to the preceding discussion if there is a mass loss from the interface due to higher rate of evaporation the vapor recoil effects due to rapid evaporation could not



Figure 3.9: Film profile variation for mass loss due to evaporation and body forces. Rupture time of $T_r=0.493$

be neglected. So in this case along with acting body force additional vapor recoil are considered to be of higher order to be included in the solving the evolution equation (2.7). The terms E, D, G are of higher order and K, M, A, S, P are considered to be of lower order assuming that the interfacial temperature remains steady even for the process of rapid evaporation. Since the interfacial temperature is considered to be constant there are no thermocapillary flows seen across the surface hence assuming them of lower order. The disjoining pressure forces through the liquid layer and surface tension across the surface are assumed to be of lower order. A typical case of higher temperature evaporation where surface tension negligibly affects the dynamics of evaporation and film being sufficiently strong to keep the van der Waals attraction at lower order.

The evolution equation (2.7) reduces to,

$$h_t + Eh^{-1} + \left[\left(-Gh^{-1} + E^2 D^{-1} \right) h_x \right]_x = 0$$
(3.41)

Simplifying further using variable transformation

$$X = \acute{x}\sqrt{\frac{E}{G}} \tag{3.42}$$

and

$$T = \acute{t}E \tag{3.43}$$

The resulting evolution equation is

$$h_T + h^{-1} - \left[h^{-1}h_X\right]_X + \frac{E^2 D^{-1}}{G} \left[h_X\right]_X = 0$$
(3.44)



Figure 3.10: Film profile variation for higher mass loss, body forces and vapor recoil effects due to rapid evaporation. Rupture time, $T_r=0.423$

The fastest growing wavelength for quasi-equilibrium film with evaporation, gravitation and vapor recoil forces dominant is

$$\lambda_{thick} = 2\pi \sqrt{\frac{2G}{E}} \tag{3.45}$$

Substituting for G and E gives

$$\lambda_{thick} = 2\pi \sqrt{\frac{2\rho g h_{fg}}{3\nu k \Delta T}} \dot{h_0}^{\frac{5}{2}}$$
(3.46)

For the case of room temperature ethanol for mean film thickness $1\mu m$ ($\sigma = 0.02N/m$, $\dot{h_0} = 10^{-6}m$, $\dot{A} = 10^{-20}J$, $h_{fg} = 8.8 \cdot 10^5 J/Kg$, $k = 0.0017 J/ms^{\circ}K$, $\nu = 5 \cdot 10^{-7} m^2/s$, $\Delta T = 10^{\circ}K$, $\rho = 790 kg/m^3$) the fastest growing wavelength is calculated as $\lambda_{thick} = 4.6\mu m$. The wavelength $\dot{\lambda}_{thick} = 4.6\mu m$ is observed for ethanol the mean film thickness of $1\mu m$ when there is considerable mass loss due to evaporation along with vapor recoil effect due to exiting vapor molecules from liquid interface. In order for gravitational

forces to affect the film dynamics, it should be sufficiently thick. The evolution equation above described conditions will show a first dry out after non-dimensional rupture time $T_r=0.423$. This means it takes non-dimensional time of 0.423 with fastest growing wavelength of $\lambda_{thick} = 4.6 \mu m$ to exhibit film rupture.

3.10 Film profile variation for whole evolution equation.

The previous all cases present film profile variations and fastest growing wavelengths for the ethanol at different film thickness from thin, transitional to thick where applicable. The different instability inducing mechanisms were neglected or included based on sole assumption or even situations close to real. The evolution equation (2.7) as presented in chapter 2 is an all inclusive instability acting mechanisms equation.

In a real space exploration system generally combination effects of all these mechanisms are observed. Mass loss due to slow evaporation could have unlikely strong effects of vapor recoil forces acting on the evaporating interface rendering the evaporating liquid layer unstable. On the same lines due this lower mass loss and higher vapor recoil effects the evaporating liquid layer could get so thin that disjoining pressure forces could get enhanced in turn showing of the glimpses of film rupture or a complete dry out. In previous all cases the interfacial degree of equilibrium was assumed to be constant making K = 0. But in actuality the interfacial temperature is variable. Due to the variable interfacial temperature there are thermocapillary flows across the liquid surface termed as Marangoni effects. Applying same process as in all previous cases the equation (2.7) simplifies using variable transformations

$$X = \acute{x}\sqrt{\frac{A}{S}} \tag{3.47}$$

and

$$T = \acute{t}\frac{A^2}{S} \tag{3.48}$$

The resulting evolution equation is

$$h_T + \frac{ES}{A^2} (h+K)^{-1} + \left(h^3 h_{XXX}\right)_X + \left(h^{-1} h_X\right)_X + \frac{E^2 D^{-1}}{A} \left(\frac{h^3}{(h+K)^3} h_X\right)_X + \frac{KMP^{-1}}{A} \left[\frac{h^2}{(h+K)^2} h_X\right]_X = 0$$
(3.49)

Using finite difference technique on 3.49 gives film profile variation with rupture time T_r of 0.444. [htbp]

The fastest growing wavelength for this combination effects is,

$$\lambda_{film} = 2\pi \sqrt{\frac{2S}{A}} \tag{3.50}$$

Substituting for S and A gives

$$\lambda_{film} = 4\pi^{\frac{3}{2}} \sqrt{\frac{\sigma \dot{h_0}^2}{\dot{A}}} \tag{3.51}$$



Figure 3.11: A typical case of film profile variation based on evolution equation.Rupture time of $T_r=0.444$

For the case of ethanol at mean film thickness of $1\mu m$ ($\sigma = 0.02N/m$, $\dot{h_0} = 10^{-6}m$, $\dot{A} = 10^{-20}J$, $h_{fg} = 8.8 \cdot 10^5 J/Kg$, $k = 0.0017 J/ms^{\circ}K$, $\nu = 5 \cdot 10^{-7} m^2/s$, $\Delta T = 10^{\circ}K$, $\rho = 790 kg/m^3$) the fastest growing wavelength is calculated as $\lambda_{film} = 31.5\mu m$. This is maximum wavelength of a typical film behavior with different stabilizing/destabilizing mechanisms controlling the dynamics of evaporating film. The film with a degree of non-equilibrium across the interface and marangoni effects over the surface shows a film dry out after non-dimensional time $T_r = 0.444$ which maximum wavelength $\lambda_{film} = 31.5\mu m$. The rupture time could vary based on variation in degree of non-equilibrium across the interface, amount of mass loss and magnitudes of vapor recoils, thermocapillary flows.

3.11 The whole evolution equation with gravitational body force

Previous discussion on evolution equation considered that there is no possibility of gravitational forces acting on the liquid layer. But there are situations when there are gravity effects in small or considerable amounts that could act as stabilizing/de-stabilizing mechanism based on the orientation of the evaporating surface. The evolution equation (2.7) is simplified by retaining the gravitational body force term inside

the whole equation. The resulting simplified evolution equation is,

$$h_{T} + \frac{ES}{A^{2}} \left(h + K\right)^{-1} + \left(h^{3} h_{XXX}\right)_{X} + \left(h^{-1} h_{X}\right)_{X} - \frac{G}{A} \left[h^{3} h_{X}\right]_{X} + \frac{E^{2} D^{-1}}{A} \left(\frac{h^{3}}{(h + K)^{3}} h_{X}\right)_{X} + \frac{KMP^{-1}}{A} \left[\frac{h^{2}}{(h + K)^{3}} h_{X}\right]_{X} + \frac{KMP^{-1}}{(h + K)^{3}} h_{X} + \frac{KMP$$

Similar to previous analysis applying finite difference technique on 3.52 gives a variation film profile with rupture time $T_r = 1.257$ [htbp]



Figure 3.12: Film profile variation for evolution equation with gravitational forces. $T_r=1.257$

The expression for fastest growing wavelength is similar to the previous case as mentioned before the fastest growing wavelengths are dependent on surface tension, density and van der waals attractions and mean film thickness. Though the mass loss due to evaporation and vapor recoil effects, thermocapillary flows degree of nonequilibrium affect the dynamics of the liquid layer. The expression for fastest growing wavelength is,

$$\lambda_{film_g} = 4\pi^{\frac{3}{2}} \sqrt{\frac{\sigma \dot{h_0}^2}{\dot{A}}} \tag{3.53}$$

For the case of ethanol at mean film thickness of $1\mu m$ ($\sigma = 0.02N/m$, $\dot{h_0} = 10^{-6}m$, $\dot{A} = 10^{-20}J$, $h_{fg} = 8.8 \cdot 10^5 J/Kg$, $k = 0.0017 J/ms^{\circ}K$, $\nu = 5 \cdot 10^{-7} m^2/s$, $\Delta T = 10^{\circ}K$, $\rho = 790 kg/m^3$) the fastest growing wavelength is calculated as $\lambda_{filmg} = 31.5\mu m$. For the same mean film thickness the maximum wavelength of the instability is $31.5\mu m$ due to additional actions of gravitational body forces and some comparable

3.11. THE WHOLE EVOLUTION EQUATION WITH GRAVITATIONAL BODY FORCE27

changes in evaporative mass loss and vapor recoil effects the dry out or rupture time increases/decreases based on magnitudes of non-dimensional terms in the evolution equation. For the current condition the first dry out is seen at $T_r = 1.094$ which can vary.

Chapter 4 An Experimental Approach

The findings of the numerical model presented in earlier chapter is aimed to be implemented in the design of the experimental test set up that goes on NASA's Reduced Gravity Aircraft. A serious understanding of different control parameters is needed before a real application test set up is constructed. In the successive sections of this chapter, the analysis about different system components is presented. The experimental approach initiated by assembling the test set up together. Its elaborate description is presented in the next sections. Followed by calibration process of the power supplies for different components, pressure sensors and guages. The data acquisition system is tested for smooth functioning before it starts real data processing.

4.1 The test set up

The exploded view of the experimental apparatus is shown in the figure (4.1). The main components of the systems contains a custom made Ace glass test chamber, thin condensation copper block and a thick evaporation copper block. There are eight thermoelectric coolers four on each side of the copper blocks. Aluminum heat sinks are required for heat rejection by the thermoelectric coolers.

The experimental test set up in figure (4.22) consists of a glass chamber, two copper blocks. The lower thick copper block is suppose to be an evaporating surface whereas top thin copper block is a condensing surface. There are two aluminum heat sinks one each on side of the copper block. There are two 12 Vdc fans one each on the outer surface of the aluminum heat sink. The evaporating copper block is 130 x 130 x 12.5 mm and condensing surface is 130 x 130 x 5 mm. There are in total eight DT series Marlow thermoelectric coolers. Four each on each side of the evaporating or condening surface.

The figure (4.22 is the)

The assembled test set up is shown in the figure (4.22)

A plan view of the whole test up is shown in figure (4.3). The whole test up with the experimental glass chamber consists of the DC power supply by Instek for the thermoelectric coolers, the DC voltage excitation power supply by Kepco.Power supplies for fan to induce forced convection for smooth heat dissipation from the



Figure 4.1: Components of the experimental apparatus.

copper block into the atmosphere. Vacuum pressure gauge for indication of pulled the amount of vacuum inside the glass chamber. The data of the this experimental set up is recorded of Keithley data acquisition module interfaced with computer. In addition to all these diagnostics there is a miniature adsolute pressure measurement sensor by Kulite.

4.2 Pressure transducer

The pressure tranducer by Kulite is a miniature pressure sensor of a size of standard pencil lead. This pressure sensor measures absolute pressure from 5 to 15 Psi by specification although 0-15 Psi absolute pressure is proportionally measured without loosing the calibration. Since the pressure transducer is of a miniature size. It is delicately used and installed carefully via compression fitting. The pictorial view is shown through figure(4.23) Before using it for real application the sensor is first tested for accuracy and repeatability. A calibration curve is calculated using the calibration provided by Kulite quality testing. The specific calibration factor of XCS-062 is 12.792 mV/Psi. The actual air pressure variation were measured without any evaporating media inside it. The calibration curve for pressure sensor for the application is shown is figure(4.5).

4.3 Vacuum Leak test

The liquid media under test has to be tested without any impurities or non-condensibles inside the system. This is possible by making the glass chamber leak free by pulling vacuum of about 28 in.of Hg(0.5 Psi air pressure) inside the system. The vacuum is


Figure 4.2: The test set up



Figure 4.3: Elaborate view of experimental test set up



Figure 4.4: Pressure transducer installation



Figure 4.5: Calibration curve for pressure transducer

pulled using a roughing vacuum pump through a compression fitting going into the glass chamber as shown in the figure(4.6). The joints and fittings are made leak free by application RTV glue and vacuum grease. The vacuum is tested for over 24 hours for possible increase in pressure. There is leak of 1 in of Hg over single day which is permissible for initial testing. The figure(4.7)shows the pressure gauge used to measure the vacuum inside the glass chamber. The pressure gauge is not calibrated so its is tested for accurate measurements using pressure sensor. The calibration curve is plotted for pressure gauge for increase in the vacuum pressure seen in the figure(4.8).

4.4 Thermocouple installation

K-type thermocouples are used in the measuring temperature data out of the different surfaces of the test set up. On the surface of the evaporating or condensing copper block the thermocouples are epoxy glued where as on the surface exposed to ambient or forced convection the thermocuples are sticked to the surface using Kapton tapes that withstand unto $200^{\circ}C$. A sample thermocouple attachment to the surface is shown in the figure(4.9). The thermocouples are tested for improper or open junction by checked the ambient temperature using thermometer against thermocouple measurement.



Figure 4.6: Compression fitting for air suction from the glass chamber



Figure 4.7: Pressure gauge for leak free vacuum chamber



Figure 4.8: Calibration curve for pressure gauge



Figure 4.9: Epoxy glued thermocouple on surface of thick copper block

4.5 Thermoelectric Coolers

The thermoelectric coolers are solid state heat pumps are used in application where temperature stabilization, temperature cycling, or cooling below ambient is required. Real application or final test design needs good deal of control over temperature stabilization and cycling of temperature. The main heating or cooling element are the evaporating/condensing copper surfaces.

A top and front views of thermoelectric coolers on thin condensing copper block are shown in the figures (4.10) and (4.11). In order to get smooth contact between



Figure 4.10: Top view of thermoelectric coolers

flat surface of thermoelectric cooler and the copper surface the copper surface is mirrored surfaces and then coolers are attached by applying vacuum grease on the both thermoelectric cooler surface and copper surface. Vacuum grease is also used to avoid air pockets in between the connecting surfaces of thermoelectric coolers and copper block. A typical thermal grease used in all our application is shown in the figure(4.12). The figure (4.13) shows the thermoelectric coolers installed in between the thin copper block and the aluminum heat sink. The heat sink is cooled using a 12 Vdc cooling fan.



Figure 4.11: Front view of thermoelectric coolers



Figure 4.12: vacuum grease used



Figure 4.13: Thermoelectric coolers installed in between copper block and heat sink

4.6 Power supplies

The thermoelectric coolers because of its high sensitivity to the changes in the current needs a stable DC power supply. For all the tests carried out an Instek GPS four channelled GPS-4303 was used. It is shown in figure (4.14).

There are four channels on the power supply. Channel 1 and channel 2 while working independently can deliver maximum voltage of 30V which initially solves our purpose. The channel 1 and channel 2 are tested for smooth functioning and calibration. The channels are verified for displayed voltage and actual current given out by the power supply. The calibration plots for two channels are presented in figure(4.15) and (4.16).

The DC voltage excitation is also required for the inducing input signals to the pressure sensor measuring pressure variations inside the chamber. Also, the sensor being very delicate and miniature it is fragile towards power fluctuations. Kepco power supply is used for providing excitation to the sensor as the amount voltage applied could be kept constant. So it was good idea to check for calibration and fluctuations through the Kepco power supply. The calibration plot for the same is presented in figure (4.18) and Kepco power supply is shown in figure(4.17).

There are two 12 Vdc fan supplies that provide forced convection to the heating sink. The fans go at the top of the heat sinks. The figure(4.19) shows the DC power supply for heat sink fans. These are integral part of the application as heat sink could get really hot burning out the thermoelectric coolers.

4.7. DATA ACQUISITION



Figure 4.14: DC power supply for Thermoelectric coolers

4.7 Data Acquisition

In order to analyse any physical phenomenon it important to observe and record all the parameters that take part in it accurately and wholistically. The Keithley 2700 module of data acquisition for the current investigation was satisfactory. It has 22 channels with 2 channels facilatating DC current measurements through the devices. This applicaton uses 14 channels for recording different data. There 9 thermocouples,2 voltage and 2 current measurement channels. The figure (4.20) shows a Keithley data acquisition system. The figure (4.21)shows the circuit board of keithley 7700 module that goes inside the Keithley 2700. The figure shows the total channels and the plan of the module.

4.8 Test of Thermoelectric coolers

The thermoelectric coolers are tested for its repeatability criteria. In total there are eight thermoelectric coolers used. Each of the thermoelectric coolers were tested for current versus voltage in conjunction with hot and cold side temperature characteristic curves. In other words, to verify that same current flows through all the thermoelectric coolers if specific voltage is applied. This ensures that each thermoelectric cooler performs in comparable fashion. The characteristics of thermoelectric coolers are comparable. The four thermoelectric coolers were coupled in series for



Figure 4.15: Calibration for channel1 providing power to thin copper block thermoelectric coolers

each copper plate. The power supplies INSTEK GPS-4303 and KEPCO DPS-40-2M were utilized as DC power source for this arrangement. The four thermoelectric coolers in series need approximately 22V and 1.8A to produce hot side temperature of $80^{\circ}C$.

4.8.1 Temperature variation across the copper blocks

In order to heat up the copper blocks four thermoelectric coolers each are used on side of each copper block. Four thermoelectric coolers are placed uniformly so that there is uniform heat conduction across and over the copper block. The uniform temperature variation was verified by constructing a two-dimensional model of the copper block and discretizing it into even number of elements as shown in the figure (4.22) and (4.23).

The temperature distribution across the plate is calculated using finite difference techniques. Centered difference is applied on the internal nodes, backward and forward difference is applied to the nodes on the edge. There were two temperature points assumed on the top surface of the copper block similar to two thermoelectric coolers equally spaced. Since, the copper has linear conductivity, the temperature variation over the surface is assumed to be linear. The finite difference techniques gives the temperature variation as seen in the figure (4.23) by solving simultaneous equations.



Figure 4.16: Calibration for channel2 providing power to thick copper block thermoelectric coolers

In real test run there is temperature difference seen in between thin and thick copper evaporating/condensing surface. As presented in the figure (4.24) there is temperature difference of $1^{\circ}C$ seen in between thermoelectric cooler heated thick copper surface and evaporating thick surface. Also about $0.5^{\circ}C$ temperature difference is in between 3 thermal points. The possible reasons for the difference in temperature might be the thermocouple locations for surface thermocouples is at the edge which could get could or heated up due to external natural convection. For the temperature difference in between 2 bottom thermocouples could be because of air pocket formation in between the kapton tape and the copper surface. A sample reading of temperature difference in between 3 thermocouples on each side of copper block is shown in figure (4.24)

The surface temperature of the copper blocks are important inorder to condense/evaporate ethanol. So it is important to know approximate current to temperature variation relationship so that according to requirements at the surface the current through the power supply could be changed. The figures (4.25)and(4.26) shown the copper surface temperature variations over the changes in the DC power supply.



Figure 4.17: constant source DC power supply excitation for the pressure transducer



Figure 4.18: Calibration for DC power supply to pressure transducer



Figure 4.19: DC power supply for the heat sink fans

4.9 Test of Resistive wire heaters

To maintain the glass chamber temperature above the saturation temperature of ethanol, it is heated using resistive rope wire heater. The resistance across the wire is 33.3Ω and 22V power supply is used to pass current through it. The wire is tested for its resistance connecting across the multimeter and checking for its resistance that is 33.3Ω . This resistance suffices the purpose of heating up the glass chamber and keeping it at constant temperature above saturation conditions of ethanol.

4.10 Temperature stabilization Thermal time scale

4.11 Test of Fan power supply

There are two fans of 12 VDC used to provide airflow across the thermoelectric cooler heat sinks used over the cold side of the thermoelectric coolers. The test is conducted to check if the power supplies provide the fans with the rated voltage.



Figure 4.20: Keithley data acquisition

4.12 Test of Thermocouples

The K-type thermocouples are used to measure temperatures at different points on the set up. The thermocouples are tested to verify that the temperature is acquired from the thermocouples and that they are calibrated correctly. There are in total eleven thermocouples used in the whole set up. Three each on two copper blocks. One each on the interior on the heat sinks. Three on glass chamber.

4.13 Pressure Transducer Test

The high sensitivity ultraminiature Kulite XCS-062 series pressure transducer is used to measure pressure inside the glass chamber. The transducer is tested for accurate and precise readings based on the pressure gauge which reads the vacuum created inside the glass chamber.



Figure 4.21: Keithley 7700 data acquisition module

4.14 System behavior

After all the pre-test criteria were fulfilled the ethanol was injected into the system in parts of 60ml and 30ml. The system was degassed to remove the air impurities and non-condensibles from ethanol and the system. The vacuum pulled on the system with ethanol liquid in it. The ethanol initially continues to boil which indicates that there are non-condensible and air pockets through the system. It took 4 hours approximately before the firt data were collected that system was totally stable with no boiling with and without vacuum applied.

Two set of data were collected to investigate the evaporating film behavior inside the degassed system. There is a difference between two data set. The first data set shown in figure(4.27) shows that there is variation in systems parameters based on changes of evaporating film copper block temperature. From the start of the evaporation to the point where system show signs of equilibrium the pressure variation shows dependence on the thick copper block surface temperature. Where as in the second set of data the system is throughout kept at the saturation temperature of ethanol at $28.3^{\circ}C$ and only the thin copper block temperature is decreased in order



Figure 4.22: Two dimensional view of thin copper block with two thermoelectric coolers



Figure 4.23: Two dimensional discretized copper plate with temperature variations



Figure 4.24: Temperature difference across the thin and thick copper block



Figure 4.25: Current versus thin copper block surface temperature variation



Figure 4.26: Current versus thin copper block surface temperature variation

of $1^{\circ}C$ or less. The system shows distinct dependability in cold side condensing plate surface temperature. The pressure shows variations on lower side unlike that seen in the first data set.

Along with recording the pressure, temperature and changes in current and voltage, there was additional counter used to record different event occuring inside the system for set of first data. The events such as first droplets formation on the condensing surface to the dry spot initiation. Number of drops falling from condensing to evaporating surface and corresponding changes in the temperature and pressure. In the events of falling drops there is a distinct drop in the system pressure observed. This kind of phenomenon recurred with each falling drop and was distinctly measured by the pressure sensor at fast sampling rate of 1.67msec and a filter of 5 counts average. Reason being that noise was not of measure interest in the present physics. The two data sets are present through figure (4.27) and (4.29). In between sample time periods of first and second data are shown through (4.28) and (4.30).

4.15 Bouyancy effects inside in the chamber

- 4.16 Constant current and constant voltage observations
- 4.16.1 Constant current
- 4.16.2 Constant pressure



Figure 4.27: System behavior when evaporating plate temperature is varied



Figure 4.28: Specific time duration from first data set for system behavior



Figure 4.29: System behavior when evaporating plate temperature is varied



Figure 4.30: Specific time duration from second data for system behavior



Figure 4.31: Periodic pressure fluctuation seen inside the glass chamber due to temperature difference between evaporating and condensing block



Figure 4.32: Periodic pressure fluctuation seen inside the glass chamber due to temperature difference between evaporating and condensing block at 10 in. of Hg vacuum



Figure 4.33: Periodic pressure fluctuation seen inside the glass chamber due to temperature difference between evaporating and condensing block on release of vacuum



Figure 4.34: The system response to constant current flow through the Thermoelectric coolers



Figure 4.35: The system response to constant current flow through the Thermoelectric coolers



Figure 4.36: The system response to constant voltage across the Thermoelectric coolers



Figure 4.37: The system response to constant voltage across the Thermoelectric coolers

References

- Bankoff S.G. Burelbach, J.P. and S.H. Davis. Nonlinear stability of evaporating/condensing liquid films. *J.Fluid Mechanics*, 195:463–494, may 1988.
- [2] Bankoff S. G. Davis S. H. Burelbach, J. P. Steady thermo capillary flows of thin liquid layers. ii. experiment. *Phys. Fluids*, 1990.
- [3] S. Som. Stability and heat transfer characteristics of an unsteady condensing film. Master's thesis, University of Washington, 2004.

Appendix A

Case 1: Van der Waals attraction and Surface tension forces

```
% THE FOLLOWING PROGRAM IS WRITTEN TO ANALYSE THE FILM PROFILE AT DIFFERENT TIMES
 OVER
% THE NONDIMENSIONAL LENGTH 'X'OVER THE INTERVAL: -Pi/Km TO Pi/Km
% WE APPLY CENTERED DIFFERENCE IN SPACE AND MID-POINT (CRANK-NICHOLSON)
% RULE IS USED IN TIME
clear all
clc
          % REPRESENTS AMPLITUDE
Am=0.1;
km=2^(-0.5); % MAXIMIZING WAVE NUMBER OF WAVE THEORY Km
lambda=2*pi/km; % WAVELENGTH
n=29; % NUMBER OF ELEMENTS TO DIVIDE WAVELENGTH IN EQUAL ELEMENTS N
dx=lambda/n; % ELEMENTS OF WIDTH
x=-pi/km; % START OF INTERVAL
tic
for i=1:n
                                     %FOR INITIAL CONDITION AT ITERATION ONE FOR
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
h(i,1)=1+(Am*sin(km*xi(i)));
temp(i,1)=h(i,1);
                                 % VARIABLE TO STORE THE VALUE OF h(X,T)OF EACH
VALUE OF i & j
end
                                     % END OF THE FIRST ITERATION FOR INITIAL
end
CONDITION
for j=2:6000
for i=1:n
xi(i)=x+(i-1)*dx;
                               % INCREMENT IN VALUE OF X
if(xi(i)<=(pi/km))</pre>
% THIS IS AN INITIAL VALUE PROBLEM SO WHEN i=1,2,28,29 AT i+1,i+2,i-1,i-2 WE DON'T
HAVE VALUES OF h.
```

```
if (i==1)
a=-(dx^-2)*((temp(i,j-1))^-1)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(28,j-1));
b=0.25*(dx^-2)*((temp(i,j-1))^-2)*((temp(i+1,j-1)-temp(28,j-1))^2);
c=-(dx^-4)*((temp(i,j-1))^3)*(temp(i+2,j-1)-4*temp(i+1,j-1)+6*temp(i,j-1)-
4*temp(28, j-1)+temp(27, j-1));
d=-0.75*(dx^-4)*((temp(i,j-1))^2)*(temp(i+2,j-1)-2*temp(i+1,j-1)+2*temp(28,j-1)
-temp(27, j-1))*(temp(i+1, j-1)-temp(28, j-1));
if(temp(i,j-1)<=0.65)
dt=0.001;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d)); % USE OF PREVIOUS VALUE OF h TO CALCULATE THE
NEXT VALUE OF h
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0002;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==2)
a=-(dx^-2)*((temp(i,j-1))^-1)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(1,j-1));
b=0.25*(dx^-2)*((temp(i,j-1))^-2)*((temp(i+1,j-1)-temp(1,j-1))^2);
c=-(dx^-4)*((temp(i,j-1))^3)*(temp(i+2,j-1)-4*temp(i+1,j-1)+6*temp(i,j-1)-
4*temp(1,j-1)+temp(28,j-1));
```
```
d=-0.75*(dx<sup>-4</sup>)*((temp(i,j-1))<sup>2</sup>)*(temp(i+2,j-1)-2*temp(i+1,j-1)+2*temp(1,j-1)
-temp(28, j-1))*(temp(i+1, j-1)-temp(1, j-1));
if(temp(i,j-1)<=0.65)
dt=0.001;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0002;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==28)
a=-(dx^-2)*((temp(i,j-1))^-1)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
b=0.25*(dx^-2)*((temp(i,j-1))^-2)*((temp(i+1,j-1)-temp(i-1,j-1))^2);
c=-(dx^-4)*((temp(i,j-1))^3)*(temp(2,j-1)-4*temp(i+1,j-1)+6*temp(i,j-1)-
4*temp(i-1,j-1)+temp(i-2,j-1));
d=-0.75*(dx^-4)*((temp(i,j-1))^2)*(temp(2,j-1)-2*temp(i+1,j-1)+2*temp(i-1,j-1))
-temp(i-2,j-1))*(temp(i+1,j-1)-temp(i-1,j-1));
if(temp(i,j-1)<=0.65)
dt=0.001;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
```

```
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0002;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if(i==29)
a=-(dx^-2)*((temp(i,j-1))^-1)*(temp(2,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
b=0.25*(dx^-2)*((temp(i,j-1))^-2)*((temp(2,j-1)-temp(i-1,j-1))^2);
c=-(dx<sup>-4</sup>)*((temp(i,j-1))<sup>3</sup>)*(temp(3,j-1)-4*temp(2,j-1)+6*temp(i,j-1)-
4*temp(i-1,j-1)+temp(i-2,j-1));
d=-0.75*(dx^-4)*((temp(i,j-1))^2)*(temp(3,j-1)-2*temp(2,j-1)+2*temp(i-1,j-1))
-temp(i-2,j-1))*(temp(2,j-1)-temp(i-1,j-1));
if(temp(i,j-1)<0.65)
dt=0.001;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0002;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
```

```
else if (temp(i, j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
 break
 end
temp(i,j)=h(i,j);
end
end
end
else a=-(dx^-2)*((temp(i,j-1))^-1)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
b=0.25*(dx^-2)*((temp(i,j-1))^-2)*((temp(i+1,j-1)-temp(i-1,j-1))^2);
c=-(dx^-4)*((temp(i,j-1))^3)*(temp(i+2,j-1)-4*temp(i+1,j-1)+6*temp(i,j-1)-
4*temp(i-1,j-1)+temp(i-2,j-1));
d=-0.75*(dx^-4)*((temp(i,j-1))^2)*(temp(i+2,j-1)-2*temp(i+1,j-1)+2*temp(i-1,j-1))
-temp(i-2,j-1))*(temp(i+1,j-1)-temp(i-1,j-1));
if(temp(i,j-1)<=0.65)
dt=0.001;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.3)
dt=0.0002;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.1)</pre>
dt=0.0001;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)+(dt*(a+b+c+d));
```

```
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
end
end
end
\operatorname{end}
end
end
end
plot(xi,temp)
realtime=toc;
```

Appendix B

Case 2: Gravitational attraction and Surface tension forces

```
clear all
clc
Am=0.1;% AMPLITUDE
km=2^(-0.5);%MAXIMIZING WAVE NUMBER OF WAVE THEORY Km
lambda=2*pi/km;%WAVELENGTH
n=29; % NUMBER OF ELEMENTS TO DIVIDE WAVELENGTH IN EQUAL ELEMENTS N
dx=lambda/n;%ELEMENTS OF WIDTH
x=-pi/km;%START OF INTERVAL
tic
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
h(i,1)=1+(Am*sin(km*xi(i)));
temp(i,1)=h(i,1);
end
end
for j=2:150
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
if (i==1)
a=((temp(i,j-1))^3)*(((dx^-4)*(temp(i+2,j-1)-4*temp(i+1,j-1)+6*temp(i,j-1))
-4*temp(28,j-1)+temp(27,j-1)))-((dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1))
+temp(28,j-1)));
b=0.75*((temp(i,j-1))^2)*(((dx^-4)*(temp(i+2,j-1)-2*temp(i+1,j-1)+
2*temp(28,j-1)-temp(27,j-1))*(temp(i+1,j-1)-temp(28,j-1)))-(dx^-2)*
((temp(i+1,j-1)-temp(28,j-1))^2));
if(temp(i,j-1)<=0.65)
dt=0.00015;
```

```
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.3)
dt=0.0002;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.00001;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==2)
a=((temp(i,j-1))^3)*(((dx^-4)*(temp(i+2,j-1)-4*temp(i+1,j-1)+6*temp(i,j-1))
-4*temp(i-1,j-1)+temp(28,j-1)))-((dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)
+temp(i-1,j-1))));
b=0.75*((temp(i,j-1))^2)*(((dx^-4)*(temp(i+2,j-1)-2*temp(i+1,j-1)+2*temp(i-1,j-1)-
temp(28, j-1))*(temp(i+1, j-1)-temp(i-1, j-1)))-(dx<sup>-2</sup>)*((temp(i+1, j-1)))
-temp(i-1,j-1))^2));
if(temp(i,j-1)<=0.65)
dt=0.00015;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
```

```
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.3)</pre>
dt=0.0002;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.1)
dt=0.00001;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==28)
a=((temp(i,j-1))^3)*(((dx^-4)*(temp(2,j-1)-4*temp(i+1,j-1)+6*temp(i,j-1))
-4*temp(i-1,j-1)+temp(i-2,j-1)))-((dx<sup>-2</sup>)*(temp(i+1,j-1)-2*temp(i,j-1))
+temp(i-1,j-1))));
b=0.75*((temp(i,j-1))^2)*(((dx^-4)*(temp(2,j-1)-2*temp(i+1,j-1)+2*temp(i-1,j-1)-
temp(i-2,j-1))*(temp(i+1,j-1)-temp(i-1,j-1)))-(dx^-2)*((temp(i+1,j-1)))
-temp(i-1,j-1))^2));
if(temp(i,j-1)<=0.65)
dt=0.00015;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0002;
```

```
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.00001;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
 break
  end
temp(i,j)=h(i,j);
end
end
end
else if(i==29)
a=((temp(i,j-1))^3)*(((dx^-4)*(temp(3,j-1)-4*temp(2,j-1)+6*temp(i,j-1))
-4*temp(i-1,j-1)+temp(i-2,j-1)))-((dx^-2)*(temp(2,j-1)-2*temp(i,j-1)))
+temp(i-1,j-1)));
b=0.75*((temp(i,j-1))^2)*(((dx^-4)*(temp(3,j-1)-2*temp(2,j-1)+2*temp(i-1,j-1)-
temp(i-2,j-1))*(temp(2,j-1)-temp(i-1,j-1)))-(dx^-2)*((temp(2,j-1)
-temp(i-1,j-1))^2));
if(temp(i,j-1)<0.65)
dt=0.00015;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0002;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
```

```
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.1)</pre>
dt=0.00001;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
     a=((temp(i,j-1))^3)*(((dx^-4)*(temp(i+2,j-1)-4*temp(i+1,j-1)+6*temp(i,j-1))
else
-4*temp(i-1,j-1)+temp(i-2,j-1)))-((dx<sup>2</sup>)*(temp(i+1,j-1)-2*temp(i,j-1))
+temp(i-1,j-1))));
b=0.75*((temp(i,j-1))^2)*(((dx^-4)*(temp(i+2,j-1)-2*temp(i+1,j-1)+2*temp(i-1,j-1)-
temp(i-2,j-1))*(temp(i+1,j-1)-temp(i-1,j-1)))-(dx^-2)*((temp(i+1,j-1)))
-temp(i-1,j-1))^2));
if(temp(i,j-1)<=0.65)
dt=0.00015;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.3)
dt=0.0002;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.1)
dt=0.00001;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
```

```
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)+(dt*(-a-b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
plot(xi,temp)
realtime=toc;
```

Appendix C

Case 3: Evaporation and Surface tension forces

```
clear all
clc
Am=0.1;
km=2^{(-0.5)};
lambda=2*pi/km;
n=29;
dx=lambda/n;
x=-pi/km;tic
S = 0.1; D = 0.01;
E = 0.01;
Esd = (E^{1.5}) * (D^{-1}) * (S^{-0.5});
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
h(i,1)=1+(Am*sin(km*xi(i)));
temp(i,1)=h(i,1);
end
end
for j=2:488
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
if (i==1)
a= (temp(i,j-1))^-1;
b=((temp(i,j-1))^3)*(dx^-4)*(temp(i+2,j-1)-4*temp(i+1,j-1)+
6*temp(i,j-1)-4*temp(28,j-1)+temp(27,j-1));
c= (0.5*(dx^-3))*(temp(i+2,j-1)-2*temp(i+1,j-1)+2*temp(28,j-1)-temp(27,j-1))
*(3*(temp(i,j-1))^2)*(0.5*(dx^-1))*(temp(i+1,j-1)-temp(28,j-1));
d= Esd*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(28,j-1));
if(temp(i,j-1)<=0.65)
```

```
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.1)</pre>
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==2)
a= ((temp(i,j-1))^-1);
b=((temp(i,j-1))^3)*(dx^-4)*(temp(i+2,j-1)-4*temp(i+1,j-1)+
6*temp(i,j-1)-4*temp(1,j-1)+temp(28,j-1));
c= (0.5*(dx^-3))*(temp(i+2,j-1)-2*temp(i+1,j-1)+2*temp(1,j-1)-temp(28,j-1))
*(3*(temp(i,j-1))^2)*(0.5*(dx^-1))*(temp(i+1,j-1)-temp(1,j-1));
d= Esd*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(1,j-1));
if(temp(i,j-1)<=0.65)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
```

```
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.3)</pre>
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==28)
a= ((temp(i,j-1))^-1);
b=((temp(i,j-1))^3)*(dx^-4)*(temp(2,j-1)-4*temp(i+1,j-1)+
6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1));
c= (0.5*(dx^-3))*(temp(2,j-1)-2*temp(i+1,j-1)+2*temp(i-1,j-1)-temp(i-2,j-1))
*(3*(temp(i,j-1))^2)*(0.5*(dx^-1))*(temp(i+1,j-1)-temp(i-1,j-1));
d= Esd*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
if(temp(i,j-1)<=0.65)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.3)</pre>
```

```
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
end
end
end
else if(i==29)
a= ((temp(i,j-1))^-1);
b=((temp(i,j-1))^3)*(dx^-4)*(temp(3,j-1)-4*temp(2,j-1)+
6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1));
c= (0.5*(dx^-3))*(temp(3,j-1)-2*temp(2,j-1)+2*temp(i-1,j-1)-temp(i-2,j-1))
*(3*(temp(i,j-1))^2)*(0.5*(dx^-1))*(temp(2,j-1)-temp(i-1,j-1));
d= Esd*(dx^-2)*(temp(2,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
if(temp(i,j-1)<0.65)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
 if(h(i,j)<=0)
break
 end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
 dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
```

```
end
 temp(i,j)=h(i,j);
 else if (temp(i,j-1) \le 0.1)
 dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
 %temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
 % temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else
a= ((temp(i,j-1))^-1);
b=((temp(i,j-1))^3)*(dx^-4)*(temp(i+2,j-1)-4*temp(i+1,j-1)+
6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1));
c= (0.5*(dx^-3))*(temp(i+2,j-1)-2*temp(i+1,j-1)+2*temp(i-1,j-1)-temp(i-2,j-1))
*(3*(temp(i,j-1))^2)*(0.5*(dx^-1))*(temp(i+1,j-1)-temp(i-1,j-1));
d= Esd*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
if(temp(i,j-1)<=0.65)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.3)
 dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.001;
```

```
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
plot(xi,temp)
realtime=toc;
```

Appendix D

Case 4: Surface tension forces

```
clear all
clc
Am=0.1;
km=2^{(-0.5)};
lambda=2*pi/km;
n=29;
dx=lambda/n;
x=-pi/km;tic
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
h(i,1)=1+(Am*sin(km*xi(i)));
temp(i,1)=h(i,1);
end
end
for j=2:1000
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
if (i==1)
a= ((temp(i,j-1))^3)*(dx^-4)*(temp(i+2,j-1)-
4*temp(i+1,j-1)+6*temp(i,j-1)-4*temp(28,j-1)+temp(27,j-1));
b= (0.5*(dx^-3))*(temp(i+2,j-1)-2*temp(i+1,j-1)+2*temp(28,j-1))
-temp(27,j-1))*(3*(temp(i,j-1))^2)*(0.5*(dx^-1))
*(temp(i+1,j-1)-temp(28,j-1));
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
```

```
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==2)
a=((temp(i,j-1))^3)*(dx^-4)*(temp(i+2,j-1)-
4*temp(i+1,j-1)+6*temp(i,j-1)-4*temp(1,j-1)+temp(28,j-1));
b= (0.5*(dx^-3))*(temp(i+2,j-1)-2*temp(i+1,j-1)+
2*temp(1,j-1)-temp(28,j-1))*(3*(temp(i,j-1))^2)*(0.5*(dx^-1))
*(temp(i+1,j-1)-temp(1,j-1));
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.3)
```

```
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==28)
a=((temp(i,j-1))^3)*(dx^-4)*(temp(2,j-1)-
4*temp(i+1,j-1)+6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1));
b= (0.5*(dx^-3))*(temp(2,j-1)-2*temp(i+1,j-1)+
2*temp(i-1,j-1)-temp(i-2,j-1))*(3*(temp(i,j-1))^2)*(0.5*(dx^-1))
*(temp(i+1,j-1)-temp(i-1,j-1));
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
```

```
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if(i==29)
a=((temp(i,j-1))^3)*(dx^-4)*(temp(3,j-1)-4*temp(2,j-1)
+6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1));
b= (0.5*(dx^-3))*(temp(3,j-1)-2*temp(2,j-1)+2*temp(i-1,j-1)-
temp(i-2,j-1))*(3*(temp(i,j-1))^2)*(0.5*(dx^-1))*(temp(2,j-1)-temp(i-1,j-1));
if(temp(i,j-1)<0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
```

```
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else
a=((temp(i,j-1))^3)*(dx^-4)*(temp(i+2,j-1)-
4*temp(i+1,j-1)+6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1));
b= (0.5*(dx^-3))*(temp(i+2,j-1)-2*temp(i+1,j-1)+
2*temp(i-1,j-1)-temp(i-2,j-1))*(3*(temp(i,j-1))^2)*(0.5*(dx^-1))
*(temp(i+1,j-1)-temp(i-1,j-1));
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
```

```
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
\operatorname{end}
end
end
end
end
end
end
end
end
%F(j-1)= getframe;
end
%movie(F)
% plot(xi,temp)
%realtime=toc;
```

Appendix E

clear all

Case 5: Evaporation, Van der Waal and Surface tension forces

```
clc
Am=0.1;
km=2^{(-0.5)};
lambda=2*pi/km;
n=29;
dx=lambda/n;
x=-pi/km;tic
E= 10^-3;S=10^-1;D=10^-3;A=10^-4;
Epsilon=0.1;%E*S*(A^-2);
Ddae=1;%(E<sup>2</sup>)*(D<sup>-1</sup>)*(A<sup>-1</sup>);
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
h(i,1)=1+(Am*sin(km*xi(i)));
temp(i,1)=h(i,1);
end
end
for j=2:10000
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
if (i==1)
a= Epsilon*(temp(i,j-1))^-1;
b= (-1*((temp(i,j-1))^-2))*(0.5*(dx^-1)*
(temp(i+1,j-1)-temp(28,j-1)))^2 +
((temp(i,j-1))^-1)*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(28,j-1));
```

```
c= Ddae*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(28,j-1));
d=(0.5*(dx^-3))*(temp(i+2,j-1)-2*temp(i+1,j-1)+2*temp(28,j-1)
-temp(27,j-1))*(3*(temp(i,j-1))^2)*(0.5*(dx^-1))*(temp(i+1,j-1)
-temp(28, j-1));
e=((temp(i,j-1))^3)*(dx^-4)*(temp(i+2,j-1)-4*temp(i+1,j-1)
+6*temp(i,j-1)-4*temp(28,j-1)+temp(27,j-1));
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
```

else if (i==2)

```
a= Epsilon*(temp(i,j-1))^-1;
b= (-1*((temp(i,j-1))^-2))*(0.5*(dx^-1)
*(temp(i+1,j-1)-temp(1,j-1)))^2 + ((temp(i,j-1))^-1)*(dx^-2)*
(temp(i+1,j-1)-2*temp(i,j-1)+temp(1,j-1));
c= Ddae*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(1,j-1));
d= (0.5*(dx^-3))*(temp(i+2,j-1)-2*temp(i+1,j-1)+2*temp(1,j-1)
-temp(28,j-1))*(3*(temp(i,j-1))^2)*(0.5*(dx^-1))*(temp(i+1,j-1)
-temp(1,j-1));
e=((temp(i,j-1))^3)*(dx^-4)*(temp(i+2,j-1)-4*temp(i+1,j-1))
+6*temp(i,j-1)-4*temp(1,j-1)+temp(28,j-1));
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
```

```
else if (i==28)
a= Epsilon*(temp(i,j-1))^-1;
b= (-1*((temp(i,j-1))^-2))*(0.5*(dx^-1)
*(temp(i+1,j-1)-temp(i-1,j-1)))^2 + ((temp(i,j-1))^-1)*(dx^-2)*
(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
c= Ddae*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
d= (0.5*(dx^-3))*(temp(2,j-1)-2*temp(i+1,j-1)+2*temp(i-1,j-1)
-temp(i-2,j-1))*(3*(temp(i,j-1))^2)*(0.5*(dx^-1))*(temp(i+1,j-1)
-temp(i-1,j-1));
e=((temp(i,j-1))^3)*(dx^-4)*(temp(2,j-1)-4*temp(i+1,j-1)+
6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1));
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
```

```
end
end
else if(i==29)
a= Epsilon*(temp(i,j-1))^-1;
b= (-1*((temp(i,j-1))^-2))*(0.5*(dx^-1)*
(temp(2,j-1)-temp(i-1,j-1)))^2 + ((temp(i,j-1))^-1)*(dx^-2)*
(temp(2,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
c= Ddae*(dx^-2)*(temp(2,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
d= (0.5*(dx^-3))*(temp(3,j-1)-2*temp(2,j-1)+2*temp(i-1,j-1))
-temp(i-2,j-1))*(3*(temp(i,j-1))^2)*(0.5*(dx^-1))*(temp(2,j-1)-
temp(i-1,j-1));
e=((temp(i,j-1))^3)*(dx^-4)*(temp(3,j-1)-4*temp(2,j-1)+
6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1));
if(temp(i,j-1)<0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
```

```
temp(i,j)=h(i,j);
end
end
end
         a= Epsilon*(temp(i,j-1))^-1;
else
b= (-1*((temp(i,j-1))^-2))*(0.5*(dx^-1)*
(temp(i+1,j-1)-temp(i-1,j-1)))^2 + ((temp(i,j-1))^-1)*
(dx<sup>-2</sup>)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
c= Ddae*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
d= (0.5*(dx^-3))*(temp(i+2,j-1)-2*temp(i+1,j-1)+
2*temp(i-1,j-1)-temp(i-2,j-1))*(3*(temp(i,j-1))^2)*
(0.5*(dx^-1))*(temp(i+1,j-1)-temp(i-1,j-1));
e=((temp(i,j-1))^3)*(dx^-4)*(temp(i+2,j-1)-4*temp(i+1,j-1)
+6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1));
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
```

Appendix F

Case 6: Evaporation and Gravitational forces

```
clear all
clc
Am=0.1;
km=2^{(-0.5)};
lambda=2*pi/km;
n=29;
dx=lambda/n;
x=-pi/km;tic
E= 10<sup>-3</sup>;G=10<sup>-11</sup>;
Eeg=1;%E*(G^-1);
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
h(i,1)=1+(Am*sin(km*xi(i)));
temp(i,1)=h(i,1);
end
end
for j=2:4930
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
if (i==1)
a= Eeg*(temp(i,j-1)^-1);
b= -(((-temp(i,j-1)^-2)*((0.5*dx^-1)*(temp(i+1,j-1)-
temp(28,j-1)))^2) + ((temp(i,j-1)^-1)*(dx^-2)*
(temp(i+1,j-1)-2*temp(i,j-1)+temp(28,j-1))));
```

```
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==2)
a= Eeg*(temp(i,j-1)^-1);
b= -(((-temp(i,j-1)^-2)*((0.5*dx^-1)*(temp(i+1,j-1)-
temp(1,j-1)))^2) + ((temp(i,j-1)^-1)*(dx^-2)*
(temp(i+1,j-1)-2*temp(i,j-1)+temp(1,j-1))));
```

```
if(temp(i,j-1)<=0.65)
```

```
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.3)</pre>
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j) \le 0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==28)
a= Eeg*(temp(i,j-1)^-1);
b= -(((-temp(i,j-1)^-2)*((0.5*dx^-1)*(temp(i+1,j-1)-
temp(i-1,j-1)))^2) + ((temp(i,j-1)^-1)*(dx^-2)*
(temp(i+1,j-1)-2*temp(i,j-1)+temp(1,j-1))));
```

```
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)</pre>
```

```
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if(i==29)
a= Eeg*(temp(i,j-1)^-1);
b= -(((-temp(i,j-1)^-2)*((0.5*dx^-1)*(temp(2,j-1)-
temp(i-1,j-1)))^2) + ((temp(i,j-1)^-1)*(dx^-2)*
(temp(2,j-1)-2*temp(i,j-1)+temp(1,j-1))));
if(temp(i,j-1)<0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
```

```
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else a= Eeg*(temp(i,j-1)^-1);
b= -(((-temp(i,j-1)^-2)*((0.5*dx^-1)*(temp(i+1,j-1)-
temp(i-1,j-1)))^2) + ((temp(i,j-1)^-1)*(dx^-2)*
(temp(i+1,j-1)-2*temp(i,j-1)+temp(1,j-1))));
```

```
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)</pre>
```

40 APPENDIX F. CASE 6: EVAPORATION AND GRAVITATIONAL FORCES

```
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
plot(xi,temp)
realtime=toc;
%kk=[1,1000,2000,3000,4000,4200,4400,4600,4800,5000];
%test=figure;plot(temp(1:17,kk));
```
Appendix G

Case 7: Evaporation, Gravitational forces and Vapor Recoil

```
clear all
clc
Am=0.1;
km=2^{(-0.5)};
lambda=2*pi/km;
n=29;
dx=lambda/n;
x=-pi/km;tic
E= 10<sup>-3</sup>; D=10<sup>-3</sup>; G=10<sup>-11</sup>;
Egd= 1;%(E<sup>2</sup>)*(D<sup>-1</sup>)*(G<sup>-1</sup>);
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
h(i,1)=1+(Am*sin(km*xi(i)));
temp(i,1)=h(i,1);
end
end
for j=2:450
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
if (i==1)
a =(temp(i,j-1)^-1);
b =(-(temp(i,j-1)^-2)*((0.5*(dx^-1))*(temp(i+1,j-1)-
temp(28,j-1)))^2)+((temp(i,j-1)^-1)*(dx^-2)*(temp(i+1,j-1)
-2*temp(i,j-1)+temp(28,j-1)));
c = Egd*(dx<sup>-2</sup>)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(28,j-1));
if(temp(i,j-1)<=0.65)
```

```
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==2)
a =(temp(i,j-1)^-1);
b =(-(temp(i,j-1)^-2)*((0.5*(dx^-1))*(temp(i+1,j-1)-
temp(1,j-1)))^2)+((temp(i,j-1)^-1)*(dx^-2)*(temp(i+1,j-1))
-2*temp(i,j-1)+temp(1,j-1)));
c = Egd*(dx<sup>-2</sup>)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(1,j-1));
if(temp(i,j-1)<=0.65)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
```

```
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.3)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.1)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==28)
a =(temp(i,j-1)^-1);
b =(-(temp(i,j-1)^-2)*((0.5*(dx^-1))*(temp(i+1,j-1)-
temp(i-1,j-1)))^2)+((temp(i,j-1)^-1)*(dx^-2)*
(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1)));
c = Egd*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
if(temp(i,j-1)<=0.65)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
```

```
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if(i==29)
a =(temp(i,j-1)^-1);
b =(-(temp(i,j-1)^-2)*((0.5*(dx^-1))*(temp(2,j-1)
-temp(i-1,j-1)))^2)+((temp(i,j-1)^-1)*(dx^-2)
*(temp(2,j-1)-2*temp(i,j-1)+temp(i-1,j-1)));
c = Egd*(dx<sup>-2</sup>)*(temp(2,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
if(temp(i,j-1)<0.65)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.3)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
```

```
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.1)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else a =(temp(i,j-1)^-1);
b =(-(temp(i,j-1)^-2)*((0.5*(dx^-1))*(temp(i+1,j-1)
-temp(i-1,j-1)))^2)+((temp(i,j-1)^-1)*(dx^-2)
*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1)));
c = Egd*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)
+temp(i-1,j-1));
if(temp(i,j-1)<=0.65)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
```

```
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.1)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(a-b+c));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
end
end
end
end
end
end
M(:,j-1)=getframe;
end
movie(M)
%plot(xi,temp)
%realtime=toc;
```

Appendix H

Case 8: Evaporation, Surface tension and Gravitational forces

```
clear all
clc
Am=0.1;
km=2^{(-0.5)};
lambda=2*pi/km;
n=29;
dx=lambda/n;
x=-pi/km;tic
E= 10<sup>-3</sup>;S=10<sup>-1</sup>;D=10<sup>-3</sup>; G=10<sup>-11</sup>;
Esg=1;%E*S*(G^-2);
Edg=1; (E<sup>2</sup>) * (D<sup>-1</sup>) * (G<sup>-1</sup>);
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
h(i,1)=1+(Am*sin(km*xi(i)));
temp(i,1)=h(i,1);
end
end
for j=2:4270
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
if (i==1)
a= Esg*(temp(i,j-1))^-1;
b= (0.5*(dx^-3))*(temp(i+2,j-1)-2*temp(i+1,j-1))
+2*temp(28,j-1)-temp(27,j-1))*(3*(temp(i,j-1))^2)
*(0.5*(dx^-1))*(temp(i+1,j-1)-temp(28,j-1));
```

```
c= ((temp(i,j-1))^3)*(dx^-4)*(temp(i+2,j-1)-
4*temp(i+1,j-1)+6*temp(i,j-1)-4*temp(28,j-1)+temp(27,j-1));
d= -( ((-(temp(i,j-1))^-2)*((0.5*(dx^-1)*(temp(i+1,j-1)-
temp(28,j-1)))^2))+ (((temp(i,j-1))^-1)*(dx^-2)
*(temp(i+1,j-1)-2*temp(i,j-1)+temp(28,j-1))));
e= Edg*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(28,j-1));
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==2)
a= Esg*(temp(i,j-1))^-1;
```

```
b= (0.5*(dx^-3))*(temp(i+2,j-1)-2*temp(i+1,j-1))
+2*temp(1,j-1)-temp(28,j-1))*(3*(temp(i,j-1))^2)
*(0.5*(dx^-1))*(temp(i+1,j-1)-temp(1,j-1));
c= ((temp(i,j-1))^3)*(dx^-4)*(temp(i+2,j-1)-
4*temp(i+1,j-1)+6*temp(i,j-1)-4*temp(1,j-1)+temp(28,j-1));
d= -( ((-(temp(i,j-1))^-2)*((0.5*(dx^-1)*(temp(i+1,j-1)-
temp(1,j-1)))^2))+ (((temp(i,j-1))^-1)*(dx^-2)*
(temp(i+1,j-1)-2*temp(i,j-1)+temp(1,j-1))));
e= Edg*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(1,j-1));
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.3)</pre>
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.1)</pre>
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==28)
```

```
a= Esg*(temp(i,j-1))^-1;
b= (0.5*(dx^-3))*(temp(2,j-1)-2*temp(i+1,j-1)+
2*temp(i-1,j-1)-temp(i-2,j-1))*(3*(temp(i,j-1))^2)
*(0.5*(dx^-1))*(temp(i+1,j-1)-temp(i-1,j-1));
c= ((temp(i,j-1))^3)*(dx^-4)*(temp(2,j-1)-4
*temp(i+1,j-1)+6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1));
d= -( ((-(temp(i,j-1))^-2)*((0.5*(dx^-1)*(temp(i+1,j-1)-
temp(i-1,j-1)))^2))+ (((temp(i,j-1))^-1)*(dx^-2)*
(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1))));
e= Edg*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
```

```
end
else if(i==29)
a= Esg*(temp(i,j-1))^-1;
b= (0.5*(dx^-3))*(temp(3,j-1)-2*temp(2,j-1)+
2*temp(i-1,j-1)-temp(i-2,j-1))*(3*(temp(i,j-1))^2)*
(0.5*(dx^-1))*(temp(2,j-1)-temp(i-1,j-1));
c= ((temp(i,j-1))^3)*(dx^-4)*(temp(3,j-1)-
4*temp(2,j-1)+6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1));
d= -( ((-(temp(i,j-1))^-2)*((0.5*(dx^-1)*(temp(2,j-1))
-temp(i-1,j-1)))^2))+ (((temp(i,j-1))^-1)*(dx^-2)
*(temp(2,j-1)-2*temp(i,j-1)+temp(i-1,j-1))));
e= Edg*(dx^-2)*(temp(2,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
if(temp(i,j-1)<0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.1)</pre>
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
```

```
end
end
else
a= Esg*(temp(i,j-1))^-1;
b= (0.5*(dx^-3))*(temp(i+2,j-1)-2*temp(i+1,j-1))
+2*temp(i-1,j-1)-temp(i-2,j-1))*(3*(temp(i,j-1))^2)
*(0.5*(dx^-1))*(temp(i+1,j-1)-temp(i-1,j-1));
c= ((temp(i,j-1))^3)*(dx^-4)*(temp(i+2,j-1)
-4*temp(i+1,j-1)+6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1));
d= -( ((-(temp(i,j-1))^-2)*((0.5*(dx^-1)*(temp(i+1,j-1)-
temp(i-1,j-1)))^2))+ (((temp(i,j-1))^-1)*(dx^-2)*
(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1))));
e= Edg*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1));
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(a+b+c+d+e));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
```

Appendix I

Case 9: Evolution equation

```
clear all
clc
Am=0.1;
km=2^{(-0.5)};
lambda=2*pi/km;
n=29;
dx=lambda/n;
x=-pi/km;tic
%E=0.1; S=0.1;A=10^-4;D=100;
K=1;%M=1;P=10;
AA=1000; %E*S*(A^{-2});
BB=1;%(E<sup>2</sup>)*(D<sup>-1</sup>)*(A<sup>-1</sup>);
CC=0.1;%K*M*(P^-1)*(A^-1);
%Egd= 1;%(E<sup>2</sup>)*(D<sup>-1</sup>)*(G<sup>-1</sup>);
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
h(i,1)=1+(Am*sin(km*xi(i)));
temp(i,1)=h(i,1);
end
end
for j=2:16
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
if (i==1)
B=AA*((temp(i, j-1))+K)^{-1};
C=((temp(i,j-1)^3)*(dx^-4)*(temp(i+2,j-1)-4*temp(i+1,j-1))
+6*temp(i,j-1)-4*temp(28,j-1)+temp(27,j-1)))+(0.5*(dx^-3)
```

```
*(temp(i+2,j-1)-2*temp(i+1,j-1)+2*temp(28,j-1)-temp(27,j-1))
*(3*(temp(i,j-1)^2))*(0.5*(dx^-1)*(temp(i+1,j-1)-temp(28,j-1))));
D=((dx^-2)*(temp(i,j-1)^-1)*(temp(i+1,j-1)-2*temp(i,j-1))
+temp(28,j-1)))+(-1)*(temp(i,j-1)^-2)*((0.5*(dx^-1)
*(temp(i+1,j-1)-temp(28,j-1)))^2);
E=BB*(((temp(i,j-1)^3)*(((temp(j,j-1))+K)^-3)*(dx^-2)
*(temp(i+1,j-1)-2*temp(i,j-1)+temp(28,j-1)))+((0.5*(dx^-1)
*(temp(i+1,j-1)-temp(28,j-1)))^2)*(((temp(i,j-1)^3)*(-3))
*((((temp(j,j-1))+K)^-4)))+((((temp(j,j-1))+K)^-3)*3*(temp(i,j-1)^2))));
F=CC*(((temp(i,j-1)^2)*(((temp(j,j-1))+K)^-2)*(dx^-2)*
(temp(i+1,j-1)-2*temp(i,j-1)+temp(28,j-1)))+((0.5*(dx^-1)))
*(temp(i+1,j-1)-temp(28,j-1)))^2)*(((temp(i,j-1)^2)*
(-2)*(((temp(j,j-1))+K)^-3))+((((temp(j,j-1))+K)^-2)*2*(temp(i,j-1)))));
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
```

```
end
end
end
```

```
else if (i==2)
B=AA*((temp(i,j-1))+K)^-1;
C=((temp(i,j-1)^3)*(dx^-4)*(temp(i+2,j-1)-4*temp(i+1,j-1))
+6*temp(i,j-1)-4*temp(1,j-1)+temp(28,j-1)))+(0.5*(dx^-3)
*(temp(i+2,j-1)-2*temp(i+1,j-1)+2*temp(1,j-1)-temp(28,j-1))
*(3*(temp(i,j-1)^2))*(0.5*(dx^-1)*(temp(i+1,j-1)-temp(1,j-1))));
D=((dx^-2)*(temp(i,j-1)^-1)*(temp(i+1,j-1)-2*temp(i,j-1))
+temp(1,j-1)))+(-1)*(temp(i,j-1)^-2)*((0.5*(dx^-1)
*(temp(i+1,j-1)-temp(1,j-1)))^2);
E=BB*(((temp(i,j-1)^3)*(((temp(j,j-1))+K)^-3)*(dx^-2)
*(temp(i+1,j-1)-2*temp(i,j-1)+temp(1,j-1)))+((0.5*(dx^-1))
*(temp(i+1,j-1)-temp(1,j-1)))^2)*(((temp(i,j-1)^3)*(-3))
*((((temp(j,j-1))+K)^-4)))+((((temp(j,j-1))+K)^-3)*3*(temp(i,j-1)^2))));
F=CC*(((temp(i,j-1)^2)*(((temp(j,j-1))+K)^-2)*(dx^-2)
*(temp(i+1,j-1)-2*temp(i,j-1)+temp(1,j-1)))+((0.5*(dx^-1))
*(temp(i+1,j-1)-temp(1,j-1)))^2)*(((temp(i,j-1)^2)*(-2))
*(((temp(j,j-1))+K)^-3))+((((temp(j,j-1))+K)^-2)*2*(temp(i,j-1)))));
```

```
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.1)</pre>
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
```

```
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==28)
B=AA*((temp(i,j-1))+K)^-1;
C=((temp(i,j-1)^3)*(dx^-4)*(temp(2,j-1)-4*temp(i+1,j-1)
+6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1)))+(0.5*(dx^-3)
*(temp(2,j-1)-2*temp(i+1,j-1)+2*temp(i-1,j-1)-temp(i-2,j-1))
*(3*(temp(i,j-1)^2))*(0.5*(dx^-1)*(temp(i+1,j-1)-temp(i-1,j-1))));
D=((dx^-2)*(temp(i,j-1)^-1)*(temp(i+1,j-1)-2*temp(i,j-1)
+temp(i-1,j-1)))+(-1)*(temp(i,j-1)^-2)*((0.5*(dx^-1)
*(temp(i+1,j-1)-temp(i-1,j-1)))^2);
E=BB*(((temp(i,j-1)^3)*(((temp(j,j-1))+K)^-3)*(dx^-2)
*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1)))+((0.5*(dx^-1)
*(temp(i+1,j-1)-temp(i-1,j-1)))^2)*(((temp(i,j-1)^3)*(-3)
*((((temp(j,j-1))+K)^-4)))+((((temp(j,j-1))+K)^-3)*3*(temp(i,j-1)^2))));
F=CC*(((temp(i,j-1)^2)*(((temp(j,j-1))+K)^-2)*(dx^-2)*(temp(i+1,j-1)-
2*temp(i,j-1)+temp(i-1,j-1)))+((0.5*(dx^-1)*(temp(i+1,j-1)-
temp(i-1,j-1)))^2)*(((temp(i,j-1)^2)*(-2)*(((temp(j,j-1))+K)^-3))
+((((temp(j,j-1))+K)^-2)*2*(temp(i,j-1)))));
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
```

```
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.1)</pre>
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if(i==29)
B=AA*((temp(i,j-1))+K)^-1;
C=((temp(i,j-1)^3)*(dx^-4)*(temp(3,j-1)-4*temp(2,j-1))
+6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1)))+(0.5*(dx^-3)
*(temp(3,j-1)-2*temp(2,j-1)+2*temp(i-1,j-1)-temp(i-2,j-1))*(
3*(temp(i,j-1)^2))*(0.5*(dx^-1)*(temp(2,j-1)-temp(i-1,j-1))));
D=((dx^-2)*(temp(i,j-1)^-1)*(temp(2,j-1)-2*temp(i,j-1)+
temp(i-1,j-1)))+(-1)*(temp(i,j-1)^-2)*((0.5*(dx^-1)))
*(temp(2,j-1)-temp(i-1,j-1)))^2);
E=BB*(((temp(i,j-1)^3)*(((temp(j,j-1))+K)^-3)*(dx^-2)*
(temp(2,j-1)-2*temp(i,j-1)+temp(i-1,j-1)))+((0.5*(dx^-1)*
(temp(2,j-1)-temp(i-1,j-1)))^2)*(((temp(i,j-1)^3)*(-3)*
(((((temp(j,j-1))+K)^-4)))+((((temp(j,j-1))+K)^-3)*3*(temp(i,j-1)^2))));
F=CC*(((temp(i,j-1)^2)*(((temp(j,j-1))+K)^-2)*(dx^-2)*
(temp(2,j-1)-2*temp(i,j-1)+temp(i-1,j-1)))+((0.5*(dx^-1)*
(temp(2,j-1)-temp(i-1,j-1)))^2)*(((temp(i,j-1)^2)*(-2)*
(((temp(j,j-1))+K)^-3))+((((temp(j,j-1))+K)^-2)*2*(temp(i,j-1)))));
```

```
if(temp(i,j-1)<0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break</pre>
```

```
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else
B=AA*((temp(i,j-1))+K)^-1;
C=((temp(i,j-1)^3)*(dx^-4)*(temp(i+2,j-1)-4*temp(i+1,j-1)
+6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1)))+
(0.5*(dx^-3)*(temp(i+2,j-1)-2*temp(i+1,j-1)+2*temp(i-1,j-1)
-temp(i-2,j-1))*(3*(temp(i,j-1)^2))*(0.5*(dx^-1)*
(temp(i+1,j-1)-temp(i-1,j-1))));
D=((dx^-2)*(temp(i,j-1)^-1)*(temp(i+1,j-1)-2*temp(i,j-1))
+temp(i-1,j-1)))+(-1)*(temp(i,j-1)^-2)*((0.5*(dx^-1)
*(temp(i+1,j-1)-temp(i-1,j-1)))^2);
E=BB*(((temp(i,j-1)^3)*(((temp(j,j-1))+K)^-3)*(dx^-2)
*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1)))+
((0.5*(dx^-1)*(temp(i+1,j-1)-temp(i-1,j-1)))^2)*
(((temp(i,j-1)^3)*(-3)*((((temp(j,j-1))+K)^-4)))+
((((temp(j,j-1))+K)^-3)*3*(temp(i,j-1)^2))));
F=CC*(((temp(i,j-1)^2)*(((temp(j,j-1))+K)^-2)*
(dx<sup>-2</sup>)*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1)))+
```

```
((0.5*(dx^-1)*(temp(i+1,j-1)-temp(i-1,j-1)))^2)*
(((temp(i,j-1)^2)*(-2)*(((temp(j,j-1))+K)^-3))+
(((((temp(j,j-1))+K)^-2)*2*(temp(i,j-1)))));
if(temp(i,j-1)<=0.65)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.0001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D+E+F));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
end
end
end
end
end
end
%M(:,j-1)=getframe;
```

end
% movie(M)
plot(xi,temp)
%realtime=toc;

Appendix J

Case 10: Evolution equation with gravitation

```
clear all
clc
Am=0.1;
km=2^{(-0.5)};
lambda=2*pi/km;
n=29;
dx=lambda/n;
x=-pi/km;tic
E=0.1; S=0.1;A=10<sup>-4</sup>;D=100;
K=0.1;M=1;P=10;G=10^-11;
AA=0.1; %E*S*(A^-2);
BB=0;%G*(A^-1);
CC=1; (E^2) * (D^{-1}) * (A^{-1});
DD=0;%K*M*(P^-1)*(A^-1);
%Egd= 1;%(E<sup>2</sup>)*(D<sup>-1</sup>)*(G<sup>-1</sup>);
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
h(i,1)=1+(Am*sin(km*xi(i)));
temp(i,1)=h(i,1);
end
end
for j=2:2000
for i=1:n
xi(i)=x+(i-1)*dx;
if(xi(i)<=(pi/km))</pre>
if (i==1)
```

```
B=AA*((temp(i,j-1))+K)^-1;
C=((temp(i,j-1)^3)*(dx^-4)*(temp(i+2,j-1)-4*temp(i+1,j-1)+
6*temp(i,j-1)-4*temp(28,j-1)+temp(27,j-1)))+(0.5*(dx^-3)*(temp(i+2,j-1)-
2*temp(i+1,j-1)+2*temp(28,j-1)-temp(27,j-1))*(3*(temp(i,j-1)^2))
*(0.5*(dx^-1)*(temp(i+1,j-1)-temp(28,j-1))));
D=((dx^-2)*(temp(i,j-1)^-1)*(temp(i+1,j-1)-2*temp(i,j-1)+
temp(28,j-1)))+(-1)*(temp(i,j-1)^-2)*((0.5*(dx^-1)*
(temp(i+1,j-1)-temp(28,j-1)))^2);
E=BB*(((temp(i,j-1)^3)*(dx^-2)*(temp(i+1,j-1)-
2*temp(i,j-1)+temp(28,j-1)))+(3*(temp(i,j-1)^2)*
((0.5*dx*(temp(i+1,j-1)-temp(28,j-1)))^2)));
F=CC*(((temp(i,j-1)^3)*(((temp(i,j-1))+K)^-3)*(dx^-2)*
(temp(i+1,j-1)-2*temp(i,j-1)+temp(28,j-1)))+((0.5*(dx^-1)*
(temp(i+1,j-1)-temp(28,j-1)))^2)*(((temp(i,j-1)^3)*(-3)*
(((((temp(i,j-1))+K)^-4)))+((((temp(i,j-1))+K)^-3)*3*(temp(i,j-1)^2))));
G=DD*(((temp(i,j-1)^2)*(((temp(i,j-1))+K)^-2)*(dx^-2)*
(temp(i+1,j-1)-2*temp(i,j-1)+temp(28,j-1)))+((0.5*(dx^-1)*
(temp(i+1,j-1)-temp(28,j-1)))^2)*(((temp(i,j-1)^2)*(-2)*
((((temp(i,j-1))+K)^-3))+((((temp(i,j-1))+K)^-2)*2*(temp(i,j-1)))));
if(temp(i,j-1)<=0.65)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
```

```
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==2)
B=AA*((temp(i,j-1))+K)^-1;
C=((temp(i,j-1)^3)*(dx^-4)*(temp(i+2,j-1)-
4*temp(i+1,j-1)+6*temp(i,j-1)-4*temp(1,j-1)+temp(28,j-1)))
+(0.5*(dx^-3)*(temp(i+2,j-1)-2*temp(i+1,j-1)+
2*temp(1,j-1)-temp(28,j-1))*(3*(temp(i,j-1)^2))*
(0.5*(dx^-1)*(temp(i+1,j-1)-temp(1,j-1))));
D=((dx^-2)*(temp(i,j-1)^-1)*(temp(i+1,j-1)-2*temp(i,j-1))
+temp(1,j-1)))+(-1)*(temp(i,j-1)^-2)*((0.5*(dx^-1)*
(temp(i+1,j-1)-temp(1,j-1)))^2);
E=BB*(((temp(i,j-1)^3)*(dx^-2)*(temp(i+1,j-1)-2*temp(i,j-1)+
temp(1,j-1)))+(3*(temp(i,j-1)^2)*((0.5*dx*(temp(i+1,j-1)-temp(1,j-1)))^2)));
F=CC*(((temp(i,j-1)^3)*(((temp(i,j-1))+K)^-3)*(dx^-2)*
(temp(i+1,j-1)-2*temp(i,j-1)+temp(1,j-1)))+((0.5*(dx^-1)*
(temp(i+1,j-1)-temp(1,j-1)))^2)*(((temp(i,j-1)^3)*(-3)*
(((((temp(i,j-1))+K)^-4)))+((((temp(i,j-1))+K)^-3)*3*(temp(i,j-1)^2))));
G=DD*(((temp(i,j-1)^2)*(((temp(i,j-1))+K)^-2)*(dx^-2)*
(temp(i+1,j-1)-2*temp(i,j-1)+temp(1,j-1)))+((0.5*(dx^-1)*
(temp(i+1,j-1)-temp(1,j-1)))^2)*(((temp(i,j-1)^2)*(-2)*
(((temp(i,j-1))+K)^-3))+((((temp(i,j-1))+K)^-2)*2*(temp(i,j-1)))));
if(temp(i,j-1)<=0.65)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.3)
dt=0.001;
```

```
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
```

```
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if (i==28)
B=AA*((temp(i,j-1))+K)^-1;
C=((temp(i,j-1)^3)*(dx^-4)*(temp(2,j-1)-
4*temp(i+1,j-1)+6*temp(i,j-1)-4*temp(i-1,j-1)
+temp(i-2,j-1)))+(0.5*(dx^-3)*(temp(2,j-1)-
2*temp(i+1,j-1)+2*temp(i-1,j-1)-temp(i-2,j-1))
*(3*(temp(i,j-1)^2))*(0.5*(dx^-1)*(temp(i+1,j-1)-temp(i-1,j-1))));
D=((dx^-2)*(temp(i,j-1)^-1)*(temp(i+1,j-1)-
2*temp(i,j-1)+temp(i-1,j-1)))+(-1)*(temp(i,j-1)^-2)
*((0.5*(dx^-1)*(temp(i+1,j-1)-temp(i-1,j-1)))^2);
E=BB*(((temp(i,j-1)^3)*(dx^-2)*(temp(i+1,j-1)-
2*temp(i,j-1)+temp(i-1,j-1)))+(3*(temp(i,j-1)^2)
*((0.5*dx*(temp(i+1,j-1)-temp(i-1,j-1)))^2)));
F=CC*(((temp(i,j-1)^3)*(((temp(i,j-1))+K)^-3)*(dx^-2)
*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1)))+
((0.5*(dx^-1)*(temp(i+1,j-1)-temp(i-1,j-1)))^2)*
(((temp(i,j-1)^3)*(-3)*((((temp(i,j-1))+K)^-4)))+
(((((temp(i,j-1))+K)^-3)*3*(temp(i,j-1)^2))));
G=DD*(((temp(i,j-1)^2)*(((temp(i,j-1))+K)^-2)*(dx^-2)
*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1)))+
((0.5*(dx^-1)*(temp(i+1,j-1)-temp(i-1,j-1)))^2)*
(((temp(i,j-1)^2)*(-2)*(((temp(i,j-1))+K)^-3))+
(((((temp(i,j-1))+K)^-2)*2*(temp(i,j-1)))));
```

```
if(temp(i,j-1)<=0.65)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.3)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1) \le 0.1)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
else if(i==29)
B=AA*((temp(i,j-1))+K)^-1;
C=((temp(i,j-1)^3)*(dx^-4)*(temp(3,j-1)-4*temp(2,j-1))
+6*temp(i,j-1)-4*temp(i-1,j-1)+temp(i-2,j-1)))
+(0.5*(dx^-3)*(temp(3,j-1)-2*temp(2,j-1)+2*temp(i-1,j-1))
-temp(i-2,j-1))*(3*(temp(i,j-1)^2))*(0.5*(dx^-1)*
(temp(2,j-1)-temp(i-1,j-1))));
D=((dx^-2)*(temp(i,j-1)^-1)*(temp(2,j-1)-2*temp(i,j-1))
+temp(i-1,j-1)))+(-1)*(temp(i,j-1)^-2)*((0.5*(dx^-1)*
(temp(2,j-1)-temp(i-1,j-1)))^2);
```

```
E=BB*(((temp(i,j-1)^3)*(dx^-2)*(temp(2,j-1)-2*temp(i,j-1))
+temp(i-1,j-1)))+(3*(temp(i,j-1)^2)*((0.5*dx*(temp(2,j-1)
-temp(i-1,j-1)))^2)));
F=CC*(((temp(i,j-1)^3)*(((temp(i,j-1))+K)^-3)*(dx^-2)
*(temp(2,j-1)-2*temp(i,j-1)+temp(i-1,j-1)))+((0.5*(dx^-1)
*(temp(2,j-1)-temp(i-1,j-1)))^2)*(((temp(i,j-1)^3)*(-3)
*((((temp(i,j-1))+K)^-4)))+((((temp(i,j-1))+K)^-3)
*3*(temp(i,j-1)^2))));
G=DD*(((temp(i,j-1)^2)*(((temp(i,j-1))+K)^-2)*
(dx<sup>-2</sup>)*(temp(2,j-1)-2*temp(i,j-1)+temp(i-1,j-1)))
+((0.5*(dx^-1)*(temp(2,j-1)-temp(i-1,j-1)))^2)
*(((temp(i,j-1)^2)*(-2)*(((temp(i,j-1))+K)^-3))
+((((temp(i,j-1))+K)^-2)*2*(temp(i,j-1)))));
if(temp(i,j-1)<0.65)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.3)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
```

```
temp(i,j)=h(i,j);
end
end
end
else
B=AA*((temp(i,j-1))+K)^-1;
C=((temp(i,j-1)^3)*(dx^-4)*(temp(i+2,j-1)-
4*temp(i+1,j-1)+6*temp(i,j-1)-4*temp(i-1,j-1)
+temp(i-2,j-1)))+(0.5*(dx^-3)*(temp(i+2,j-1)-
2*temp(i+1,j-1)+2*temp(i-1,j-1)-temp(i-2,j-1))
*(3*(temp(i,j-1)^2))*(0.5*(dx^-1)*(temp(i+1,j-1)-temp(i-1,j-1))));
D=((dx^-2)*(temp(i,j-1)^-1)*(temp(i+1,j-1)-2*temp(i,j-1)+
temp(i-1,j-1)))+(-1)*(temp(i,j-1)^-2)*((0.5*(dx^-1)*
(temp(i+1,j-1)-temp(i-1,j-1)))^2);
E=BB*(((temp(i,j-1)^3)*(dx^-2)*(temp(i+1,j-1)-
2*temp(i,j-1)+temp(i-1,j-1)))+(3*(temp(i,j-1)^2)
*((0.5*dx*(temp(i+1,j-1)-temp(i-1,j-1)))^2)));
F=CC*(((temp(i,j-1)^3)*(((temp(i,j-1))+K)^-3)*(dx^-2)
*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1)))+((0.5*(dx^-1)
*(temp(i+1,j-1)-temp(i-1,j-1)))^2)*(((temp(i,j-1)^3)*(-3)*
(((((temp(i,j-1))+K)^-4)))+(((((temp(i,j-1))+K)^-3)*3*(temp(i,j-1)^2))));
G=DD*(((temp(i,j-1)^2)*(((temp(i,j-1))+K)^-2)*(dx^-2)
*(temp(i+1,j-1)-2*temp(i,j-1)+temp(i-1,j-1)))+
((0.5*(dx^-1)*(temp(i+1,j-1)-temp(i-1,j-1)))^2)
*(((temp(i,j-1)^2)*(-2)*(((temp(i,j-1))+K)^-3))
+((((temp(i,j-1))+K)^-2)*2*(temp(i,j-1)))));
if(temp(i,j-1)<=0.65)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i,j-1)<=0.3)
dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else if (temp(i, j-1) \le 0.1)
dt=0.001;
```

```
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
%temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
else dt=0.001;
h(i,j)=temp(i,j-1)-(dt*(B+C+D-E+F+G));
% temp(i,j+1)=h(i,j+1);
if(h(i,j)<=0)
break
end
temp(i,j)=h(i,j);
end
end
end
end
end
end
end
end
end
%M(:,j-1)=getframe;
end
% movie(M)
plot(xi,temp)
%realtime=toc;
```

Appendix K

Air Bouyancy effect detection

- end matter ———–