

A toast with Fermat, Mersenne, Violet and Don

PALINDROMIC WISHES FOR THE NEW YEAR

BY MARCO BURATTI

Continuing the tradition, here I am with my palindromic wishes for the new year. As I already pointed out, when I search for a nice palindromic representation of a number, I first look at its decomposition into primes. Well, the prime decomposition of 2023 is $7 \cdot 17^2$ which, in my opinion, is very nice though aware that some of my compatriots would strongly disagree considering the double appearance of 17. Indeed in the Italian tradition the number 17 is considered very unlucky. It seems that it dates backs to ancient Roman times. In Roman numerals 17 is XVII and an anagram for XVII is VIXI which, in Latin, means “I have lived” implying that now ... “I am dead”. Thus superstitious Italian people tend to avoid 17 in all possible ways. It may happen, for instance, that some hotels do not have the room number 17 or that some streets do not have the house number 17. Also, some Italians would never plan their wedding on the 17 of whatever month. Anyway I usually go against the flow and I am not superstitious at all. Or, maybe, it is better to say that I am “scaramantico”, an Italian behavior consisting in invoking something bad with the purpose that the bad thing will not happen at all. So it is not surprising that 17 is my favorite number! I got my Math degree on July 17 😊

But I am going off-topic. How to get a palindrome from the above factorization? An almost immediate answer might be $7 \cdot 17 \cdot 17$ which, however, is not very satisfactory. Indeed even though the sequence of the digits **71717** is a palindrome, the

signs are not symmetric. On the other hand, how can one not notice that the factors are either Fermat or Mersenne primes?! In binary, every number of the form $2^n \pm 1$ is a palindrome! In particular, the binary representations of 7 and 17 are 111 and 10001, respectively. All in all, here are my favorite wishes for the new year:

Happy 10001 · 111 · 10001 !!!



But there are palindromic wishes for all tastes. For instance, Violet Syrotiuk wished me

all the best in **7E7** !!!

Yes, in the **hexadecimal** system (base 16) the digits should be 0, 1, 2, ..., 9, A, B, C, D, E, F and the related conversion of 2023 is then **7E7**:

$$2023 = 7 \cdot 17^2 = 7 \cdot (16 + 1)^2 = 7 \cdot 16^2 + 14 \cdot 16 + 7.$$

Similarly, four years ago, in a series of palindromic wishes for a Happy 2019 I proposed, in particular, “Happy **5B5**” in **enneadecimal** system (base 19) and “Happy **3C3**” in **tetravigesimal** system (base 24).

Another friend who seems to be a bit infected by my “palindromania” of seeing everything symmetric at the cost of rewriting numbers in non-decimal bases, is our editor and production manager Don Kreher. Last September 14 he sent to me this nice puzzle:



“Today my age is a palindrome in two consecutive bases”.

Clearly, he meant a *non-trivial* palindrome, i.e., with more than one digit. Indeed every number n is a one digit palindrome in base b for every $b > n$. In the following, it will be tacitly understood that “palindrome” will mean “non-trivial palindrome”.

Of course Don cannot be 10 (which is 101 in base 3 and 22 in base 4). I could have replied “Happy 232 in base 4 and happy 141 in base 5!!!” if he had turned 46 years old. But look at him! With all best intentions, I couldn’t think that he is under 50. I could have replied “Happy 343 in base 5 and happy 242 in base 6” or “Happy 232 in base 6 and happy 161 in base 7” if he were 98 or 92 years old, respectively. But both these cases are hardly believable either; he would be in fantastic form if he was one of these ages! It remains only one possible reply. Are you able to find it?

I imagine it would be really exciting when one is able to say “Today my age is a palindrome in three consecutive bases”. The first time is when one turns 454 years in base 6, 343 years in base 7, and 262 years in base 8. I suspect that none of you already had this experience and that most of you, me included, still need to be a little patient; it will happen on the occasion of our 178th birthday in the usual decimal system. You can easily check that the next two times this will happen is on the occasions of our 300th and 373rd birthdays.

Are you so greedy to desire an age which is a palindrome in all bases from 2 to 10?! Well, maybe it will never happen, not even if you are immortal. Indeed, just when I was about to propose the definition of “Kreher numbers of order n ” as numbers greater than 1 which are palindromes in n consecutive bases, I discovered a chat on “Mathematics Stack Exchange” where they even question their existence for $n \geq 4$... Who is content with the least, is the richest of all!