Group-supermagic labeling of Cartesian products of two odd cycles

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Abstract. A Γ-supermagic labeling of a graph $G = (V, E)$ with $|E| = k$ is a bijection from E to an Abelian group Γ of order k such that the sum of labels of all incident edges of every vertex $x \in V$ is equal to the same element $\mu \in \Gamma$. An existence of a Γ-supermagic labeling of Cartesian product of two cycles, $C_n \square C_m$ for every $n, m \geq 3$ by Z_{2mn} was proved recently. In this paper we present a labeling method for all Abelian groups of order $2mn$ where m, n are odd and greater than one.

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