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On tight 6-cycle and tight 9-cycle decompositions of complete 3-uniform hypergraphs minus a 1-factor

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Abstract. The complete 3-uniform hypergraph of order v, denoted by $K_v^{(3)}$, has a set V of size v as its vertex set and the set of all 3-element subsets of V as its edge set. If $v \equiv 0 \pmod 3$, then the edge set of $K_v^{(3)}$ contains a collection I of $v/3$ vertex-disjoint edges, called a 1-factor. Let $K_v^{(3)} - I$ denote any hypergraph isomorphic to the one obtained by removing the edge set of a 1-factor from that of $K_v^{(3)}$. For $m > 3$, a 3-uniform tight m-cycle, denoted TC_m, is any hypergraph isomorphic to the one with vertex set $\mathbb{Z}_m$ and edge set $\{\{i, i+1, i+2\} : i \in \mathbb{Z}_m\}$. Necessary and sufficient conditions for the existence of TC_6- and TC_9-decompositions of $K_v^{(3)}$ have previously been found. We show that there exists a TC_6-decomposition of $K_v^{(3)} - I$ if and only if $v \equiv 0$, 3, or 6 $\pmod{12}$ and that there exists a TC_9-decomposition of $K_v^{(3)} - I$ if and only if $v \equiv 0 \pmod 3$ and $v \neq 6$. Results similar to ours were obtained independently and simultaneously by Keszler and Tuza (*Spectrum of 3-uniform 6- and 9-cycle systems over* $K_v^{(3)} - I$, arXiv:2212.11058.)

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