



Optimal ordered covering arrays via an exact algorithm

IRENE HIESS, LUDWIG KAMPEL^{*}LKAMPEL@SBA-RESEARCH.ORG AND
DIMITRIS E. SIMOS

Abstract. Ordered covering arrays (orCAs) are combinatorial objects that recently raised interest as they can be used for the generation of covering codes in Niederreiter-Rosenbloom-Tsfasman (NRT) spaces. We present an exact algorithm for the generation of orCAs and use this algorithm to determine sizes of some optimal orCAs. With the calculated orCAs and a theorem from Castoldi et al., *J. Combin. Des.*, 2023, we can phrase inequalities giving upper bounds on the size of covering codes in NRT spaces. We discuss these in the context of bounds that can be derived based on results existing in the literature.

References

- [1] A.G. Castoldi and E.L. Monte Carmelo, The covering problem in Rosenbloom-Tsfasman spaces, *Electron. J. Combin.*, **22** (2015), paper 3.30, <https://doi.org/10.37236/4974>.
- [2] A.G. Castoldi, E.L. Monte Carmelo and R. da Silva, Partial sums of binomials, intersecting numbers, and the excess bound in Rosenbloom-Tsfasman space, *Comput. Appl. Math.*, **38** (2019), article 55, <https://doi.org/10.1007/s40314-019-0828-2>.
- [3] A.G. Castoldi, E.L. Monte Carmelo, L. Moura, D. Panario and B. Stevens, Bounds on covering codes in RT spaces using ordered covering arrays, *Lecture Notes in Comput. Sci.*, **11545** (2019), 100–111.
- [4] A.G. Castoldi, E.L. Monte Carmelo, L. Moura, D. Panario and B. Stevens, Ordered covering arrays and upper bounds on covering codes, *J. Combin. Des.*, **31** (2023), 304–329.
- [5] G. Cohen, I. Honkala, S. Litsyn and A. Lobstein, *Covering codes*, Elsevier, 1997.
- [6] C.J. Colbourn, Combinatorial aspects of covering arrays, *Matematiche (Catania)*, **59**(1,2) (2004), 125–172.
- [7] C.J. Colbourn and J.H. Dinitz, *Handbook of combinatorial designs*, Taylor & Francis Group, CRC Press, 2007.

- [8] C.J. Colbourn, G. Kéri, P.P. Rivas Soriano and J.-C. Schlage-Puchta, Covering and radius-covering arrays: Constructions and classification, *Discrete Appl. Math.*, **158** (2010), 1158–1180.
- [9] M. Gebser, B. Kaufmann and T. Schaub, Conflict-driven answer set solving: From theory to practice, *Artificial Intelligence*, **187–188** (2012), 52–89.
- [10] A.S. Hedayat, N.J.A. Sloane and J. Stufken, *Orthogonal arrays: theory and applications*, Springer, 2012.
- [11] B. Hnich, S.D. Prestwich, E. Selensky and B.M. Smith, Constraint models for the covering test problem, *Constraints*, **11** (2006), 199–219.
- [12] I. Izquierdo-Marquez and J. Torres-Jimenez, New optimal covering arrays using an orderly algorithm, *Discrete Math. Algorithms Appl.*, **10** (2018), 1850011.
- [13] L. Kampel and D.E. Simos, A survey on the state of the art of complexity problems for covering arrays, *Theoret. Comput. Sci.*, **800** (2019), 107–124.
- [14] L. Kampel, I. Hiess, I.S. Kotsireas and D.E. Simos, Balanced covering arrays: A classification of covering arrays and packing arrays via exact methods, *J. Combin. Des.*, **31** (2023), 205–261.
- [15] T. Krikorian, *Combinatorial constructions of ordered orthogonal arrays and ordered covering arrays*, M.S. thesis, Department of Mathematics, Ryerson University, 2011.
- [16] D.R. Kuhn, R.N. Kacker and Y. Lei, *Introduction to Combinatorial Testing*, Taylor & Francis Group, 2013.
- [17] K.M. Lawrence, A combinatorial characterization of (t, m, s) -nets in base b , *J. Combin. Des.*, **4** (1996), 275–293.
- [18] W.J. Martin and D.R. Stinson, A Generalized Rao Bound for Ordered Orthogonal Arrays and (t, m, s) -Nets, *Canad. Math. Bull.*, **42** (1999), 359–370.
- [19] MATRIS, Webpage: Optimal Ordered Covering Arrays, <https://srd.sba-research.org/data/orcas/> Accessed: 2024-03-15.
- [20] G.L. Mullen and W.Ch. Schmid, An equivalence between (t, m, s) -nets and strongly orthogonal hypercubes, *J. Combin. Theory Ser. A*, **76** (1996), 164–174.
- [21] T. Nanba, T. Tsuchiya and T. Kikuno, Using satisfiability solving for pairwise testing in the presence of constraints, *IEICE TRANSACTIONS on Fundamentals of Electronics, Communications and Computer Sciences*, **E95-A**(9) (2012), 1501–1505.
- [22] D. Panario, M. Saaltink, B. Stevens and D. Wevrick, A general construction of ordered orthogonal arrays using LFSRs, *IEEE Trans. Inform. Theory*, **65** (2019), 4316–4326.

- [23] S. Raaphorst, L. Moura and B. Stevens, Variable strength covering arrays, *J. Combin. Des.*, **26** (2018), 417–438.