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On some extensions of mutually orthogonal graph squares

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Abstract. A decomposition $\mathcal{G}$ =$\{G_0, G_1, \ldots, G_{n-1}\}$ of a graph $K_{n,n}$ is a partition of the edge set of $K_{n,n}$ into edge disjoint subgraphs $G_0, \ldots, G_{n-1}$ (called *pages*) in which all G_i, $i \in \{0, 1, \ldots, n-1\}$ are isomorphic to a specific graph G, and $\mathcal{G}$ is called a decomposition of $K_{n,n}$ by G. A family of decompositions $\{\mathcal{G}_0, \mathcal{G}_1, \ldots, \mathcal{G}_{k-1}\}$ of a complete bipartite graph $K_{n,n}$ is a collection of k mutually orthogonal graph squares ($MOGS$) if $\mathcal{G}_i$ and $\mathcal{G}_j$ are orthogonal for all $i, j \in \{0, 1, \ldots, k-1\}$ and $i \neq j$. For any subgraph G of $K_{n,n}$ with n edges, $N(n, G)$ represents the greatest number k in the largest feasible set $\{\mathcal{G}_0, \mathcal{G}_1, \ldots, \mathcal{G}_{k-1}\}$ of ($MOGS$) of $K_{n,n}$ by G. In this paper, we present several novel results pertaining to mutually orthogonal graph squares of the complete bipartite graph. Our focus lies in exploring starter functions of ($MOGS$), as well as utilizing the technique of Kronecker product of ($MOGS$) to construct new mutually orthogonal sets of disjoint union stars.

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