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On some extensions of mutually orthogonal graph squares

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Abstract. A decomposition $\mathcal{G}=\{G_0, G_1, \ldots, G_{n-1}\}$ of a graph $K_{n,n}$ is a partition of the edge set of $K_{n,n}$ into edge disjoint subgraphs $G_0, \ldots, G_{n-1}$ (called *pages*) in which all G_i, $i \in \{0, 1, \ldots, n-1\}$ are isomorphic to a specific graph G, and $\mathcal{G}$ is called a decomposition of $K_{n,n}$ by G. A family of decompositions $\{\mathcal{G}_0, \mathcal{G}_1, \ldots, \mathcal{G}_{k-1}\}$ of a complete bipartite graph $K_{n,n}$ is a collection of k mutually orthogonal graph squares (MOGS) if $\mathcal{G}_i$ and $\mathcal{G}_j$ are orthogonal for all $i, j \in \{0, 1, \ldots, k-1\}$ and $i \neq j$. For any subgraph G of $K_{n,n}$ with n edges, $N(n, G)$ represents the greatest number k in the largest feasible set $\{\mathcal{G}_0, \mathcal{G}_1, \ldots, \mathcal{G}_{k-1}\}$ of MOGS of $K_{n,n}$ by G. In this paper, we present several novel results pertaining to mutually orthogonal graph squares of the complete bipartite graph. Our focus lies in exploring starter functions of MOGS, as well as utilizing the technique of Kronecker product of MOGS to construct new mutually orthogonal sets of disjoint union stars.

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