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On some extensions of mutually orthogonal graph squares

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Abstract. A decomposition $\mathcal{G}=\{G_0, G_1, \ldots, G_{n-1}\}$ of a graph $K_{n,n}$ is a partition of the edge set of $K_{n,n}$ into edge disjoint subgraphs $G_0, \ldots, G_{n-1}$ (called *pages*) in which all G_i, $i \in \{0, 1, \ldots, n-1\}$ are isomorphic to a specific graph G, and $\mathcal{G}$ is called a decomposition of $K_{n,n}$ by G. A family of decompositions $\{\mathcal{G}_0, \mathcal{G}_1, \ldots, \mathcal{G}_{k-1}\}$ of a complete bipartite graph $K_{n,n}$ is a collection of k mutually orthogonal graph squares ($MOGS$) if $\mathcal{G}_i$ and $\mathcal{G}_j$ are orthogonal for all $i, j \in \{0, 1, \ldots, k-1\}$ and $i \neq j$. For any subgraph G of $K_{n,n}$ with n edges, $N(n, G)$ represents the greatest number k in the largest feasible set $\{\mathcal{G}_0, \mathcal{G}_1, \ldots, \mathcal{G}_{k-1}\}$ of ($MOGS$) of $K_{n,n}$ by G. In this paper, we present several novel results pertaining to mutually orthogonal graph squares of the complete bipartite graph. Our focus lies in exploring starter functions of ($MOGS$), as well as utilizing the technique of Kronecker product of ($MOGS$) to construct new mutually orthogonal sets of disjoint union stars.

References

[1] B. Alspach, K. Heinrich and G. Liu, *Orthogonal factorizations of graphs*, in Contemporary Design Theory", J.H. Dinitz and D.R. Stinson, eds. Wiley, New York, (1992), 13–40.

[2] G. Appa, D. Magos and I. Mourtos, An LP-based proof for the non-existence of a pair of orthogonal Latin squares of order 6, *Oper. Res. Lett.*, **32** (2004) 336–344.

[3] C.J. Colbourn and J.H. Dinitz, Mutually orthogonal Latin squares: a brief survey of constructions, *J. Statist. Plann. Inference*, **95** (2001), 9–48.

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[4] C.J. Colbourn and J.H. Dinitz (eds.), *Handbook of Combinatorial Designs, Second Edition*, Chapman & Hall/CRC, London, Boca Raton, FL, 2007.

[5] P.J. Dukes and C.M. van Bommel, Mutually orthogonal Latin squares with large holes, *J. Statist. Plann. Inference*, **159** (2015), 81–89.

[6] A. El-Mesady, Y.S. Hamed and H. Shabana, On the decomposition of circulant graphs using algorithmic approaches. *Alex. Eng. J.*, **61** (2022), 8263–8275.

[7] A. El-Mesady, Y.S. Hamed and K.M. Abualnaja, A novel application on mutually orthogonal graph squares and graphorthogonal arrays, *AIMS Math.*, **7** (2022), 7349–7373.

[8] R. El-Shanawany, *Orthogonal Double Covers of Complete Bipartite Graphs*, Ph.D. Thesis, Universitat Rostock, 2002.

[9] R. El-Shanawany, On mutually orthogonal graph-path squares, *Open J. Discrete Math.*, **6** (2016), 7–12.

[10] R. El-Shanawany, On mutually orthogonal disjoint copies of graph square, *Note Math.*, **36** (2016), 89–98.

[11] R. El-Shanawany and A. El-Mesady, Mutually orthogonal graph squares for disjoint union of stars, *Ars Combin.*, **149** (2020), 83–91.

[12] R. El-Shanawany, S. Nada, A. Elrokh and E. Sallam, A novel construction of mutually orthogonal three disjoint union of certain trees squares, *Appl. Math. Inf. Sci.* **17** (2033), 7–11.

[13] E. Gilbert, F. MacWilliams and N. Sloane, Codes which detect deception, *Bell System Tech. J.* **53** (1974), 405–424.

[14] H.F. MacNeish, Euler squares, *Ann. of Math.*, **23** (1922), 221–227.

[15] R. Sampathkumar and S. Srinivasan, Mutually orthogonal graph squares, *J. Combin. Des.* **17** (2009), 369–373.

[16] I.M. Wanless and B.S. Webb, The existence of Latin squares without orthogonal mates, *Des. Codes Cryptogr.* **40** (2006), 131–135.

[17] X. Wang, N. Jin, M. Cao and Y. Wen, Construction of authentication codes based on orthogonal array and Latin square *Comput. Appl. Math.*, **41** (2022), Paper no. 161, 22pp.