Mixed hexagon systems

Robert Gardner* and Simeon Ignace

Dedication. In memory of the late, great Dean Hoffman of Auburn University. He was the Master's thesis advisor and a mentor for the first author. He once commented to the first author that mixed triple systems were "a cute idea!"

Abstract. A decomposition of the complete mixed graph on v vertices into a partial orientation of the 6-cycle with two edges and four arcs is a *mixed hexagon system* of order v. Necessary and sufficient conditions for the existence of a mixed hexagon system of order v are given for each of the 25 such partial orientations of the 6-cycle.

References

[1] B. Alspach and H. Gavlas, Cycle decompositions of K_n and $K_n - I$, *J. Combin. Theory Ser. B*, **81** (2001), 77-99.

[2] M. Buratti, Rotational k-cycle systems of order $v < 3k$; another proof of the existence of odd cycle systems, *J. Combin. Des.*, **11**(6) (2003), 433–441.

[3] R. Gardner, Triple systems from mixed graphs, *Bull. Inst. Combin. Appl.*, **27** (1999), 95–100.

[4] F. Harary and E. Palmer, Enumeration of mixed graphs, *Proc. Amer. Math. Soc.*, **17**(3) (1966), 682–687.

[5] S. Hung and N. Mendelsohn, Directed triple systems, *J. Combin. Theory Ser. A*, **14** (1973), 310–318.

[6] C.C. Lindner and C.A. Rodger, *Design Theory* Second Edition, Discrete Mathematics and Its Applications Series, CRC Press (2008).

---

*Corresponding author: gardnerr@etsu.edu

[7] N. Mendelsohn, A natural generalization of Steiner triple systems, *Computers in Number Theory*, eds. A.O. Atkin and B. Birch, Academic Press, London, 1971.

[8] M. Meszka and A. Rosa, Cyclic and rotational six-cycle systems, *Bull. Inst. Combin. Appl.*, **87** (2019), 41–46.

[9] M. Meszka and A. Rosa, Six-cycle systems, *Math. Slovaca*, **71**(3) (2021), 543–564.

[10] A. Rosa, On cyclic decompositions of the complete graph into $(4m+2)$-gons, *Mat.-Fyz. Časopis SAV*, **16** (1966), 349–352.

[11] A. Rosa and C. Huang, Another class of balanced graph designs: Balanced circuit designs, *Discrete Math.*, **12** (1975), 269–293.

[12] M. Šajna, Cycle decompositions III: Complete graphs and fixed length cycles, *J. Combin. Des.*, **10**(1) (2002), 27–78.