



# Peakless Motzkin paths of bounded height

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**Abstract.** There was recent interest in Motzkin paths without peaks (peak: up-step followed immediately by down-step); additional results about this interesting family are worked out. The new results are the enumeration of such paths that live in a strip  $[0..ℓ]$ , and as consequence the asymptotics of the average height, which is given by  $2 \cdot 5^{-1/4} \sqrt{\pi n}$ . Methods include the kernel method and singularity analysis of generating functions.

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