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On cyclic matroids and their applications

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Abstract. A matroid is a combinatorial structure that captures and generalizes the algebraic concept of linear independence under a broader and more abstract framework. Matroid theory is closely related to many other topics in discrete mathematics, such as graphs, matrices, codes, and projective geometries. In this work, we define *cyclic matroids* as matroids over a ground set of size n whose automorphism group contains an n-cycle. We study the properties of such matroids, with special focus on the minimum size of their basis sets. For this, we broadly employ two different approaches: the multiple basis exchange property and an orbit-stabilizer method developed by analyzing the action of the cyclic group of order n on the set of bases. We further present some applications of our theory to algebra and geometry, illustrating connections to cyclic projective planes, cyclic codes, and k-normal elements.

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