



# Some imbalanced hypergraph Zarankiewicz numbers

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**Abstract.** The Zarankiewicz number  $z(m, n; a, b)$  is the maximum number of edges  $|E|$  among all bipartite graphs  $G = (X \dot{\cup} Y, E)$  satisfying  $|X| = m$ ,  $|Y| = n$ , and that no  $a$  vertices of  $X$  and  $b$  vertices of  $Y$  induce a copy of the complete bipartite graph  $K_{a,b}$  as a subgraph of  $G$ . For  $m \geq (a-1)\binom{n}{b}$ , Čulík proved  $z(m, n; a, b) = (a-1)\binom{n}{b} + (b-1)m$ . We extend this result to hypergraphs of a similarly imbalanced variety. Our key will be a construction employing Baranyai's theorem on hyperclique matching decompositions.

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