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Some imbalanced hypergraph Zarankiewicz numbers

Eion Mulrenin and Brendan Nagle*

Abstract. The Zarankiewicz number $z(m,n;a,b)$ is the maximum number of edges $|E|$ among all bipartite graphs $G=(X\dot{\cup}Y,E)$ satisfying $|X|=m$, $|Y|=n$, and that no a vertices of X and b vertices of Y induce a copy of the complete bipartite graph $K_{a,b}$ as a subgraph of G. For $m \geq (a-1)\binom{n}{b}$, Čulík proved $z(m,n;a,b)=(a-1)\binom{n}{b}+(b-1)m$. We extend this result to hypergraphs of a similarly imbalanced variety. Our key will be a construction employing Baranyai's theorem on hyperclique matching decompositions.

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*Corresponding author: bnagle@usf.edu

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