



A combinatorial proof for the Fibonacci dying rabbits problem

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Abstract. We consider the generalized Fibonacci counting problem with rabbits that become fertile at age f and die at age d , with $1 \leq f \leq d$, and d finite or infinite. We provide a combinatorial proof of a recurrence relation for the number of rabbits at each generation. The proof is based exclusively on a counting argument and uses only elementary mathematics. The recurrence relation generalizes both the original Fibonacci sequence, and several other Fibonacci-related sequences, such as the Padovan sequence, and the Tribonacci, Tetranacci, and alike sequences.

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