



# A combinatorial proof for the Fibonacci dying rabbits problem

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**Abstract.** We consider the generalized Fibonacci counting problem with rabbits that become fertile at age  $f$  and die at age  $d$ , with  $1 \leq f \leq d$ , and  $d$  finite or infinite. We provide a combinatorial proof of a recurrence relation for the number of rabbits at each generation. The proof is based exclusively on a counting argument and uses only elementary mathematics. The recurrence relation generalizes both the original Fibonacci sequence and several other Fibonacci-related sequences, such as the Padovan sequence and the Tribonacci, Tetranacci, and alike sequences.

## References

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