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On S-magic labeling of graph products

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Abstract. Let $G = (V, E)$ be a graph and let S be a set of positive integers with $|S| = |V|$. The graph G is said to be S-magic if there exists a bijection $l\colon V \to S$ such that the weight of any vertex u, which is defined as the sum of labels on vertices adjacent to u, is a constant k for all $u \in V$. The constant k is called an S-magic constant. The set of all S-magic constants of G for different labeling sets is denoted by $M(G)$. In this paper, we study S-magic labelings of various graph products like lexicographic products of graphs with C_4, direct products of graphs with C_4, Cartesian products of graphs with C_4, corona products of graphs, and joins of graphs. We find various classes of the above graph products that do not admit an S-magic labeling. We also give S-magic labeling conditions for several classes of the above graph products that do admit S-magic labelings, and we determine $M(G)$ for these classes of graphs.

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