



Book thickness of path and cycle powers

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Abstract. Book embeddings with minimum page number are given for powers of path and cycle graphs. We find minimum book embeddings with *star forest pages* using at most two additional pages.

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