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An algorithmic approach in constructing infinitely many even size graphs with local antimagic chromatic number 3

Dedicated to Prof. S. Arumugam on the occasion of his 80th birthday

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Abstract. An edge labeling of a connected graph $G = (V, E)$ is said to be local antimagic if it is a bijection $f\colon E \to \{1, \dots, |E|\}$ such that for any pair of adjacent vertices x and y, $f^+(x) \neq f^+(y)$, where the induced vertex label $f^+(x) = \sum f(e)$, with e ranging over all the edges incident to x. The local antimagic chromatic number of G, denoted by $\chi_{la}(G)$, is the minimum number of distinct induced vertex labels over all local antimagic labelings of G. In this paper, we first introduce an algorithmic approach to construct a family of infinitely many even size non-regular tripartite graphs with $t \geq 1$ component(s) in which every component is of odd order $p \geq 9$ and of size $q = n(p+1)$ for $n \geq 2$. We show that every graph in this family has local antimagic chromatic number 3. We then allow the m-th component to have order $p_m \geq 9$ and size $n_m(p_m + 1)$ for $n_m \geq 2$, $1 \leq m \leq t$. We prove that every such graph with all components having same order and size also has local antimagic chromatic number 3. Lastly, we construct another family of infinitely many graphs such that different components may have different order and size all of which having local antimagic chromatic number 3. Consequently, many other families of (possibly disconnected) graphs with local antimagic chromatic number 3 are also constructed.

Key words and phrases: Local antimagic chromatic number, disconnected, non-regular, tripartite.
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