



# The extreme case of 3-PGDD's with block size 4 and 2 groups

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**Abstract.** We introduce a generalization of group divisible 3-designs with 2 groups, 3-GDDs with 2 associate classes, into partial group divisible 3-designs, 3-PGDDs with 3 associate classes. A *partial group divisible 3-design*, 3-PGDD( $n, 2, k; \lambda, \mu_{21}, \mu_{12}$ ), is a pair  $(G_1 \cup G_2, \mathcal{B})$  where  $G_1$  and  $G_2$  are called *groups* of size  $n$ , and  $\mathcal{B}$  is a collection of  $k$ -subsets, called *blocks*, of  $G_1 \cup G_2$  such that every 3-subset of  $G_i$  occurs in  $\lambda$  blocks in  $\mathcal{B}$  and every  $i$  elements of  $G_1$  and  $j$  elements of  $G_2$  occur together in  $\mu_{ij}$  blocks in  $\mathcal{B}$  for  $i \neq j \in \{1, 2\}$ . Our study focuses on the case  $k = 4$ . We study obvious necessary conditions for the existence of a 3-PGDD( $n, 2, 4; \lambda, \mu_{21}, 0$ ), and prove that they are sufficient whenever  $(n - 2)\lambda = n\mu_{21}$ . Our construction technique relies on the existence of 3- $(n, 4, \lambda)$  designs and some large sets of triple systems.

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