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Further results on the metric dimension of circulant graphs with 2 or 3 generators

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Abstract. Let $G = (V, E)$ be a connected graph. A subset $W = \{v_1, \ldots, v_k\} \subseteq V$ is called a resolving set of G if $r(u \,|\, W) \neq r(v \,|\, W)$ for every $u, v \in V$, $u \neq v$, where $r(u \,|\, W) = \big(d(u, v_1), \ldots, d(u, v_k)\big)$ and $d(u, v_i)$ is the graph distance between u and v_i, $1 \leq i \leq k$. The cardinality of a smallest possible resolving set of G is called the metric dimension of G, which is denoted by $\beta(G)$. Extensive research has been conducted on the metric dimension of various graph classes, including circulant graphs, which exhibit interesting properties. Let n, m and $a_1, a_2, \ldots, a_m$ be positive integers such that $1 \leq a_1 < a_2 < \cdots < a_m \leq \frac{n}{2}$. A circulant graph $C_n(a_1, a_2, \ldots, a_m)$ consists of vertices $v_0, v_1, \ldots, v_{n-1}$ and edges $v_i v_{i+a_j}$ for $i = 0, \ldots, n-1$ and $j = 1, \ldots, m$ where the indices are modulo n. In this paper, we sharpen an upper bound of $\beta\big(C_n(1, 3)\big)$ found by Javaid, Azhar, and Salman [9], and we disprove an exact value of $\beta\big(C_n(1, 2, 4)\big)$ given by Imran and Bokhary [7], both by providing a smaller resolving set, respectively.

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