



On generalized Turán problems with bounded matching number

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Abstract. Given a graph H and a family of graphs $\mathcal{F}$, the generalized Turán number $\text{ex}(n, H, \mathcal{F})$ is the maximum number of copies of H in an n -vertex graphs that do not contain any member of $\mathcal{F}$ as a subgraph. Recently there has been interest in studying the case $\mathcal{F} = \{F, M_{s+1}\}$ for arbitrary F and $H = K_r$. We extend these investigations to the case H is arbitrary as well.

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