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Generating all Eulerian trails avoiding forbidden transitions

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Abstract.

Let G be a multigraph without loops, and H a graph possibly with loops. We say that G is an H-colored multigraph whenever there exists a function $c : E(G) \to V(H)$. A walk (respectively, path, trail) $W = (v_0, e_0, v_1, e_1, \ldots, e_{k-1}, v_k)$ in G is an H-walk (respectively, H-path, H-trail) if and only if $(c(e_0), c(e_1), \ldots, c(e_{k-2}), c(e_{k-1}))$ is a walk in H. W is a *closed H-walk* (respectively, closed H-trail) if and only if W is an H-walk (respectively, H-trail) such that $v_0 = v_k$, and $c(e_{k-1})c(e_0) \in E(H)$. Notice that W is a properly colored trail whenever H is a complete graph without loops, in particular when H is K_2 we have that W is a properly 2-colored trail.

In 1995 Pevzner defined the order transformations, which allow us to generate all properly colored Eulerian trails in a 2-colored multigraph, starting with a fixed one. This result has been fundamental for the study of DNA physical mapping.

In this paper we give sufficient conditions on an H-edge coloring of G to generate all Eulerian H-trails of G, starting with a fixed one. As a consequence of the main result we obtain a polynomial time algorithm to do it.

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