



# Flattened Catalan words

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**Abstract.** In this work, we define flattened Catalan words as Catalan words whose runs of weak ascents have leading terms that appear in weakly increasing order. We provide generating functions, formulas, and asymptotic expressions for the number of flattened Catalan words based on the number of runs of ascents (descents), runs of weak ascents (descents),  $\ell$ -valleys, valleys, symmetric valleys,  $\ell$ -peaks, peaks, and symmetric peaks.

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