BULLETIN OF THE ICA
Article in press.

The metamorphoses of maximum packings of $2K_n$ with triples to maximum packings of $2K_n$ with 4-cycles for $n \equiv 2, 3, 6, 7,$ or 10 $(mod\ 12)$

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Abstract. Gionfriddo and Lindner detailed the idea of the metamorphosis of 2-fold triple systems with no repeated triples into 2-fold 4-cycle systems of all orders where each system exists (2003). Couch expanded on this idea for $n \equiv 5, 8,$ or 11 $(mod\ 12)$, proving that when $n \equiv 11$ $(mod\ 12)$, a maximum packing of $2K_n$ with triples has a metamorphosis into a maximum packing of $2K_n$ with 4-cycles, with the leave of a double edge being preserved throughout the metamorphosis, and for $n \equiv 5$ or 8 $(mod\ 12)$, a maximum packing of $2K_n$ with triples has a metamorphosis into a a 2-fold 4-cycle system of order n, except for when $n = 5$ or 8, when no such metamorphosis is possible (2016). In this paper, we prove that all remaining orders, i.e. $n \equiv 2, 3, 6, 7,$ or 10 $(mod\ 12)$, can be similarly addressed. For $n \equiv 3, 6, 7,$ or 10 $(mod\ 12)$, a 2-fold triple system (moreover, a hinge system) of order n has a metamorphosis to a maximum packing of $2K_n$ with 4-cycles with the leave a double edge, except for $n = 3$, $n = 6$, and $n = 7$, where no such metamorphosis is possible. When $n \equiv 2$ $(mod\ 12)$, a maximum packing of $2K_n$ with triples (and as before, with hinges) has a metamorphosis into a maximum packing of $2K_n$ with 4-cycles, with the leave of a double edge being preserved throughout the metamorphosis.

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Received: 22 May 2024
Accepted: 1 March 2025

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