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On factorisations of complete multigraphs into line graphs of complete graphs

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Abstract. A connection between residuals of biplanes and factorisations of complete multigraphs into isomorphic copies of line graphs of complete graphs is presented. A biplane with blocks of size $n+1$ can be used to construct a factorisation of $4K_{\binom{n}{2}}$ into $n+1$ copies of the line graph of K_n, thus establishing existence of such factorisations for $n \in \{8, 10, 12\}$. Together with the Hall-Connor Theorem, the connection also gives a new proof of the result that, for $n > 3$, there is no factorisation of $2K_{\binom{n}{2}}$ into copies of the line graph of K_n.

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