**BULLETIN OF THE ICA**
Article in press.

# A 2-person game decomposing 2-manifolds

David R. Berman and Lee O. Leonard Jr

*This paper is dedicated to Lee Leonard (1951-2025), my dear friend of more than 60 years. Many years ago, Lee conceived of the crazy idea of playing games on 2-manifolds and worked out much of the analysis. With persistence he eventually convinced me to join the project.*

**Abstract.** Two players play a game by alternately splitting a surface of a compact 2-manifold along a simple closed curve that is not null-homotopic and attaching disks to the resulting boundary; the last player who can move wins. Starting from an orientable surface, the $G$-series is $01\dot{2}\dot{0}$ according to increasing genus. Starting from a nonorientable surface, the $G$-series is $012\dot{4}60\dot{3}$ according to increasing genus. Nim addition determines the $G$-values of the remaining compact 2-manifolds.

## References

[1] E. R. Berlekamp, J. H. Conway, and R. K. Guy, *Winning Ways for Your Mathematical Plays*, Academic Press, 1982.

[2] C. Drutu and M. Lackenby, Webpage: Oxford Part A Topology lecture notes: HT 2018, p. 47, `https://www.maths.ox.ac.uk/system/files/attachments/toplectnotes17.pdf` Accessed: 2024-06-30.

[3] R. K. Guy and C. A. B. Smith, The G-Values of Various Games, Proc. Cambridge Philos. Soc. **52** (1956), 514–526.

[4] J. Greever, *Theory and Examples of Point Set Topology*, Brooks/Cole Publishing Company, 1968.

[5] W. Hurewicz and H. Wallman, *Dimension Theory* (*PMS-4*) *Volume 4*, Princeton University Press, 1941.

[6] W. S. Massey, *Algebraic Topology: An Introduction*, Harcourt, New York, 1967.

[7] E. E. Moise, *Geometric Topology in Dimensions* 2 *and* 3, (*GTM-47*), Springer, 1978.

[8] J. W. Vick, *Homology Theory, An Introduction to Algebraic Topology* (*GTM-145*), Springer, 1994.

Received: 30 June 2024
Accepted: 25 May 2025

David R. Berman
Wilmington, NC
bermand@uncw.edu

Lee O. Leonard Jr
Austin, TX
lleonardjr@icloud.com