Some results on the signless Laplacian permanental polynomial and star degree of balanced trees

AQIB KHAN, PRATIMA PANIGRAHI, SWARUP KUMAR PANDA

Abstract. Let $Q(G)$ be the signless Laplacian matrix of a simple connected graph G with n vertices. Let *per* denotes the permanent of a matrix and I_n be the identity matrix of order n. The polynomial $per(xI_n - Q(G))$ is defined as the signless Laplacian permanental polynomial of G. The multiplicity of root 1 of the polynomial $per(xI_n - Q(G))$ is equal to the star degree $SD(G)$ of G. In this paper, we consider a balanced tree $T(h, d)$ of height h, in which all the non-pendant vertices have degree $d(> 2)$ and all the leaves have the same depth. We prove that, among all possible graphs with the same degree sequence, $T(h, d)$ has the maximum star degree. Further, we show that $T(2, d)$ is determined by its signless Laplacian permanental polynomial. By applying the known fact that the Laplacian permanental polynomial and signless Laplacian permanental polynomial are the same for a bipartite graph G; one gets that $T(2, d)$ is also determined by its Laplacian permanental polynomial.

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Aqib Khan⋆, Pratima Panigrahi, and Swarup Kumar Panda
Department of Mathematics,
Indian Institute of Technology,
Kharagpur-721302, India
aqib@iitkgp.ac.in,pratima@maths.iitkgp.ac.in,spanda@maths.iitkgp.ac.in