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The deleted edge numbers $de(P_m, K_{1,n})$ and $de(K_{1,3}, K_{m,n})$

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Abstract. In the paper [*Australas. J. Combin.* 74 (3) (2019), 476-485], Budden and Dejonge defined the *k-deleted Ramsey number,* where $k \in \mathbb{N}$, and the *deleted edge number* in order to study the destruction of Ramsey property by the removal of edges incident at any fixed vertex. Given simple graphs G and H, the *k-deleted Ramsey number* $D_k(G, H)$ is the least $n \in \mathbb{N}$ such that every 2-edge-colouring of $K_n - E(K_{1,k})$, $k \leq n-1$, contains a copy of G in colour 1 or a copy of H in colour 2. The *deleted edge number* $de(G, H)$ is defined as the least $k \in \mathbb{N}$ such that $D_{k-1}(G, H) < D_k(G, H)$. In this paper, we give bounds to $de(G, H)$, when $(G, H) = (P_m, K_{1,n})$ for certain values of m and n; and when $(G, H) = (K_{1,3}, K_{m,n})$ for all values of m and n, where $n \geq m \geq 2$. Also, we prove that $de(P_m, K_{1,n}) = R(P_m, K_{1,n}) - 1$, when $n \equiv 1(mod\ m-1)$.

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